Question 1 RUSE X2 TRIAL 2007 **Marks**

(a) (i) Find the real numbers a, b and c such that $\frac{1}{x(4+x^2)}=\frac{a}{x}+\frac{bx+c}{4+x^2}$. **2**

(ii) Find $\int\frac{1}{x(4+x^2)}dx$. **2**

(b) Evaluate $\int_0^2 x\sqrt{2-x}\,dx$, leaving your answer in exact form. **3**

(c) Find the zeros of $P(x)=x^4-5x^3+7x^2+3x-10$ over the complex field if $2-i$ is a zero. **3**

(d) Given that $I_{2n+1}=\int_0^1 x^{2n+1}e^{x^2}\,dx$ where n is a positive integer, show that **2**

$$I_{2n+1}=\frac{1}{2}e-nI_{2n-1}.$$

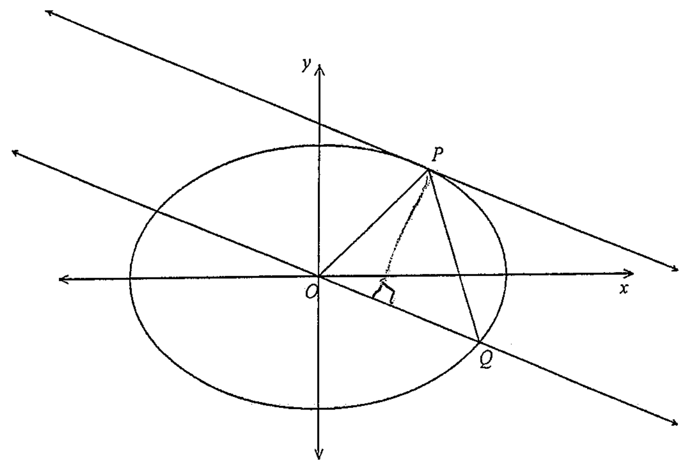
Hence, or otherwise, evaluate $\int_0^1 x^5e^{x^2}\,dx$. **3**

Question 2 (15 Marks) [START A NEW PAGE] **Marks**

(a)(i) Given that $z^2=-3-4i$, find z. **4**

(ii) Solve the equation $x^2-3x+3+i=0$ over the complex field. **3**

(b)



In the diagram above, $P(a\cos\theta, b\sin\theta)$ is a point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where P lies in the first quadrant.

A straight line through the origin parallel to the tangent at P meets the ellipse at the point Q, where P and Q both lie on the same side of the y-axis.

(i) Prove that the equation of the line OQ is $xb\cos\theta+ya\sin\theta=0$. **2**

(ii) Find the coordinates of the point Q given that Q lies in the fourth quadrant. **3**

(iii) Prove that the area of ΔOPQ is independent of the position of P. **3**

Question 3 (15 Marks) [START A NEW PAGE] **Marks**

(a) A particle is projected from the origin with a speed V and an angle of elevation α on level ground. **3**

A vertical wall of "unlimited" height is a distance d from the origin, and the plane of the wall is perpendicular to the plane of the particle's trajectory.

If $d < \dfrac{V^2}{g}$, show that the particle will strike the wall before it hits the ground provided that $\beta < \alpha < \dfrac{\pi}{2} - \beta$ where $\beta = \dfrac{1}{2}\sin^{-1}\left[\dfrac{gd}{V^2}\right]$.

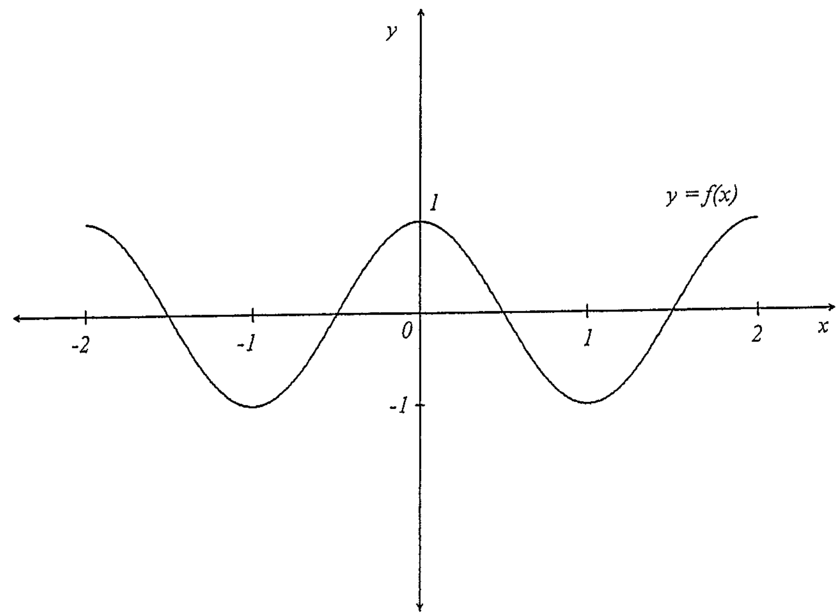
You may assume that the range on the horizontal plane from the point of projection is $\dfrac{V^2 \sin 2\alpha}{g}$.

