JAMES RUSE A.H.S. - 2004 - YR 11 MATHS

QUESTION 1: 15 MARKS (START A NEW PAGE)

a) Write in simplest form: 2

$$\frac{x+8}{x^{-1}+8^{-1}}$$

b) Simplify, leaving your answer in index form: 3

$$\frac{25^{-1}}{\left(5^{n}\right)^{3}\times(125)^{1-n}}$$

c) Solve the following inequation and graph the solution on a number line: 3

$$|4-5x|< 11$$

d) Find the exact length of the diagonal of a rectangle if it makes an angle of 30° with the longer side, which is 48 *cm* in length. 2

e) A, B, C are the vertices of a triangle with coordinates (-2, -3), (4, 3) and (k, 5) respectively. Find:

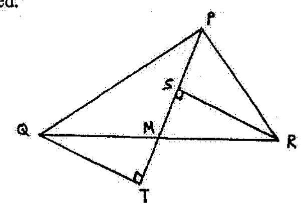
(i) the coordinates of the midpoint of AB, 1
(ii) the gradient of the interval AB, 1
(iii) the equation of the perpendicular bisector of AB, 1
(iv) hence, or otherwise, find k if ΔABC is isosceles having base AB. 2

QUESTION 2: 15 MARKS (START A NEW PAGE)

a) Differentiate with respect to x:

(i) $4\sin(3-x)$ 1
(ii) $\sqrt{1+x^2}$ 2
(iii) $3xe^{2x}$ 2
(iv) $\log_e(\log_e x)$ 1

b) In ΔPQR, M is the midpoint of QR, QT and RS are perpendicular lines drawn from Q and R to the line PM produced.



(i) Prove ΔQMT $\equiv$ ΔRMS, giving reasons. 3
(ii) What type of quadrilateral is QSRT, giving reasons. 2

c) For what value(s) of k will the following equation have equal roots? 2

$$x^2-(k+4)x+7+k=0$$

d) Evaluate the following limit: $\lim\limits_{x\to 1}\frac{x^2-x}{1-x}$ 2

QUESTION 3: 15 MARKS (START A NEW PAGE)

a) Determine the set of values of x for which $\frac{dy}{dx}>0$ in the function: $y=-x^3+13x^2-35x+5$ 3

b) (i) Solve for x: $1-2\cos x=0, \quad 0\le x\le 360°$. 2

(ii) Sketch: $y=1-2\cos x, \quad 0\le x\le 360°$. 2

c) (i) Find the equation of the tangent to the curve $y=(2x-1)^2$ at the point A where $x=-1$. 3

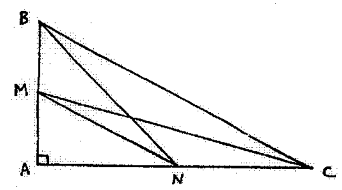
(ii) The tangent intersects the y axis at B. Find the area of the triangle OAB, where O is the origin. 2

d) The minute and hour hands of a clock are respectively 9cm and 6cm long. Find the distance between the ends of the hands when the time is twenty past two. (Give your answer to 1 decimal place). 3

QUESTION 4: 15 MARKS (START A NEW PAGE)

a) In this figure, $\angle BAC = 90°$, M and N are the midpoints of BA and AC respectively.
Let AM = a units and AN = b units.

Prove: $BN^2+CM^2=5\times MN^2$ 4



b) A and B are the points (-3, -1) and (7, 3) respectively. The point P (x, y) moves so that $\angle APB = 90°$.

