JAMES RUSE – TRIAL HSC – 3U – 2011

Question 1:

(a)(i) Find the derivative of $x^2 \cos x$. 2

(ii) Evaluate $\int_1^6 \frac{x}{x^2+4}\,dx$. 2

(b)(i) Sketch $y=|x+1|$. 2

(ii) Hence or otherwise solve $|x+1|=3x$. 1

(c) If $f(x)=2\sin^{-1}(3x)$, find

(i) the domain and range of $f(x)$, 2

(ii) $f\left(\frac{1}{6}\right)$, 1

(iii) $f'\left(\frac{1}{6}\right)$. 2

QUESTION 2: (START A NEW PAGE)

(a) P(-7,3), Q(9,15) and B(14,0) are three points and A divides the interval PQ in the ratio 3:1. Prove that PQ is perpendicular to AB. 3

(b) By using the substitution $u^2=x+1$ evaluate $\int_0^3 \frac{x+2}{\sqrt{x+1}}\,dx$. 3

(c) Water flows from a hole in the base of a cylindrical vessel at a rate given by 6

$$\frac{dh}{dt}=-k\sqrt{h}$$

where k is a constant and h mm is the depth of water at time t minutes.
If the depth of water falls from 2500mm to 900mm in 5 minutes, find how much longer it will take to empty the vessel.

QUESTION 3: (START A NEW PAGE)

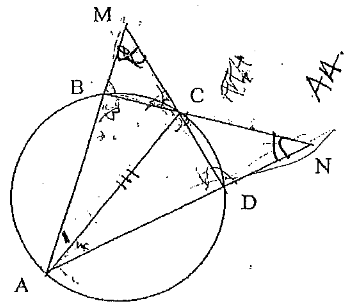
(a) Find the value of the constant term in the expansion of $\left(3x+\frac{2}{\sqrt{x}}\right)^6$. 3

(b) Three boys (Adam, Bruce, Chris) and three girls (Debra, Emma, Fay) form a single queue at random in front of the school canteen window. Find the probability that:

(i) the first two to be served are Emma and Adam in that order, 2

(ii) a boy is at each end of the queue, 1

(iii) no two girls stand next to each other. 1



(c) In the figure ABM, DCM, BCN and ADN are straight lines and $\angle AMD = \angle BNA$.

(i) Copy the diagram onto your answer sheet and prove that $\angle ABC = \angle ADC$. 3

(ii) Hence prove that AC is a diameter. 2

QUESTION 4: (START A NEW PAGE)

(a)(i) Given that $\sin^2 A+\cos^2 A=1$, prove that $\tan^2 A=\sec^2 A-1$. 2

(ii) Sketch the curve $y=4\tan^{-1}x$ clearly showing its range. 2

(iii) Find the volume of the solid formed when the area bounded by the curve $y=4\tan^{-1}x$, the y-axis and the line $y=\pi$ is rotated one revolution about the y-axis. 2

(b)(i) An object has velocity $v\ ms^{-1}$ and acceleration $\ddot{x}\ ms^{-2}$ at position $x\ m$ from the origin, show that $\frac{d}{dx}\left(\frac{1}{2}v^2\right)=\ddot{x}$. 2

(ii) The acceleration (in ms^{-2}) of an object is given by $\ddot{x}=2x^3+4x$.

(α) If the object is initially 2 m to the right of the origin traveling with velocity 6 ms^{-1}, find an expression for v^2 (the square of its velocity) in terms of x. 2

(β) What is the minimum speed of the object? (Give a reason for your answer) 2

QUESTION 5: (START A NEW PAGE)

(a) The curves $y = e^{-2x}$ and $y = 3x + 1$ meet on the y-axis. Find the size of the acute angle between these curves at the point where they meet. 3

(b)(i) Sketch the function $y = f(x)$ where $f(x) = (x-1)^2 - 4$ clearly showing all intercepts with the co-ordinate axes. (Use the same scale on both axes) 2

(ii) What is the largest positive domain of f for which $f(x)$ has an inverse $f^{-1}(x)$? 1

(iii) Sketch the graph of $y = f^{-1}(x)$ on the same axes as (i). 1

(c) In tennis a player is allowed a maximum of two serves when attempting to win a point. If the first serve is not legal it is called a fault and the server is allowed a second serve. If the second serve is also illegal then it is called a double fault and the server loses the point. The probability that Pat Smash's first serve will be legal is 0.4. If Pat Smash needs to make a second serve then the probability that it will be legal is 0.7.