(b) Express $z = \dfrac{\sqrt{2}}{1-i}$ in the modulus-argument form and hence find z^5 in the form of $x + yi$. **4**

Question 3 continues on page 4

Question 3 cont'd **Marks**

(c)



The diagram shows the graph of the continuous function $y = f(x)$. Critical points occur at $x = -2, -1, 0, 1, 2$.

On the sheets provided draw separate sketches of the graphs of the following :

(i) $y = |f(x)|$ **1**

(ii) $y = \dfrac{1}{f(x)}$ **2**

(iii) $y = \sqrt{f(x)}$ **2**

(iv) $y = xf(x)$ **3**

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Question 4 (15 Marks) [START A NEW PAGE]

Marks

(a) Find $\int \frac{1}{x(\ln x)^2}\,dx$. 2

(b) Five letters are chosen from the letters of the word MOBILITY. These five letters are then placed alongside each other to form a five-letter arrangement. 4

Find the number of different arrangements which are possible.

(c) $P\left(3p, \frac{3}{p}\right)$ and $Q\left(3q, \frac{3}{q}\right)$ are points on different branches of the hyperbola $xy = 9$.

(i) Find the equation of the tangent at P. 2

(ii) Find the point of intersection T, of the tangents at P and Q. 3

(iii) If the chord PQ passes through the point (0, 2), find the locus of T. 3

(iv) Find the restriction on the locus of T. 1

Question 5 (15 Marks) [START A NEW PAGE]

(a) (i) Find the volume generated when the area bounded by $y = \sin x$ and the x-axis, for $0 \le x \le \pi$, is rotated about the x-axis. 3

redo 5

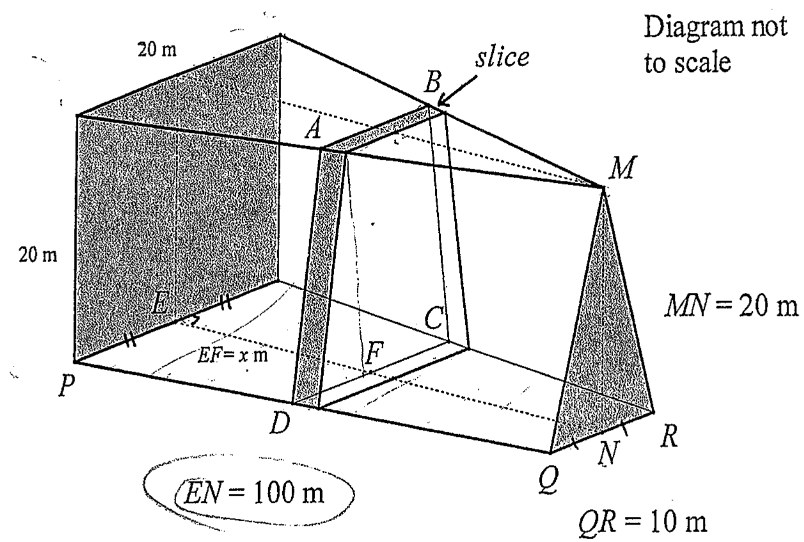
(ii) The area described in (i) is now rotated about the line $x = 2\pi$. Find the volume of the solid formed. 4

Question 5 continues on page 6

Question 5 con'td

Marks

(b) A boat showroom is built on level ground. The length of the showroom is 100m. At one end of the showroom the shape is a square measuring 20m by 20m and at the other end an isosceles triangle of height 20m and base 10m.



(i) If EF is x m in length, show that the length of DC is $\left[20 - \frac{x}{10}\right]$m. 2

(ii) By considering trapezoidal slices parallel to the ends of the showroom, find the volume enclosed by the showroom in m^3. 6

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Question 6 (15 Marks) [START A NEW PAGE] **Marks**

(a) Find $\int \frac{dx}{x^2 - 6x + 13}$. **2**

(b) A food parcel of 1 kg is dropped from a helicopter which is hovering 800 metres above a group of stranded bushwalkers. After 10 seconds a parachute opens automatically. Air resistance is neglected for the first 10 seconds but the effect of the open parachute is to cause a retardation of $2v$ newtons where v ms^{-1} is the velocity of the parcel after t seconds, $(t \geq 10)$.

Take the position of the helicopter as the origin, the downwards direction as positive and the value of g, the acceleration due to gravity as 10m s^{-2} .

(i) Write down the equation of motion of the parcel before the parachute opens. **1**

(ii) Determine the velocity and the distance fallen by the parcel at the end of the 10 seconds. **4**

(iii) Write down the equation of motion for $t \geq 10$. **1**

(iv) What is the terminal velocity of the parcel? **1**

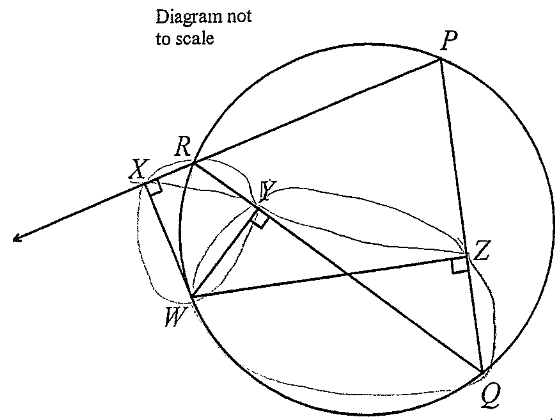
(v) Show that the velocity of the parcel after the parachute has opened is given by : **3**

$$v = 5 + 95e^{-2(t-10)}, \quad t \geq 10 .$$

(vi) Find the distance fallen, x, as a function of t and hence find the distance the parcel has fallen ~~t~~ minutes after it leaves the helicopter. **3**