(i) Derive the equation of the locus of P. 2
(ii) Find the centre and radius of the above circle. 3

c) (i) A wheel has a radius of 15 cm. Through what angle (in radians) does a point on the wheel rotate if the wheel rolls 150 cm along a horizontal path? 1

(ii) A point on the wheel was initially in contact with the ground. In rolling 150 cm show that the point on the wheel has completed more than $1\frac{1}{2}$ revolutions. 2

(iii) What is the height above the ground of the point described in part (ii) after the wheel rolls 150 cm. (Give answer to 3 decimal places). 3

End of Exam Paper

Solutions to Yr 11 – HALF YEARLY 2004

(2U.) MATHEMATICS LEVEL

4 × 15 marks each = 60 marks total

Question 1 : 15 marks — Marks

a) $\frac{x+8}{x^{-1}+8^{-1}} = \frac{x+8}{\frac{1}{x}+\frac{1}{8}} = \frac{x+8}{\frac{8+1}{8x}} = 8x$ ① ① — 2

b) $\frac{25^{-1}}{(5^n)^3 \times (125)^{1-n}} = \frac{5^{-2}}{5^{3n} \times 5^{3-3n}} = 5^{-2-\cancel{3n}-3+\cancel{3n}}$ ① ①

$= 5^{-5}$ ① — 3.

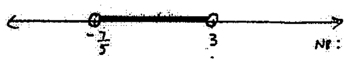
c) $|4-5x| < 11$

$4-5x < 11$ OR $4-5x > -11$

$-5x < 7$ $-5x > -15$

$x > -\frac{7}{5}$ ① $x < 3$ ① — 3.

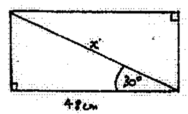
OR $-\frac{7}{5} < x < 3$



①

NB: $-\frac{1}{2}$ if closed circles
1 if graphed solution is correct from incorrect answers above.
0 if arrows are pointing wrong way.

d)



$\frac{x}{48} = \sec 30°$ ① or equivalent first step.

$x = 48 \times \sec 30°$

$= 48 \times \frac{2}{\sqrt{3}}$

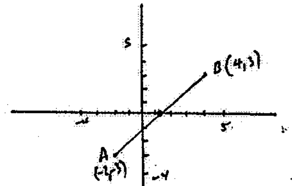
$= \frac{96}{\sqrt{3}}$ ✱ — 2

$= \frac{96\sqrt{3}}{3}$

$= 32\sqrt{3}$ ①

✱ $-\frac{1}{2}$ if not rationalised

e)



(i) Midpoint AB $= \left(\frac{-2+4}{2}, \frac{-3+3}{2}\right) = (1, 0)$ — 1

($\frac{1}{2}$) for x coord.
($\frac{1}{2}$) for y coord.

(ii) $m_{AB} = \frac{3+3}{4+2} = \frac{6}{6} = 1$ — 1

(iii) $\perp m_{AB} = -1$ ($\frac{1}{2}$) — 1

$\therefore y - 0 = -1(x-1)$

$\underline{y = -x+1}$ ($\frac{1}{2}$) for any equivalent equation

(iv) The $\perp$ bisector will pass through $C(k, 5)$, so satisfies the equation $y = -x+1$. ① — 2

$\therefore 5 = -k+1$

$\underline{k = -4}$ ①

Question 2 (15 marks)

a) (i) $\frac{d}{dx}\left(4\sin(3-x)\right) = -1 \times 4\cos(3-x)$

$= -4\cos(3-x)$ 1

$-\frac{1}{2}$ if no negative

(ii) $\frac{d}{dx}\sqrt{1+x^2} = \frac{d}{dx}(1+x^2)^{\frac{1}{2}}$

① ①

$= \frac{1}{2}(1+x^2)^{-\frac{1}{2}} \times 2x$ 2

$= x(1+x^2)^{-\frac{1}{2}}$

(iii) $\frac{d}{dx}\left(3x\, e^{2x}\right) = 3x \cdot 2e^{2x} + e^{2x} \cdot 3$ ① ①

$= 6x e^{2x} + 3e^{2x}$ 2

OR

$= 3e^{2x}(2x+1)$

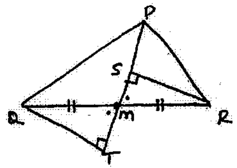
(iv) $\frac{d}{dx}\left(\log_e(\log_e x)\right) = \frac{\frac{1}{x}}{\log_e x} = \frac{1}{x\log_e x}$ 1

①

↑

$-\frac{1}{2}$ if left as this

b)



(i) In Δ's QMT & SRM :

RM = QM (given) ½

$\angle RSM = \angle QTM = 90°$ (given) ½

$\angle SMR = \angle QMT$ (vertically opposite) 1 * 3.