(i) Find the probability that Pat Smash will serve a double fault when trying to win a point. 2

(ii) If Pat Smash attempts to win six points, what is the probability that he will serve at least two double faults? (Give answer correct to 2 decimal places) 3

QUESTION 6: (START A NEW PAGE)

(a) A spherical bubble is expanding so that its volume is increasing at $10\ cm^3 s^{-1}$. Find the rate of increase of its radius when the surface area is $500\ cm^2$. 3

(Volume $= \frac{4}{3}\pi r^3$, Surface area $= 4\pi r^2$)

(b) Prove by Mathematical Induction that: 4

$2(1!) + 5(2!) + 10(3!) + \ldots + (n^2+1)n! = n(n+1)!$ for positive integers $n \geq 1$.

(c) If $y = \dfrac{\log_e x}{x}$ find $\dfrac{dy}{dx}$ and hence show that $\displaystyle\int_e^{e^2} \frac{1-\log_e x}{x \log_e x}\, dx = \log_e 2 - 1$. 5

QUESTION 7: (START A NEW PAGE)

(i) By considering the expansion of $\sin(X+Y) - \sin(X-Y)$ prove that 3

$\sin A - \sin B = 2\cos\left(\dfrac{A+B}{2}\right)\sin\left(\dfrac{A-B}{2}\right)$.

(ii) Also given that $\cos A - \cos B = 2\sin\left(\dfrac{A+B}{2}\right)\sin\left(\dfrac{B-A}{2}\right)$ prove that 2

$\dfrac{\sin A - \sin B}{\cos A - \cos B} = -\cot\left(\dfrac{A+B}{2}\right)$.

(iii) Prove that the position of a projectile t seconds after projection from ground level with initial horizontal and vertical velocity components of $V\cos\alpha$ and $V\sin\alpha$ respectively is given by $x = Vt\cos\alpha$ and $y = -\frac{1}{2}gt^2 + Vt\sin\alpha$. (Assume that there is no air resistance) 2

(iv) Two objects P and Q are projected from the same ground position at the same time with initial speed $V\ ms^{-1}$ at angles α and β respectively (β > α).

(α) If at time t seconds the line joining P and Q makes an acute angle θ with the horizontal prove that $\tan\theta = \left|\dfrac{\sin\beta - \sin\alpha}{\cos\beta - \cos\alpha}\right|$. 3

(β) Hence show that $\theta = \frac{1}{2}(\pi - \alpha - \beta)$. 2

THIS IS THE END OF THE EXAMINATION PAPER

Worked Solutions

James Ruse 3U Trial 2011

Question one

(a) (i) $\frac{d}{dx} x^2\cos x$

$= 2x \times \cos x + x^2 \times -\sin x$

$= 2x\cos x - x^2\sin x$ ✓

(ii) $\int_1^6 \frac{x}{x^2+4}\,dx$

$= \frac{1}{2}\int_1^6 \frac{2x}{x^2+4}\,dx$

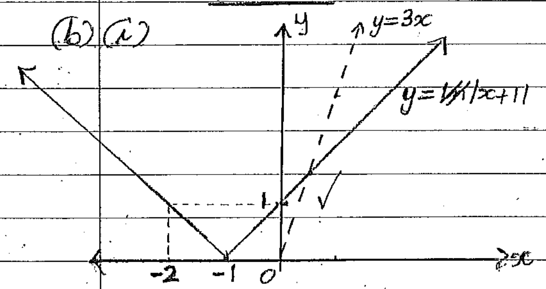
$= \frac{1}{2}\left[\ln(x^2+4)\right]_1^6$