Question 7 (15 Marks) [START A NEW PAGE] **Marks**

(a)



PQR is a triangle inscribed in a circle. W is a point on the arc QR.
From W, perpendiculars are drawn to PR (produced), QR and PQ, meeting them at X, Y and Z respectively.

Copy the diagram.

(i) Explain why $WXRY$ and $WYZQ$ are cyclic quadrilaterals. **2**

(ii) Prove that the points X, Y and Z are collinear. **4**

(b)(i) By considering the expansion of $(\cos\theta + i\sin\theta)^5$ and by using De Moivre's Theorem show that **2**

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta .$$

(ii) Hence finds all the four roots of the equation **2**

$$16x^4 - 20x^2 + 5 = 0 .$$

(iii) Hence or otherwise, show that $\cos\frac{\pi}{10}\cos\frac{3\pi}{10} = \frac{\sqrt{5}}{4}$. **3**

(iv) Find the exact value of $\sin\frac{3\pi}{5}\sin\frac{6\pi}{5}$. **2**

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Question 8 (15 Marks) [START A NEW PAGE] **Marks**

(a) The region R in the Argand diagram is defined by:

$$|z-1| \le |z-i| \text{ and } |z-2-2i| \le 1.$$

(i) Sketch the region R. **3**

(ii) If z describes the boundary of the region R, find **2**

(α) the value of $|z|$ when $\arg(z)$ has the smallest value.

(β) z in the form of $a+ib$ when $\arg(z-1) = \frac{\pi}{4}$. **3**

(b) A certain type of merry go-round consists of seats hung from pivots attached to the rim of a horizontal circular disc. The disc is rotated by a motor attached to the vertical axle. As the angular velocity increases, the seats swing out and move up. The seat is represented by a point with mass m kg suspended by a rod of length h metres below the pivot, which is a metres from the axle of rotation.

Neglecting the mass of the rod, assume that when the disc rotates with constant angular velocity w radians per second, there is an equilibrium position such that the rod makes an angle θ with the vertical as shown in the diagram on the following page.

Question 8 continues on page 10

Question 8 cont'd **Marks**

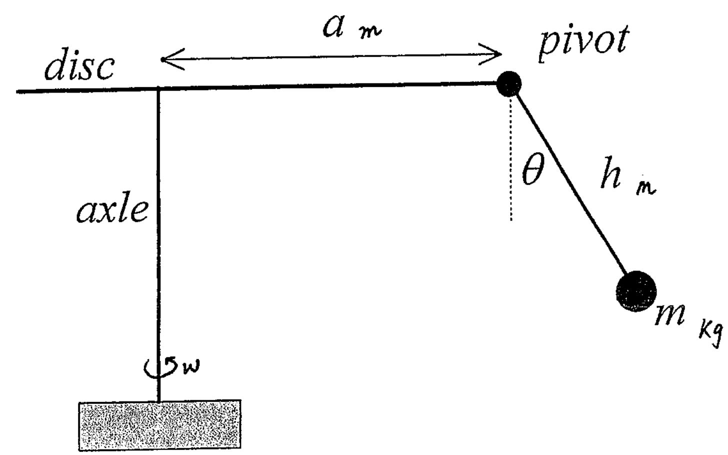

Diagram not to scale

(i) Show that w and θ satisfy the equation **3**

$$(a + h\sin\theta)w^2 = g\tan\theta$$

where g is the acceleration due to gravity.

(ii) Use graphical method to show that for a given w, there is only one value of θ in the domain $0 \le \theta \le \frac{\pi}{2}$, which satisfies the above equation. **3**

(iii) Given $a = 4$, $h = 2.5$, $\theta = 30^0$ and using $g = 10\text{ms}^{-2}$, find the angular velocity w correct to 3 significant figures when the merry-go-round is in equilibrium. **1**

END

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(a)(i) $\frac{a}{x} + \frac{bx+c}{4+x^2} = \frac{a+c \qquad x^2+}{x(4+x^2)}$ use

$\therefore 4a = 1 \Rightarrow a = \frac{1}{4}$ #

$a + b = 0 \qquad b = -\frac{1}{4}$ # 1

$c = 0$ #

(ii) $\int \frac{dx}{x(4+x^2)} = \int \frac{dx}{4x} + \int \frac{-\frac{1}{4}x\,dx}{4+x^2}$

