

**TRIAL HIGHER SCHOOL CERTIFICATE  
EXAMINATION 2008**

**MATHEMATICS**

*Time Allowed - 3 Hours  
(Plus 5 minutes Reading Time)*

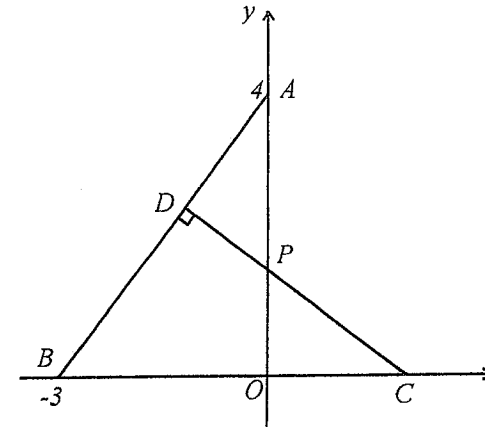
- All questions may be attempted
- All questions are of equal value
- Department of Education approved calculators are permitted
- In every question, show all necessary working
- Marks may not be awarded for careless or badly arranged work
- No grid paper is to be used unless provided with the examination paper

**The answers to all questions are to be returned in separate stapled bundles clearly labeled Question 1, Question 2, etc. Each question must show your Candidate Number.**

**Question 1**

- (a) Solve for  $t$ :  $7 - 4t > 12$ . 2
- (b) Simplify:  $\frac{x}{2} - \frac{2x+5}{6}$ . 2
- (c) Solve  $2\sin \theta = -1$  for  $0 \leq \theta \leq 2\pi$ . 2
- (d) Differentiate with respect to  $x$ :
- (i)  $y = \frac{6}{\sqrt{x}}$ . 1
- (ii)  $y = x^2 \ln x$ . 2
- (e) Evaluate  $\int_1^5 \left(3 + \frac{2}{x}\right) dx$ . 3

**Question 2 (START A NEW PAGE)**



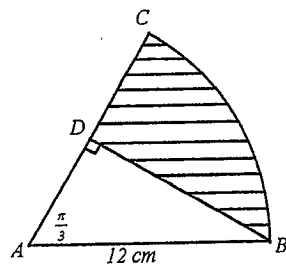
In the diagram  $AB = BC$  and  $CD$  is perpendicular to  $AB$ .  $CD$  intersects the  $y$ -axis at  $P$ .

- (i) Find the length of  $AB$ . 1
- (ii) Hence show that the co-ordinates of point  $C$  are  $(2, 0)$ . 1
- (iii) Show that the equation of  $CD$  is  $3x + 4y = 6$ . 3
- (iv) Show that the co-ordinates of  $P$  are  $(0, 1\frac{1}{2})$ . 1
- (v) Show that the length of  $CP$  is  $2\frac{1}{2}$ . 1
- (vi) Prove that  $\triangle ADP$  is congruent to  $\triangle COP$ . 3
- (vii) Hence calculate the area of the quadrilateral  $DPOB$ . 2

**Question 3 (START A NEW PAGE)**

(a) Find the equation of the tangent to the curve  $y = x^3 - 4x - 1$  at the point  $T(2, -1)$ .

(b)



$ABC$  is a sector with  $\angle BAC = \frac{\pi}{3}$  and  $AC = AB = 12$  cm.

(i) Calculate the area of sector  $ABC$ .

(ii) Calculate the area of the shaded region.

(c) (i) The equation of a parabola is given as  $y = \frac{1}{8}x^2 - x$ . Rewrite the equation in the form  $(x - x_0)^2 = b(y + y_0)$  where  $x_0$ ,  $y_0$  and  $b$  are constants.

(ii) Hence write down the focal length of the parabola and the co-ordinates of its vertex and focus.

**Question 4 (START A NEW PAGE)**

(a) The price of gold,  $\$P$ , was studied over a period of  $t$  years.

(i) Throughout the period of study  $\frac{dP}{dt} > 0$ . What does this say about the price of gold?

(ii) It was also observed that the rate of change of the price of gold decreased over the period of study.

What does this statement imply about  $\frac{d^2P}{dt^2}$ ?

(b) A pool is being drained at a rate of  $240t - 9600$  litres/minute.

(i) How long does it take before the draining of the pool stops?