$= \frac{1}{2}\left[\ln 40 - \ln 5\right]$ ✓

$= \frac{1}{2}\ln 8 = \frac{1}{2}\ln 2^3$ ✓

$= \frac{3}{2}\ln 2$ ✓

(b) (i)



(ii) $|x+1| = 3x$ ✓

$x+1 = 3x$ (from graph)

$1 = 2x$

$x = \frac{1}{2}$ ✓

(c) $f(x) = 2\sin^{-1}(3x)$

(i) $-1 \le 3x \le 1$ ✓

$-\frac{1}{3} \le x \le \frac{1}{3}$ (Domain)

$-\pi \le f(x) \le \pi$ ✓ (Range)

(ii) $f(\frac{1}{6}) = 2\sin^{-1}(\frac{1}{2})$

$= 2 \times \frac{\pi}{6}$

$= \frac{\pi}{3}$ ✓

(iii) $f'(x) = \frac{6}{\sqrt{1-9x^2}}$ ✓

$f'(\frac{1}{6}) = 6/\sqrt{3/4}$

$= 12/\sqrt{3}$

$= 4\sqrt{3}$ ✓

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Question Two

(a) $A(x_1, y_1)$ $P(-7,3)$ $Q(9,15)$ [Ratio 3 : 1]

$x_1 = \frac{3\times 9 + 1\times -7}{3+1}$ $\quad y_1 = \frac{3\times 15 + 1\times 3}{3+1}$

$= 20/4$ $\quad = 48/4$ ✓

$= 5$ ✓ $\quad = 12$

$A(5,12)$ $B(14,0)$

$AB_{gradient} = \frac{0-12}{14-5}$ $\quad PQ_{gradient} = \frac{15-3}{9+7}$

$= -12/9$ ✓ $\quad = 12/16$ ✓

$= -4/3$ $\quad = 3/4$

$AB_{gradient} \times PQ_{gradient} = -4/3 \times 3/4 = -1$ ✓

$\therefore AB \perp PQ$ ✓

(b) $\int_0^3 \frac{x+2}{\sqrt{x+1}}\,dx$ $\quad$ Let $u^2 = x+1 \Rightarrow u^2+1 = x+2$

$2u\,du = dx$ ✓

$= \int_1^2 \frac{u^2+1}{u} \times 2u\,du$ ✓ $\quad u = (x+1)^{1/2}$

$= 2\int_1^2 (u^2+1)\,du$ $\quad x=0,\ u=1$

$= 2\left[\frac{1}{3}u^3 + u\right]_1^2$ $\quad x=3,\ u=2$

$= 2\left[(\frac{1}{3}\times 2^3 + 2) - (\frac{1}{3}\times 1^3 + 1)\right]$ ✓

$= 2\left[\frac{14}{3} - \frac{4}{3}\right]$

$= 20/3$ ✓

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(c) $\frac{dh}{dt} = -k\sqrt{h}$

$\int dt = -\frac{1}{k}\int \frac{1}{\sqrt{h}}\,dh$

$t + C = -\frac{2}{k}\sqrt{h}$ $\quad (t=0,\ h=2500)$ ✓

$0 + C = -\frac{2}{k} \times \sqrt{2500}$

$C = -\frac{2}{k} \times 50 = -100/k$

$t = 100/k + -2/k\sqrt{h}$ ✓

$= 2/k\,(50 - \sqrt{h})$ $\quad (t=5,\ h=900)$

$5 = 2/k\,(50 - \sqrt{900})$

$= 2/k \times 20 = 40/k$ ✓

$k = 8$

$t = \frac{2}{8}\cdot\frac{1}{4}(50 - \sqrt{h})$; when vessel is empty, $h=0 \Rightarrow t = \frac{50}{4} = 12\frac{1}{2}$

It will take another $7\frac{1}{2}$ minutes for the vessel to empty.