$= \frac{1}{4}\ln|x| - \frac{1}{8}\int \frac{2x}{x^2+4} + c$

$= \frac{1}{4}\ln|x| - \frac{1}{8}\ln|x^2+4| + c$ # 2

(b) $\int_0^2 x\sqrt{2-x}\,dx \qquad u = x \quad dV = \sqrt{2-x}$

$du = dx \quad v = -\frac{2}{3}(2-x)^{3/2}$

$= \left[-\frac{2}{3}x(2-x)^{3/2}\right]_0^2 + \frac{2}{3}\int_0^2 (2-x)^{3/2}dx$ 1

$= 0 + \frac{2}{3}\cdot\left[(2-x)^{5/2}\left(-\frac{2}{5}\right)\right]_0^2$ 1

$= \frac{4}{15}\,2^{5/2}$ # or $\frac{16\sqrt{2}}{15}$ # 1

(c) Since all coeff are real and $2-i$ is a zero, $2+i$ is also a zero 1

$(x-2+i)(x-2-i) = x^2-4x+5$

$$\begin{array}{r} x^2 - x - 2 \quad 1 \\ x^2-4x+5 \overline{\big) x^4 - 5x^3 + 7x^2 + 3x - 10} \\ x^4 - 4x^3 + 5x^2 \\ \hline -x^3 + 2x^2 + 3x \\ -x^3 + 4x^2 - 5x \\ \hline -2x^2 + 8x - 10 \\ -2x^2 + 8x - 10 \\ \hline \end{array}$$

$x^2 - x - 2 = (x-2)(x+1)$

Zeros are $2, -1, 2+i, 2-i$ # 1

(a)(i) $I_{2n+1} = \int_0^1 x^{2n+1} e^{x^2} dx \qquad u = x^{2n} \quad v' = xe^{x^2}$

$= \int_0^1 x^{2n}.x.e^{x^2}dx \qquad \frac{du}{dx} = 2n\,x^{2n-1} \quad v = \frac{e^{x^2}}{2}$

$= \left[\frac{x^{2n}e^{x^2}}{2}\right]_0^1 - \int_0^1 \frac{1}{2}e^{x^2}2n\,x^{2n-1}dx$ 1

$= \frac{e}{2} - n\int_0^1 x^{2n-1}e^{x^2}dx$

$= \frac{1}{2}e - n\,I_{2n-1}$ # 1

(ii) $I_5 = \frac{e}{2} - 2I_3 = \frac{e}{2} - 2\left[\frac{e}{2} - I_1\right]$
(n=2)

$I_5 = \frac{e}{2} - e + 2I_1 = -\frac{e}{2} + 2I_1$ 1

$I_1 = \int_0^1 x e^{x^2}dx = \frac{1}{2}\int_0^1 2x e^{x^2}dx = \left[\frac{e^{x^2}}{2}\right]_0^1 = \frac{1}{2}(e-1)$ 1
(n=0)

$\therefore I_5 = -\frac{e}{2} + \frac{2}{2}(e-1) = -\frac{e}{2} + e - 1 = \frac{e}{2} - 1$ # 1

Question 2

a(i) Let $z = x + yi$

$z^2 = x^2 + 2xyi - y^2 = -3-4i$

$x^2 - y^2 = -3 \quad (1)$ 1

$2xy = -4 \quad \therefore xy = -2 \Rightarrow y = -\frac{2}{x} \quad (2)$

$x^2 - \frac{4}{x^2} = -3$

$x^4 + 3x^2 - 4 = 0$

$(x^2+4)(x^2-1) = 0 \quad \therefore x = \pm 2i \text{ or } \pm 1$ 1

But $x \in R \therefore x = \pm 1$

when $x = 1, \quad y = -2$

$x = -1, \quad y = 2$

$\therefore z = 1-2i$ or $-1+2i$ # 1

b(i) $x = a\cos\theta \quad \frac{dx}{d\theta} = -a\sin\theta$

$y = b\sin\theta \quad \frac{dy}{d\theta} = b\cos\theta \quad \Big\} \frac{dy}{dx} = \frac{-b\cos\theta}{a\sin\theta}$ 1

Eq of OQ passing thru (0,0)

$y = \frac{-b\cos\theta}{a\sin\theta}x$

$b\cos\theta x + a\sin\theta y = 0$ is the eq of OQ 1

P is in 1^{st} Quad $\therefore a>0, \ b>0$

$\frac{x^2}{a^2} + \frac{b^2\cos^2\theta}{b^2a^2\sin^2\theta}x^2 = 1$ 1

$\frac{x^2}{a^2} + \frac{\cos^2\theta}{a^2\sin^2\theta}x^2 = 1$

$x^2(\sin^2\theta + \cos^2\theta) = a^2\sin^2\theta$

$x^2 = a^2\sin^2\theta$

But Q is in 4^{th} Quad $\therefore x_Q > 0$ 1

$x_Q = a\sin\theta$

$y_Q = \frac{-b\cos\theta}{a\sin\theta}(a\sin\theta)$

$y_Q = -b\cos\theta$

$\therefore Q = (a\sin\theta, -b\cos\theta)$ 1

(ii) Perpendicular distance from Q to OP

$d = \frac{|ab\cos^2\theta + ab\sin^2\theta|}{\sqrt{a^2\sin^2\theta + b^2\cos^2\theta}}$ 1