(ii) If the pool initially held 1920000 litres of water, find the volume of water in the pool after 15 minutes.

(c) In a hat are 5 red and 3 green jellybeans. Jason reaches into the hat and randomly selects two jellybeans. Find the probability that:

(i) both selected jellybeans are red.

(ii) the chosen jellybeans are different colours.

(d) The tangent to the curve  $y = 4\sqrt{x}$  at a point  $P$  has a slope of 2. Find the co-ordinates of point  $P$ .

**Marks**  
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**Question 5 (START A NEW PAGE)**

**Marks**

(a)



Pattern #1



Pattern #2



Pattern #3

The above patterns are made using small sticks. Pattern #1 requires 6 sticks, pattern #2 requires 11 sticks and pattern #3 requires 16 sticks.

(i) Write a formula for the number of sticks,  $U_n$ , needed to construct pattern number  $n$ .

(ii) What is the largest pattern that can be constructed from 200 sticks?

(iii) How many sticks would be needed to construct all the patterns from pattern #1 to pattern #20?

(b) (i) Show that the curves  $y = 1 + \sqrt{x}$  and  $y = 7 - x$  meet at the point  $(4, 3)$ .

(ii) Sketch, on the same set of axes, the curves  $y = 1 + \sqrt{x}$  and  $y = 7 - x$  and shade the region bounded by these curves and the  $y$ -axis.

(iii) Find the volume of the solid formed when the area bounded by the  $y = 1 + \sqrt{x}$ ,  $y = 7 - x$  and the  $y$ -axis is rotated one revolution about the  $y$ -axis.

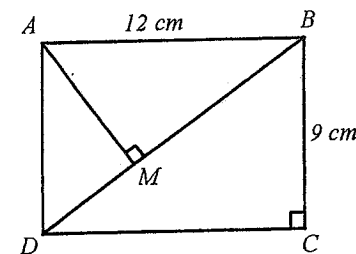
**Question 6 (START A NEW PAGE)**

(a) (i) Show that the curves  $y = \sin 2x$  and  $y = \cos x$  meet at the point with abscissa  $x = \frac{\pi}{6}$ .

(ii) Sketch, on the same set of axes, the curves  $y = \sin 2x$  and  $y = \cos x$  in the domain  $0 \leq x \leq \frac{\pi}{2}$  and shade the region bounded only by these curves.

(iii) Find the area of the shaded region described in (ii).

(b)



(Diagram not drawn to scale)

$ABCD$  is a rectangle with  $AB = 12$  cm,  $BC = 9$  cm and  $AM$  perpendicular to  $BD$ .

(i) Copy the diagram onto your answer sheet and find the length of  $BD$ .

(ii) Prove that  $\triangle ABM$  is similar to  $\triangle BDC$ .

(iii) Hence find the length of  $BM$ . (Give reasons)

**Question 7 (START A NEW PAGE)**

- (a) In an experiment, the amount of crystals,  $x$  grams, that dissolved in a solution after  $t$  minutes was given by:

$$x = 200(1 - e^{-kt})$$

- (i) After 3 minutes it was found that 50 grams of crystals had dissolved, hence find the value of  $k$ .
- (ii) At what rate were the crystals dissolving after 5 minutes? Give your answer to the nearest gram/minute.

**Marks**

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- (b) Two particles  $A$  and  $B$  start from the origin at the same time and move along a straight line so that their velocities in m/s at any time  $t$  seconds are given by:

$$v_A = t^2 + 2 \text{ and } v_B = 8 - 2t$$

- (i) Show that the two particles never move with the same acceleration.
- (ii) Write a formula for the position of particle  $A$  at time  $t$  seconds.
- (iii) After leaving the origin, find when the two particles are again at the same position.

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**Question 8 (START A NEW PAGE)**

- (i) Given that  $f(x) = x^2\sqrt{10-x}$ , show that  $f'(x) = \frac{5x(8-x)}{2\sqrt{10-x}}$ .
- (ii) State the domain of the function  $y = x^2\sqrt{10-x}$ .
- (iii) Find all the stationary points on the curve  $y = x^2\sqrt{10-x}$  and determine their nature.
- (iv) Sketch the curve  $y = x^2\sqrt{10-x}$  indicating its intercepts with the co-ordinate axes and the position of all stationary points.