$(12\frac{1}{2} - 5 = 7\frac{1}{2})$ ✓

Question Three

(a) $(3x+2x^{-1/2})^6$

$T_{k+1} = {}^6C_k \times (3x)^{6-k} \times (2x^{-1/2})^k$

$= {}^6C_k \times 3^{6-k} \times x^{6-k} \times 2^k \times x^{-k/2}$

$= {}^6C_k \times 3^{6-k} \times 2^k \times x^{6-\frac{3}{2}k}$

In the constant term $x^{6-\frac{3}{2}k} = 1 = x^0$

$6-\frac{3}{2}k=0$

$k=4$

∴ The constant term is ${}^6C_4 \times 3^2 \times 2^4$

$= \underline{2160}$

(b)(i) $[E\ A] \times {}^4P_4$

$1 \times 1 \times {}^4P_4 = 24$ (Number of arrangements satisfying condition)

$6! = 720$ (Number of arrangements with no restriction)

Probability $= \frac{24}{720} = \frac{1}{30}$

(ii) $3 \times 2 \times {}^4P_4$

$= 6 \times 24 = 144$ (Arrangements satisfying condition)

Probability $= \frac{144}{720} = \frac{1}{5}$

(iii) case 1: BGBGBG $= 3\times3\times2\times2\times1\times1 = 36$

case 2: GBGBGB $= 3\times3\times2\times2\times1\times1 = 36$

Probability $= \frac{36+36}{720} = \frac{1}{10}$

(c) $\angle MCB = \angle NCD$ (vert. opp. ∠s)

$\angle BMC = \angle DNC$ (given)

∴ $\triangle BMC \mathbin{|||} \triangle DNC$ (AAA)

$\angle MBC = \angle NDC$ (corr. ∠s of ||| Δs)

Let $\angle MBC = \theta$

$\angle MBC + \angle ABC = 180°$ (∠s on straight line)

$\angle ABC = 180° - \angle MBC = 180° - \theta$

$\angle NDC + \angle ADC = 180°$ (∠s on straight line)

$\angle ADC = 180° - \angle NDC = 180° - \theta$ ⇒ ∴ $\angle ABC = \angle ADC$

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(ii) ABCD is a cyclic quad.

$\angle ABC + \angle ADC = 180°$ (opp. ∠s of cyclic quad are supplementary)

But $\angle ABC = \angle ADC$ (proved in part (i))

∴ $\angle ABC = \angle ADC = 90°$

∴ AC is a diameter ($\angle ABC$, $\angle ADC$ are right ∠s in a semi-circle.)

Question Four

(a)(i) $\sec^2 A - 1$

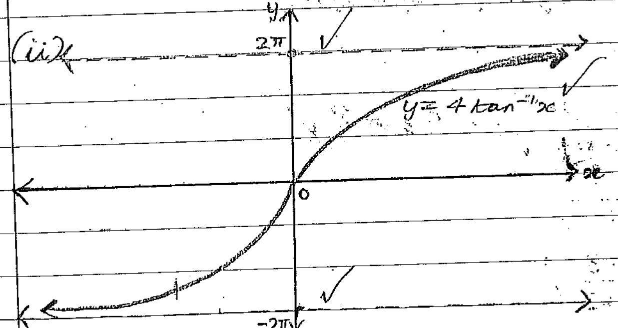
$= \frac{1}{\cos^2 A} - 1$

$= \frac{1}{\cos^2 A} - \frac{\cos^2 A}{\cos^2 A}$

$= \frac{1-\cos^2 A}{\cos^2 A}$

$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$

(ii)