$d = \frac{|ab|}{\sqrt{a^2\sin^2\theta + b^2\cos^2\theta}}$

Area $\triangle OQP = \frac{1}{2}\times\frac{|ab|}{\sqrt{a^2\sin^2\theta+b^2\cos^2\theta}} \times OQ$

$= \frac{|ab|}{2\sqrt{a^2\sin^2\theta+b^2\cos^2\theta}}\sqrt{a^2\sin^2\theta+b^2\cos^2\theta}$ 1

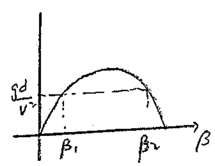
$= \frac{|ab|}{2}$ independent of position of P 1 #

Q.3

a) For the particle to hit the wall, the wall must be at a distance d less than the range for angle β

$d < \frac{V^2\sin 2\beta}{g}$ 1

$\sin 2\beta > \frac{gd}{V^2}$



at β_1, β_2 $\sin 2\beta = \frac{gd}{V^2}$

$2\beta = n\pi + (-1)^n\sin^{-1}\left(\frac{gd}{V^2}\right)$ 1

n=0 $\beta_1 = \frac{1}{2}\sin^{-1}\left(\frac{gd}{V^2}\right)$

n=1 $\beta_2 = \frac{\pi}{2} - \frac{1}{2}\sin^{-1}\left(\frac{gd}{V^2}\right) = \frac{\pi}{2} - \beta_1$

$\therefore$ the particle will hit wall at an angle α such that

$\frac{1}{2}\sin^{-1}\left(\frac{gd}{V^2}\right) < \alpha < \frac{\pi}{2} - \frac{1}{2}\sin^{-1}\left(\frac{gd}{V^2}\right)$

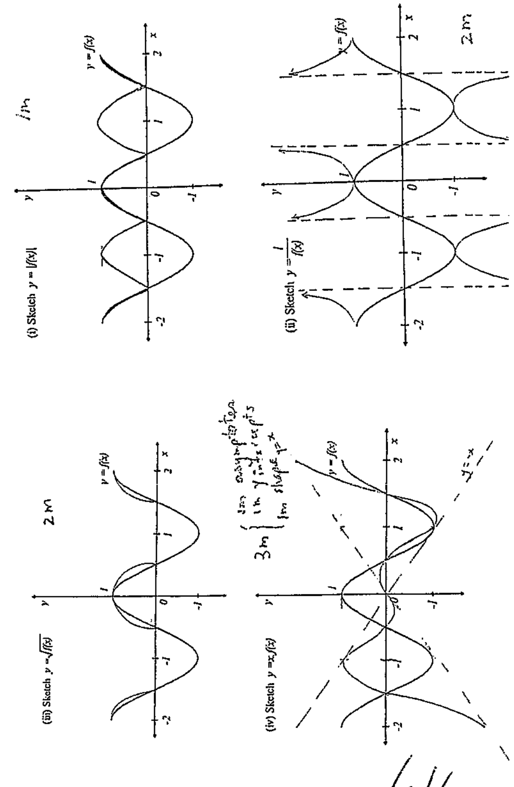
b) $z = \frac{\sqrt{2}}{1-i}\cdot\frac{1+i}{1+i} = \frac{\sqrt{2}(1+i)}{2} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$ 1

$z = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}\ \text{cis}\frac{\pi}{4}$

$z = 1.\text{cis}\frac{\pi}{4}$ 1

$z^5 = 1^5\,\text{cis}\frac{5\pi}{4} = \cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}$ 1

$z^5 = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$ # 1



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(ii) $u = \ln x \quad du = \frac{1}{x}dx$

$\int \frac{du}{u^2} = -u^{-1} + c = -\frac{1}{\ln x} + c$ #

(b) No. of ways to choose the 5 letters:

No "i" MOBLTY $^6C_5 = 6$

1 "i" MOBILTY $^6C_4 = 15$

2 "i" MOBILITY $\frac{^6C_3}{2!} = 10$

To arrange the 5 letters = 5!

∴ Total No. of different arrangements $= (6+15+10) \times 5! = 3720$ #

c(i) $\frac{dy}{dx} = \frac{dy}{dp} \times \frac{dp}{dx} = \frac{-3}{p^2} \times \frac{1}{3} = -\frac{1}{p^2}$

Eq of tangent at P: $y - \frac{3}{p} = -\frac{1}{p^2}(x - 3p)$

$y - \frac{3}{p} = \frac{-x}{p^2} + \frac{3}{p}$

$p^2 y + x = 6p$ # ——①

ii) Similarly eq. of tangent at Q:

$q^2 y + x = 6q$ ——②

solving ① & ② simultaneously

$y(p^2 - q^2) = 6(p-q) \quad \therefore y = \frac{6}{p+q}$

$x = 6p - p^2 y = 6p - \frac{6p^2}{p+q} = \frac{6p^2 + 6pq - 6p^2}{p+q}$

$x = \frac{6pq}{p+q}$

$\therefore T = \left(\frac{6pq}{p+q}, \frac{6}{p+q}\right)$

(iii) slope of chord $PQ = \frac{3(\frac{1}{p} - \frac{1}{q})}{3(p-q)}$

$= \frac{\frac{q-p}{pq}}{p-q} = -\frac{1}{pq}$

Eq of PQ: $y - \frac{3}{p} = -\frac{1}{pq}(x - 3p)$

$y - \frac{3}{p} = -\frac{x}{pq} + \frac{3}{q}$

$pqy + x = 3(p+q)$

PQ passes thro' (0,2)