$\therefore \Delta QMT \equiv \Delta SRM$ (AAS) 1

* Alternatively:

SR ∥ QT (since the alternate angles RSM and MTQ are equal then the lines are parallel)

$\therefore \angle SRM = \angle TQM$ (alternate angles equal; SR ∥ QT)

(ii) There are several Alternatives possible.

Solution 1 :

① SR ∥ QT (since the alternate angles $\angle RSM$ and $\angle QTM$ are equal, then the lines SR and QT are parallel)

① SR = QT (corresponding sides of congruent Δ's) 2

$\therefore$ QSRT is a parm (one pair of opposite sides equal and parallel) ½

Solution 2 :

SM = MT (corresponding sides of congruent Δ's)

QM = MT (given)

$\therefore$ QSRT is a parm (diagonals ST and QR of a parm bisect each other)

c) For equal roots, $\Delta = 0$

ie $(k+4)^2 - 4(7+k) = 0$ ①

$k^2 + 8k + 16 - 28 - 4k = 0$

$k^2 + 4k - 12 = 0$ ½ 2

$(k+6)(k-2) = 0$

$k = -6$ or 2 ½

d) $\lim_{x \to 1} \frac{x(x-1)}{1-x}$

$= \lim_{x \to 1} -x$ ① 2

$= -1$ ①

Question 3 (15 marks)

a) $y = -x^3 + 13x^2 - 35x + 5$

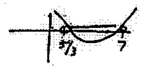
$\frac{dy}{dx} = -3x^2 + 26x - 35$ (1)

$-3x^2 + 26x - 35 > 0$

$3x^2 - 26x + 35 < 0$

$(3x-5)(x-7) < 0$ (1)

$\frac{5}{3} < x < 7$ (1)



OR $x > \frac{5}{3}$ and $x < 7$ 3.

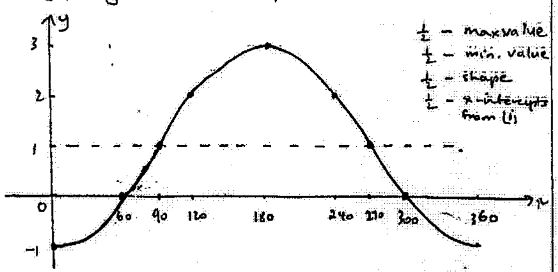
b) (i) $1 - 2\cos x = 0$, $0 \le x \le 360°$

$\cos x = \frac{1}{2}$ (1)

$x = 60°, \ 300°$

($\frac{1}{2}$) ($\frac{1}{2}$) 2

(ii) $y = 1 - 2\cos x$, $0 \le x \le 360°$



$\frac{1}{2}$ - max value

$\frac{1}{2}$ - min. value

$\frac{1}{2}$ - shape

$\frac{1}{2}$ - x-intercepts from (i) 2

c)(i) $y = (2x-1)^2$

$\frac{dy}{dx} = 2(2x-1)\cdot 2 = 4(2x-1)$ ($\frac{1}{2}$)

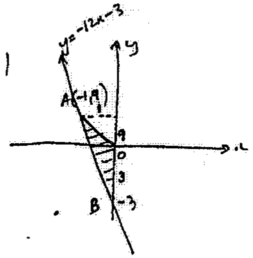
at $x = -1$, $\frac{dy}{dx} = 4(2x-1-1) = -12$ ($\frac{1}{2}$)

at $x = -1$, $y = (2\times -1 - 1)^2 = 9$ (1)

$\therefore$ Eqn of tangent: $y - 9 = -12(x+1)$ (1)

$y = -12x - 3$ 3.

(ii)

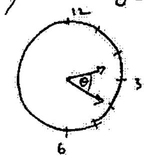


(1)

Area $\triangle OAB = \frac{1}{2} \times 12 \times 1 =$

$= \underline{\underline{6 \text{ units}^2}}$ (1) 2

d) Need angle between the hands at 2:20

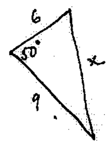


$\frac{360}{12} = 30°$ for each 5 minutes.