(iii) $y = 4\tan^{-1}x$

$x = \tan\left(\frac{y}{4}\right)$

$x^2 = \tan^2\left(\frac{y}{4}\right)$

$= \sec^2\left(\frac{y}{4}\right) - 1$

$V = \pi \int_a^b x^2\,dy$

$= \pi \int_0^{\pi} \left(\sec^2\left(\frac{y}{4}\right) - 1\right) dy$

$= \pi \left[4\tan\left(\frac{y}{4}\right) - y\right]_0^{\pi}$

$= \pi\left[\left(4\tan\frac{\pi}{4} - \pi\right) - (0-0)\right]$

$= \pi[4-\pi]$

$= 4\pi - \pi^2$

The volume of the solid formed is $4\pi - \pi^2$ units^3

(b)(i) $\frac{d}{dx}\left(\frac{1}{2}v^2\right)$

$= \frac{d}{dv}\left(\frac{1}{2}v^2\right) \times \frac{dv}{dx}$

$= \frac{1}{2} \times 2 \times v \times \frac{dv}{dx}$

$= v\frac{dv}{dx}$

$= \frac{dx}{dt} \times \frac{dv}{dx} = \frac{dv}{dt} = \ddot{x}$

(d)

(ii) $\ddot{x} = 2x^3+4x$

$\frac{d}{dx}(\frac{1}{2}v^2) = 2x^3+4x$

$\int dv^2 = 4\int(x^3+2x)dx$ ✓

$v^2+c = x^4+4x^2 \quad (x=2, v=6)$

$6^2+c = 2^4+4\times2^2$

$36+c = 32 \Rightarrow c=-4$

$v^2 = x^4+4x^2+4$

$= (x^2+2)^2$ ✓

(β) $|v^2_{min}|$ occurs when $x=0$

$|v^2_{min}| = 2^2$ ✓

$|v_{min}| = 2$ ✓

Minimum speed of the object is $2ms^{-1}$ ✓

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Question Five

(a) Intersect at $(0,1)$ ✓ [By inspection]

$y=e^{-2x}$

$\frac{dy}{dx} = -2e^{-2x}$

at $x=0$, $m_1 = -2e^0$, $m_2 = 3$ ✓

$= -2$ ✓

$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$= \left|\frac{-2-3}{1+-2\times3}\right|$

$= 1$ ✓

$\theta = \frac{\pi}{4}$ or $45°$ ✓ The acute angle between the curves at the point where ~~the~~ they meet is $45°$. ✓

(b) (i) y-intercept: $x=0$

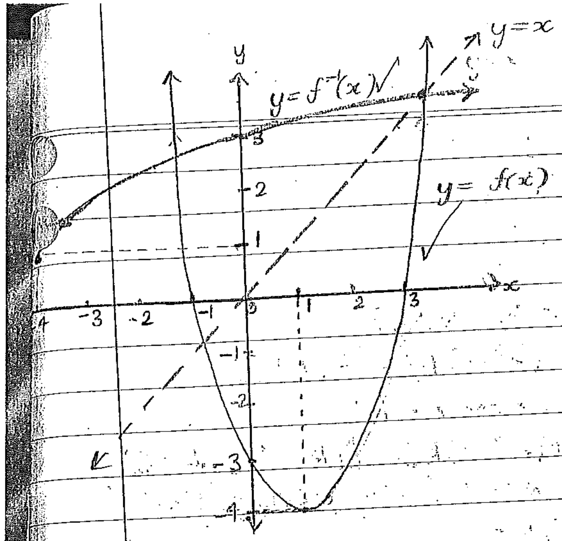
$f(0) = 1^2-4 = -3$ ✓

x-intercepts: $f(x)=0 \Rightarrow (x-1)^2 = 4$

$x-1 = \pm2$ ✓

$x = -1, 3$

Vertex of parabola $= (1,-4)$ ✓



(ii) $x \geq 1$ ✓

(iii) See left.

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(c) (i) $(1-0.4)\times(1-0.7)$

$= 0.6\times0.3$

$= 0.18$ ✓

The probability that Pat Smash will serve a double fault is 0.18.