$pq(2) + 0 = 3(p+q)$

$2pq = 3(p+q)$

$\therefore \frac{6pq}{p+q} = 6 \times \frac{3}{2} = 9$

$\frac{6}{p+q} = 6 \times \frac{3}{2pq} = \frac{9}{pq}$

∴ Locus of T is $x = 9$

iv) Since P, Q are on different branches of the rectangular hyperbola

$p \cdot q < 0$

Restriction on the locus of T: $x = 9$ and $y < 0$

∴ T must be in 4th Quad. #

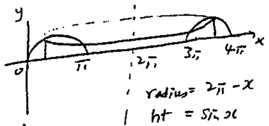
Question 5:

a i) $V = \pi \int_0^{\pi} (\sin x)^2 dx$

$= \pi \int_0^{\pi} \left(\frac{1 - \cos 2x}{2}\right) dx$

$= \frac{\pi}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi} = \frac{\pi}{2}(\pi - 0) = \frac{\pi^2}{2}$ unit³ #

ii) $V = \int_0^{\pi} 2\pi(2\pi - x)\sin x \, dx$



$= \int_0^{\pi} (4\pi^2 \sin x - 2\pi x \sin x) dx$

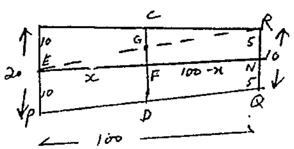
$= 4\pi^2(-\cos x)\Big]_0^{\pi} - 2\pi \int_0^{\pi} x \sin x \, dx$ (u dv)

$= -4\pi^2(-1-1) - 2\pi\left[-x\cos x\right]_0^{\pi} - \int_0^{\pi} \cos x \, dx$

$= 8\pi^2 - 2\pi(\pi) - [\sin x]_0^{\pi}$

$= 6\pi^2$ #

b) Join ER cutting CF at G



Base Plan

$\frac{FG}{5} = \frac{x}{100}$

$FG = \frac{x}{20}$

$\frac{CG}{10} = \frac{100-x}{100}$

$CG = \frac{100-x}{10}$

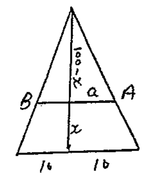
$CF = CG + FG = \frac{x}{20} + \frac{100-x}{10} = \frac{x + 200 - 2x}{20}$

$= \frac{200-x}{20}$

$CD = 2 \times CF = \frac{200-x}{20} \times 2 = \frac{200-x}{10} = 20 - \frac{x}{10}$ m #

Q5

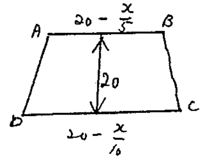
$\frac{a}{10} = \frac{100-x}{100}$



$a = \frac{100-x}{10} = 10 - \frac{x}{10}$

$AB = 2a = 20 - \frac{x}{5}$

Each trapezoidal slice:



$\text{Area} = \frac{20}{2}\left[20 - \frac{x}{5} + 20 - \frac{x}{10}\right]$

$= 10\left(40 - \frac{3x}{10}\right)$

$= 400 - 3x$

ΔV (volume slice) $= (400 - 3x)\Delta x$

Volume of the showroom $= \lim_{\Delta x \to 0} \sum \Delta V$

$= \int_0^{100} (400 - 3x) dx = \left[400x - \frac{3x^2}{2}\right]_0^{100}$

$= 25000 \text{ m}^3$ #

Question 6

(a) $\int \frac{dx}{(x^2 - 6x + 9) + 4} = \int \frac{dx}{(x-3)^2 + 4}$ let $u = x - 3$, $du = dx$

$= \int \frac{du}{u^2 + 2^2} = \frac{1}{2}\tan^{-1}\left(\frac{u}{2}\right) + c = \frac{1}{2}\tan^{-1}\left(\frac{x-3}{2}\right) + c$ #

(b) i) $\ddot{y} = 10$

ii) $\dot{y} = 10t + C_1$ $\quad t = 0, \dot{y} = 0, \therefore C_1 = 0$

$\dot{y} = 10t$

$y = 5t^2 + C_2$ $\quad t = 0, y = 0, \therefore C_2 = 0$

$y = 5t^2$

$t = 10$, $\dot{y} = 10 \times 10 = 100$ m/s #

$y = 5 \times 10^2 = 500$ m #

iii) For $t \geq 10$ $\downarrow 10$ $\uparrow 2v$ $\quad \ddot{y} = \frac{dv}{dt} = 1(10 - 2v) = 10 - 2v$