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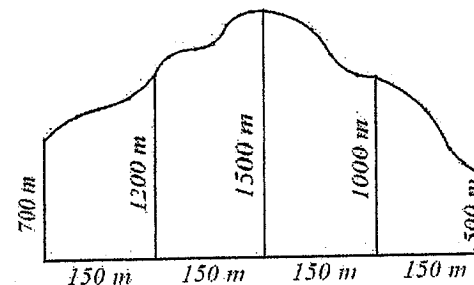
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**Question 9 (START A NEW PAGE)**

**Marks**

- (a) Prime land along a foreshore is to be reclaimed and developed as part of a housing estate. A plan of the land to be reclaimed is shown below:

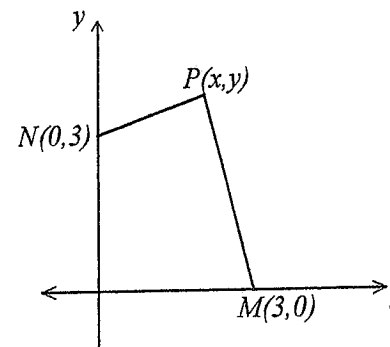
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(Diagram not drawn to scale)

Use Simpson's rule with five function values to find the approximate area, expressed in hectares, of the land to be reclaimed.

- (b)



The point  $P(x,y)$  moves so that its distance from the point  $M(3,0)$  is always twice its distance from the point  $N(0,3)$ .

- (i) Show that the equation of the locus of the point  $P(x,y)$  is  $x^2 + 2x + y^2 - 8y + 9 = 0$ .
- (ii) Give a geometric description of the locus.
- (c) (i) Graph  $y = f(x)$  if  $f(x) = \begin{cases} -4x & \text{for } x < 0 \\ x^2 + 3 & \text{for } x \geq 0 \end{cases}$
- (ii) Evaluate  $f(-2) + f(2)$ .
- (iii) Solve for  $a > 0$ :  $f(a) = f(-a)$ .

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**Question 10 (START A NEW PAGE)**

**Marks**

(a) Abigale decides to save for a car. She creates a savings account into which she deposits \$100 at the beginning of each week. The account pays interest at a rate of 0.16% per week calculated on the value of the account at the end of each week.

(i) Show that Abigale's account is worth  $62600(1.0016^n - 1)$  dollars at the end of  $n$  weeks. 3

(ii) Find the value of Abigale's account at the end of 52 weeks. Give your answer correct to the nearest dollar. 1

(iii) Find the number of weeks needed for Abigale's account to accumulate \$20000. (Give your answer to nearest week) 3

(b) When a ship is travelling at a speed of  $v$  km/hr, its rate of consumption of fuel in tonnes per hour is given by  $125 + 0.004v^3$ .

(i) Show that on a voyage of 5000km at a speed of  $v$  km/hr the formula for the total fuel used,  $T$  tonnes, is given by: 1

$$T = \frac{625000}{v} + 20v^2$$

(ii) Hence find the speed for the greatest fuel economy and the amount of fuel used at this speed. (Justify your answer.) 4



This is the END of the examination paper



Question 1

(a) Solve for  $t$ :  $7 - 4t > 12$ .  
 $-4t > 5$   
 $t < -1\frac{1}{4}$

(b) Simplify:  $\frac{x}{2} - \frac{2x+5}{6}$ .

$$\begin{aligned} \frac{x}{2} - \frac{2x+5}{6} &= \frac{3x}{6} - \frac{2x+5}{6} \\ &= \frac{3x-2x-5}{6} \\ &= \frac{x-5}{6} \end{aligned}$$

(c) Solve  $2\sin\theta = -1$  for  $0 \leq \theta \leq 2\pi$ .

$$\begin{aligned} \sin\theta &= -\frac{1}{2} \\ \theta &= \pi + \frac{\pi}{6} \text{ or } 2\pi - \frac{\pi}{6} \\ \theta &= \frac{7\pi}{6} \text{ or } \frac{11\pi}{6} \end{aligned}$$

(d) Differentiate with respect to  $x$

(i)  $y = \frac{6}{\sqrt{x}}$

$$\begin{aligned} y &= 6x^{-\frac{1}{2}} \\ y' &= -3x^{-\frac{3}{2}} \\ &= -\frac{3}{x\sqrt{x}} \end{aligned}$$