(ii) 0 double faults $\Rightarrow (1-0.18)^6$

1 double fault $\Rightarrow {}^6C_1\times(0.18)^1\times(1-0.18)^5$

Probability of at least two double faults is $1-[(1-0.18)^6 + {}^6C_1\times(0.18)^1\times(1-0.18)^5]$

$= 0.2955...$

$= 0.30$ (2 d.p.) ✓

Question Six

(a) $\frac{dr}{dt} = \frac{dr}{dV}\times\frac{dV}{dt}$

$= \frac{dr}{dV}\times10$ $[\frac{dV}{dt} = 10$ ✓ is given$]$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2 = 500$ (Surface area)

$\frac{dr}{dV} = \frac{1}{500}$ ✓

$\frac{dr}{dt} = \frac{1}{500}\times10$

$= \frac{1}{50}$ ✓

The radius is increasing by $0.02 cm s^{-1}$ ✓ when surface area is $500cm^2$.

Prove true for $n=1$

(b) LHS $2(1!)$ $= 2\times 1$ $= 2$ ✓ RHS $1(1+1)!$ $= 1\times 2!$ $= 2$ ✓

LHS = RHS, true for $n=1$.

Assume true for $n=k$ $(k\in\mathbb{N}, k\geq 1)$ ✓

$2(1!)+5(2!)+10(3!)+\ldots+(k^2+1)k! = k(k+1)!$

Use assumption to prove true for $n=k+1$.

LHS $2(1!)+5(2!)+10(3!)+\ldots+(k^2+1)k! + [(k+1)^2+1](k+1)!$

$= k(k+1)! + (k^2+2k+2)(k+1)!$ [Using assumption]

~~$= k(k+1)! + (k+1)(k+2)(k+$~~

$= (k+1)!(k^2+3k+2)$ ✓

$= (k+1)!(k+2)(k+1)$

$= (k+1)(k+2)!$ ✓

$=$ RHS

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True for $n=k+1$, if it is true for $n=k$. Since it is true for $n=1$, it must be true for $n=2$, ∴ true for $n=3$ and so on ∴ it is true for all positive integers $n\geq 1$. ✓

(c) $y = x^{-1}\ln x = \frac{\ln x}{x}$

$\frac{dy}{dx} = -x^{-2}\ln x + x^{-1}\times\frac{1}{x}$

$= -x^{-2}\ln x + x^{-2}$

$= \frac{1-\ln x}{x^2}$ ✓

$\int_e^{e^2}\frac{1-\ln x}{x\ln x}dx$

$= \int_e^{e^2}\left[\frac{1}{x\ln x}-\frac{1}{x}\right]dx$ $\int_e^{e^2}\frac{1-\ln x}{x^2}\times\frac{x}{\ln x}dx$

$= \cancel{\left[\ln(\ln x)-\ln x\right]_e^{e^2}}$ $= \int_e^{e^2}\left[\left(\frac{1-\ln x}{x^2}\right)/\left(\frac{\ln x}{x}\right)\right]dx$ ✓

$= \left[\ln\left(\frac{\ln x}{x}\right)\right]_e^{e^2}$ [Using result of differentiation] ✓

$= \ln\left(\frac{\ln e^2}{e^2}\right) - \ln\left(\frac{\ln e}{e}\right)$

$= \ln(\ln e^2) - \ln e^2 - \ln(\ln e) + \ln e$

$= \ln(2\ln e) - 2\ln e - \ln(1) + \cancel{\ln e}\ 1$

$= \ln 2 - 2 - 0 + 1$ ✓

$= \ln 2 - 1$

Question Seven.

(i) $\sin(X+Y)-\sin(X-Y)$

$= (\sin X\cos Y+\cos X\sin Y)-(\sin X\cos Y-\sin Y\cos X)$

$= \cancel{2\sin Y\cos X}\ 2\cos X\sin Y$

sub $X=\frac{A+B}{2}$, $Y=\frac{A-B}{2}$ ✓

$\sin\left[\frac{A+B}{2}+\frac{A-B}{2}\right]-\sin\left[\frac{A+B}{2}-\frac{A-B}{2}\right]$ ✓

$= \sin A-\sin B$

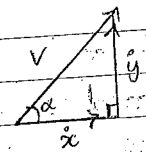
$= 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ [as required] ✓