$\ddot{y} = 10 - 2v$ #

iv) terminal velocity when $\ddot{y} = 0$

$10 - 2V = 0$ $\quad V = 5$ m/s #

$\frac{dv}{dt} =$ [illegible] v

$\int \frac{dv}{10 - 2v} = \int dt$

$\int \frac{dv}{2v - 10} = -\int dt$

$\ln|v - 5| = -2t + K$

$t = 10, \ V = 100$

$\ln|100 - 5| = -20 + K$

$K = 20 + \ln 95$

$\ln(v - 5) = -2t + 20 + \ln 95$

$\frac{v-5}{95} = e^{-2(t-10)}$

$v = 5 + 95e^{-2(t-10)}$, $t \geq 10$

vi) $\frac{dx}{dt} = 5 + 95e^{-2(t-10)}$

$x = 5t + \frac{95e^{-2(t-10)}}{-2} + c_2$

$t = 10$, $x = 500$

$500 = 50 - \frac{95}{2}e^0 + c_2$

$497.5 = c_2$

$\therefore x = 5t - \frac{95}{2}e^{-2(t-10)} + 497.5$

After 1 minute ($t = 60$)

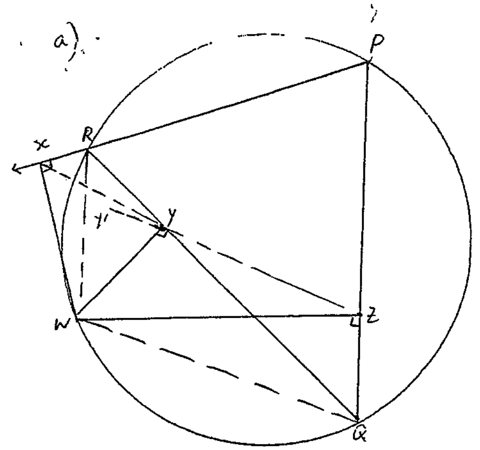
$x = 5 \times 60 - \frac{95}{2}e^{-2(50)} + 497.5$

$x = 797.5$ m #

∴ The particle has fallen 797.5 m after 1 min.

65

a)



$\angle RXW = \angle QYW = 90°$ (given)

(i) $WXRY$ is a cyclic quad.
(exterior angle equals interior opposite angle) # 1

$\angle WYQ = \angle WZQ = 90°$ (given)
$WYZQ$ is a cyclic quad.
(line interval WQ subtends equal angles at Y, Z, on the same side of the line interval, the 4 end points $WQZY$ are then concyclic) # 1

b) $(\cos\theta + i\sin\theta)^5$

$= \cos^5\theta + 5i\cos^4\theta\sin\theta + 10i^2\cos^3\theta\sin^2\theta$
$+ 10i^3\cos^2\theta\sin^3\theta + 5i^4\cos\theta\sin^4\theta + i^5\sin^5\theta$ 1

$= \cos^5\theta + 5i\cos^4\theta\sin\theta - 10\cos^3\theta(1-\cos^2\theta)$
$-10i\cos^2\theta\sin^3\theta + 5\cos\theta(1-\cos^2\theta)^2 + i\sin^5\theta$

$= \cos^5\theta + 5i\cos^4\theta\sin\theta - 10\cos^3\theta + 10\cos^5\theta$
$-10i\cos^2\theta\sin^3\theta + 5\cos\theta(1-2\cos^2\theta+\cos^4\theta) + i\sin^5\theta$

By De Moivre's Theorem,
$(\cos\theta + i\sin\theta)^5 = \cos 5\theta + i\sin 5\theta$

$\therefore \cos 5\theta = \cos^5\theta - 10\cos^3\theta + 10\cos^5\theta +$
$5\cos\theta - 10\cos^3\theta + 5\cos^5\theta$

1 $\cos 5\theta = \underline{16\cos^5\theta - 20\cos^3\theta + 5\cos\theta}$

Let $\cos\theta$ Then $\theta =$ ans
$x^4 - 20x^2 + 5) = 0$

Hence the roots of $16x^4 - 20x^2 + 5 = 0$ are the non-zero values of $\cos\theta$, where θ is a solution of $\cos 5\theta$

$\cos 5\theta = 0$ when $5\theta = 2n\pi \pm \frac{\pi}{2}$, $n \in \mathbb{Z}$ 1

There are 4 distinct non-zero values of $\cos\theta$, namely $\frac{\pi}{10}, \frac{3\pi}{10}, \frac{7\pi}{10}, \frac{9\pi}{10}$

$\therefore$ the four roots are $\cos\frac{\pi}{10}, \cos\frac{3\pi}{10}, \cos\frac{7\pi}{10}, \cos\frac{9\pi}{10}$ 1 #

iii) Products of all roots $= \left(\cos\frac{\pi}{10}\cos\frac{3\pi}{10}\cos\frac{7\pi}{10}\cos\frac{9\pi}{10}\right)$
$= \frac{e}{a} = \frac{5}{16}$ 1

But $\cos\frac{9\pi}{10} = -\cos\frac{\pi}{10}$, $\cos\frac{7\pi}{10} = -\cos\frac{3\pi}{10}$ 1