(ii)  $y = x^2 \ln x$

$$\begin{aligned} y' &= 2x \ln x + x^2 \cdot \frac{1}{x} \\ &= 2x \ln x + x \end{aligned}$$

(e) Evaluate  $\int_1^5 \left(3 + \frac{2}{x}\right) dx$ .

$$\begin{aligned} \int_1^5 \left(3 + \frac{2}{x}\right) dx &= [3x + 2 \ln x] \\ &= (15 + 2 \ln 5) - (3 + 2 \ln 1) \\ &= 12 + 2 \ln 5 \end{aligned}$$

Marks  
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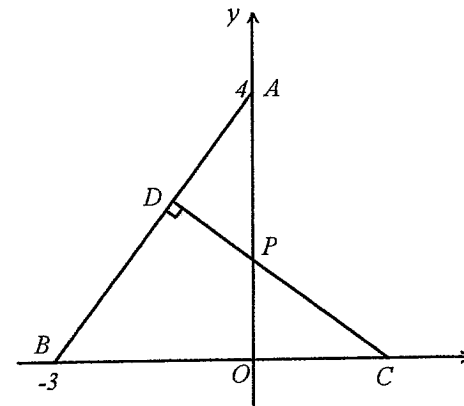
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Question 2 (START A NEW PAGE)



In the diagram  $AB = BC$  and  $CD$  is perpendicular to  $AB$ .  $CD$  intersects the  $y$ -axis at  $P$ .

(i) Find the length of  $AB$

$$\begin{aligned} AB^2 &= 3^2 + 4^2 \text{ (Pythagoras' Theorem)} \\ AB &= 5 \end{aligned}$$

(ii) Hence show that the co-ordinates of point  $C$  are  $(2, 0)$ .

$$\begin{aligned} BC &= AB \\ \therefore BC &= 5 \\ OB + OC &= 5 \\ 3 + OC &= 5 \\ OC &= 2 \\ \therefore C \text{ is } (2, 0) \end{aligned}$$

(iii) Show that the equation of  $CD$  is  $3x + 4y = 6$ .

$$\begin{aligned} m(AB) &= \frac{4}{3} \\ m(CD) &= -\frac{3}{4} \\ \text{Eqn. } CD: y - 0 &= -\frac{3}{4}(x - 2) \\ 4y &= -3x + 6 \\ 3x + 4y &= 6 \end{aligned}$$

(iv) Show that the co-ordinates of  $P$  are  $(0, 1\frac{1}{2})$ .

$$\begin{aligned} 3x + 4y &= 6 \\ \text{at } P \quad x &= 0 \\ \therefore 0 + 4y &= 6 \\ y &= 1\frac{1}{2} \end{aligned}$$

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- (v) Show that the length of  $CP$  is  $2\frac{1}{2}$ .  
 $CP^2 = (1\frac{1}{2})^2 + 2^2$  (Pythagoras' Theorem)  
 $= 6\frac{1}{4}$   
 $CP = 2\frac{1}{2}$

- (vi) Prove that  $\triangle ADP$  is congruent to  $\triangle COP$

In  $\triangle ADP$  and  $\triangle COP$   
 $\angle ADP = \angle COP$  (both  $90^\circ$ )  
 $\angle DPA = \angle OPC$  (vertically opposite angles are equal)  
 $AP = PC$  (both  $= 2\frac{1}{2}$ )  
 $\therefore \triangle ADP \cong \triangle COP$  (AAS)

- (vii) Hence calculate the area of the quadrilateral  $DPOB$ .

$$\begin{aligned} \text{area } BDPO &= \text{area } \triangle BAO - \text{area } \triangle ADP \\ &= \text{area } \triangle BAO - \text{area } \triangle OPC \quad (\triangle OPC \cong \triangle ADP) \\ &= \frac{1}{2}(3)(4) - \frac{1}{2}(2)(1\frac{1}{2}) \text{ u}^2 \\ &= 4\frac{1}{2} \text{ u}^2 \end{aligned}$$

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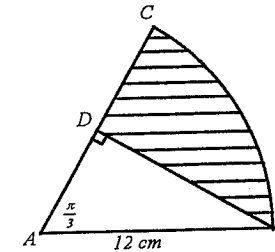
**Question 3 (START A NEW PAGE)**