$\frac{4}{12} \times 30° = 10°$.

$\therefore \ \theta = 30 + 30 - 10$

$\underline{\underline{\theta = 50°}}$ (1) 3.



$x^2 = 6^2 + 9^2 - 2.6.9.\cos 50°$ * (1)

$= 47.57891815$

$x \doteqdot 6.89774877.4$ (1)

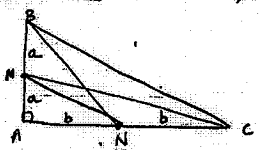
$\underline{x = 6.9 \ (1dp)}$

$-\frac{1}{2}$ if not correct to 1 dp

* If angle of 50° is incorrect, max marks possible is (2)/3.

Question 4 (15 marks)

a)



$MN^2 = a^2 + b^2$

In ΔAMN: $a^2 + b^2 = MN^2$ (1)

In ΔBAN: $(2a)^2 + b^2 = BN^2$

$4a^2 + b^2 = BN^2$ (1)

In ΔCAM: $a^2 + (2b)^2 = CM^2$

$a^2 + 4b^2 = CM^2$ (1) — 4

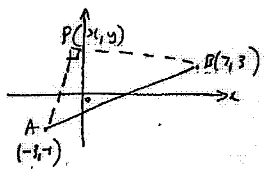
$\therefore BN^2 + CM^2 = 4a^2 + b^2 + 4b^2 + a^2$ (1/2)

$= 5a^2 + 5b^2$

$= 5(a^2 + b^2)$ (1/2)

$= 5 \times MN^2$

b)



(i) Condition required: $m_{PA} \times m_{PB} = -1$

$\frac{y+1}{x+3} \times \frac{y-3}{x-7} = -1$ (1)

* Any of these are possible for (1) mark.

$\frac{y^2 - 2y - 3}{x^2 - 4x - 21} = -1$

$y^2 - 2y - 3 = -x^2 + 4x + 21$

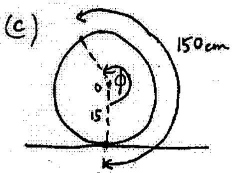
$y^2 - 2y + 1 + x^2 - 4x + 4 = 21 + 3 + 1 + 4$

$(y-1)^2 + (x-2)^2 = 29$ — 2

(ii) $(x-2)^2 + (y-1)^2 = 29$ (1) — 3

Centre of circle is $(2,1)$ (1), radius is $\sqrt{29}$ (1)

(c)



(i) $l = r\theta$ — 1

$150 = 15 \times \theta$

$\theta = 10$ rads

($-\frac{1}{2}$) if wrong units used

$\therefore$ Wheel rotates angle of $\underline{10 \text{ rads}}$

(ii) 2π rads $=$ 1 revolution

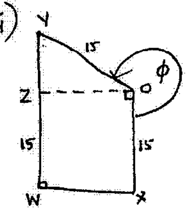
1 rad $= \frac{1}{2\pi}$ revolutions

$\therefore$ 10 rads $= \frac{10}{2\pi}$ revolution (1) — 2

$\therefore$ 10 rads $= 1.59$ revolutions

Thus, the point has made 1.59 revolutions which is more than 1.5 revolutions. (1)

(iii)



$\phi = (10 - 2\pi)$ rads (1)

$(\doteqdot 3.716814693)$

$\angle YOZ = 2\pi - \phi - \frac{\pi}{2}$

$= 2\pi - (10 - 2\pi) - \frac{\pi}{2}$

$= 4\pi - 10 - \frac{\pi}{2}$

$\therefore \angle YOZ = \left(\frac{7\pi}{2} - 10\right)$ rads (1) — 3

$\frac{YZ}{15} = \sin \angle YOZ$

$\therefore YZ = 15 \sin\left(\frac{7\pi}{2} - 10\right)$

$= 12.58607294$ (1/2)

$\therefore YW = 15 + 12.58607294$

$= \underline{27.586}$ (3dp) (1/2)

$\therefore$ The height of point above ground is $\underline{27.586 \text{ cm}}$