(ii) $\frac{\sin A-\sin B}{\cos A-\cos B}$

$= \frac{2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}{2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)}$ ✓

$= \frac{2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}{-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}$

$= \frac{-\cos\left(\frac{A+B}{2}\right)}{\sin\left(\frac{A+B}{2}\right)} = -\cot\left(\frac{A+B}{2}\right)$ ✓

(iii) 

$\dot{y}(t=0)=V\sin\alpha$

$\dot{x}(t=0)=V\cos\alpha$

$\ddot{x}=0$

$\dot{x}=C_1$ $(t=0\ \dot{x}=V\cancel{\sin\alpha}\cos\alpha)$

$= \cancel{V\sin\alpha}\ V\cos\alpha$

$x = Vt\cancel{\sin\alpha}\cos\alpha + C_2$ $(t=0, x=0)$

$= \cancel{Vt\sin\alpha}\ Vt\cos\alpha$ ✓

$\ddot{y}=-g$

$\dot{y}=-gt+C_3$ $(t=0\ \dot{y}=V\sin\alpha)$

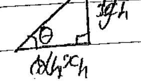
$= V\sin\alpha-gt$ ✓

$y = Vt\sin\alpha-\frac{1}{2}gt^2+C_4$ $(t=0, y=0)$

$= Vt\sin\alpha-\frac{1}{2}gt^2$

(iv) (α) $x_p = Vt\cos\alpha$ $x_q = Vt\cos\beta$

$y_p = Vt\sin\alpha-\frac{1}{2}gt^2$ $y_q = Vt\cos\beta-\frac{1}{2}gt^2$



$\tan\theta = \frac{y_h}{x_h}$ where y_h is the difference between the y-coordinates of P & Q at time t and x_h is the difference between x-coordinates of P & Q at time t.

So, $x_h = |x_p - x_Q|$

$= |Vt\cos\alpha - Vt\cos\beta|$

$= Vt\,|\cos\alpha - \cos\beta| = Vt\,|\cos\beta - \cos\alpha|$

$y_h = |y_p - y_Q|$

$= |(Vt\sin\alpha - \frac{1}{2}gt^2) - (Vt\cos\beta - \frac{1}{2}gt^2)|$

$= |Vt\sin\alpha - Vt\sin\beta|$

$= Vt\,|\sin\alpha - \sin\beta| = Vt\,|\sin\beta - \sin\alpha|$

$\tan\theta = \frac{y_h}{x_h}$

$= \frac{Vt\,|\sin\beta - \sin\alpha|}{Vt\,|\cos\beta - \cos\alpha|}$

$= \frac{|\sin\beta - \sin\alpha|}{|\cos\beta - \cos\alpha|}$ (as required)

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(β) $\tan\theta = \left|-\cot\left(\frac{\alpha+\beta}{2}\right)\right|$ [Using result in (ii)]

$= \cot\left(\frac{\alpha+\beta}{2}\right)$ [$0 < \alpha+\beta < \pi$]

$\frac{\sin\theta}{\cos\theta} = \frac{\cos(\frac{1}{2}(\alpha+\beta))}{\sin(\frac{1}{2}(\alpha+\beta))}$

$\sin(\frac{\pi}{2} - x)$	$\cos(\frac{\pi}{2} - x)$
$= \sin\frac{\pi}{2}\cos x - \cos\frac{\pi}{2}\sin x$	$= \cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x$
$= \cos x$	$= \sin x$

$\sin\left(\frac{\pi}{2} - \frac{1}{2}(\alpha+\beta)\right) = \cos\left(\frac{1}{2}(\alpha+\beta)\right)$

$\cos\left(\frac{\pi}{2} - \frac{1}{2}(\alpha+\beta)\right) = \sin\left(\frac{1}{2}(\alpha+\beta)\right)$

$\therefore \theta = \frac{\pi}{2} - \frac{1}{2}(\alpha+\beta)$ [Equating LHS & RHS]

$= \frac{1}{2}(\pi - \alpha - \beta)$ [as required]

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