$\therefore \left(\cos\frac{\pi}{10}\cos\frac{3\pi}{10}\right)^2 = \frac{5}{16}$

$\cos\frac{\pi}{10}\cos\frac{3\pi}{10} = \frac{+\sqrt{5}}{4}$ # 1 $\left(\begin{array}{l}\text{Since } \cos\frac{\pi}{10} > 0 \\ \cos\frac{3\pi}{10} > 0 \\ \therefore \cos\frac{\pi}{10}\cos\frac{3\pi}{10} > 0\end{array}\right)$

iv) $\sin\frac{3\pi}{5} = \cos\left(\frac{\pi}{2} - \frac{3\pi}{5}\right) = \cos\left(-\frac{\pi}{10}\right) = \cos\frac{\pi}{10}$ 1

$\sin\frac{6\pi}{5} = \cos\left(\frac{\pi}{2} - \frac{6\pi}{5}\right) = \cos\left(-\frac{7\pi}{10}\right) = \cos\frac{7\pi}{10}$
$= -\cos\frac{3\pi}{10}$

$\therefore \sin\frac{3\pi}{5}\cdot\sin\frac{6\pi}{5} = \cos\frac{\pi}{10}\left(-\cos\frac{3\pi}{10}\right) = -\frac{\sqrt{5}}{4}$ # (from part iii) 1

Q7a ii) Construction: Join XY, WQ, RW
Join ZY and extend it to Y'

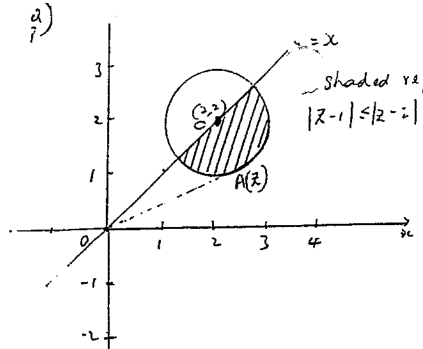
Proof: $\angle XRW = \angle XYW$ (angles at circumference in same segment) 1

$\angle XRW = \angle ZQW$
(exterior angle equals to interior opposite angle in cyclic quadrilateral $RPQW$) 1

Similarly, $\angle Y'YW = \angle ZQW$ since in part (i) we have proved $WYZQ$ is a cyclic quadrilateral 1

$\therefore \angle XRW = \angle XYW = \angle Y'YW$
Since $\angle XYW = \angle Y'YW$, X, Y, Z must be collinear. 1

a) i)



shaded region (R):
$|z-1| \le |z-i|$ and $|z-2-2i| \le 1$

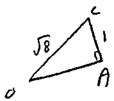
1m $y=x$
1m circle
1m shaded Area

Let Point A represents z.

ii) α) When $\arg(z)$ has the smallest value, OA is tangent to the circle 1

$OC = \sqrt{2^2+2^2} = \sqrt{8}$

$\therefore OA = \sqrt{8-1} = \sqrt{7}$ 1

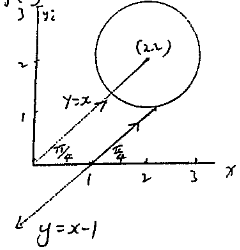


$\therefore |z| = \sqrt{7}$ when $\arg(z)$ has the smallest value.

β)

$\arg(z-1) = \frac{\pi}{4}$ is the line $y = x - 1$ ①

Pt of intersection with the circle
$(x-2)^2 + (y-2)^2 = 1$



$(x-2)^2 + (x-3)^2 = 1$

$x^2 - 4x + 4 + x^2 - 6x + 9 = 1$

$2x^2 - 10x + 13 = 1$

$x^2 - 5x + 6 = 0$

$(x-3)(x-2) = 0$

$\therefore x = 2$ or 3 1

When $x = 2$, $y = 1$
$x = 3$, $y = 2$

$z = 2 + i$ or $3 + 2i$ 1

b, i) $T\cos\theta = mg$

$T = \frac{mg}{\cos\theta}$ —— ①

$mrw^2 = T\sin\theta$

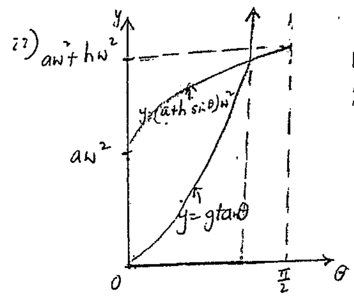
$m(a + h\sin\theta)w^2 = T\sin\theta$ —— ②

Sub ① into ②

$\cancel{m}(a + h\sin\theta)w^2 = \frac{\cancel{m}g}{\cos\theta}\sin\theta$

$(a + h\sin\theta)w^2 = g\tan\theta$ # 1

ii)



1m $y = (a + h\sin\theta)$
1m $y = g\tan\theta$
1m for pt of intersection

As there is only 1 pt of intersection there is only one value of θ that satisfies $(a + h\sin\theta)w^2 = g\tan\theta$

iii) $(a + h\sin\theta)w^2 = g\tan\theta$

$(4 + 2.5\sin 30°)w^2 = 10\tan 30°$

$5.25w^2 = \frac{10\sqrt{3}}{3}$

$w^2 = 1.0997$

$w = 1.04867$

$w = \underline{1.05 \text{ radian/second}}$ 1