**Marks**  
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- (a) Find the equation of the tangent to the curve  $y = x^3 - 4x - 1$  at the point  $T(2, -1)$ .

$$\begin{aligned} y' &= 3x^2 - 4 \\ \text{when } x = 2, y' &= 3(2)^2 - 4 \\ &= 8 \\ \text{eqn. of tangent} \\ y + 1 &= 8(x - 2) \\ y &= 8x - 17 \end{aligned}$$

- (b)



$ABC$  is a sector with  $\angle BAC = \frac{\pi}{3}$  and  $AC = AB = 12 \text{ cm}$

- (i) Calculate the area of sector  $ABC$

$$\begin{aligned} \text{area} &= \frac{1}{2}(12)^2 \left(\frac{\pi}{3}\right) \text{ cm}^2 \\ &= 24\pi \text{ cm}^2 \end{aligned}$$

- (ii) Calculate the area of the shaded region.

$$\frac{AD}{12} = \cos\left(\frac{\pi}{3}\right) \text{ cm}$$

$$\begin{aligned} AD &= 12 \times \frac{1}{2} \text{ cm} \\ &= 6 \text{ cm} \end{aligned}$$

$$\text{area } \triangle ADB = \frac{1}{2} \times 6 \times 12 \times \sin\left(\frac{\pi}{3}\right) \text{ cm}^2$$

$$= 36 \times \frac{\sqrt{3}}{2} \text{ cm}^2$$

$$= 18\sqrt{3} \text{ cm}^2$$

$$\text{shaded area} = (24\pi - 18\sqrt{3}) \text{ cm}^2$$

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- (c) (i) The equation of a parabola is given as  $y = \frac{1}{8}x^2 - x$ . Rewrite the equation in the form  $(x - x_0)^2 = b(y + y_0)$  where  $x_0$ ,  $y_0$  and  $b$  are constants.

$$y = \frac{1}{8}x^2 - x$$

$$8y = x^2 - 8x$$

$$8y + 16 = x^2 - 8x + 16$$

$$(x - 4)^2 = 8(y + 2)$$

- (ii) Hence write down the focal length of the parabola and the co-ordinates of its vertex and focus.

$$\text{focal length} = 2$$

$$\text{vertex } (4, -2)$$

$$\text{focus } (4, 0)$$

2

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#### Question 4 (START A NEW PAGE)

- (a) The price of gold, \$P, was studied over a period of  $t$  years.

- (i) Throughout the period of study  $\frac{dP}{dt} > 0$ . What does this say about the price of gold? 1

Price of gold is increasing

- (ii) It was also observed that the rate of change of the price of gold decreased over the period of study. 1

What does this statement imply about  $\frac{d^2P}{dt^2}$ ?

$$\frac{d^2P}{dt^2} < 0$$

- (b) A pool is being drained at a rate of  $240t - 9600$  litres/minute.

- (i) How long does it take before the draining of the pool stops? 1

$$240t - 9600 = 0$$

$$t = \frac{9600}{240}$$

$$= 40$$

$$\text{time} = 40 \text{ minutes}$$

- (ii) If the pool initially held 1920000 litres of water, find the volume of water in the pool after 15 minutes. 3

$$\frac{dV}{dt} = 240t - 9600$$

$$V = 120t^2 - 9600t + c$$

$$\text{when } t = 0, V = 1920000$$

$$1920000 = 0 - 0 + c$$

$$c = 1920000$$

$$V = 120t^2 - 9600t + 1920000$$

$$\text{when } t = 15$$

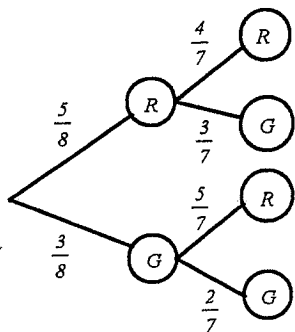
$$V = 120(15)^2 - 9600(15) + 1920000$$

$$= 1803000$$

$$\text{volume} = 1803000 \text{ l}$$

- (c) In a hat are 5 red and 3 green jellybeans. Jason reaches into the hat and randomly selects two jellybeans. Find the probability that:

(i) both selected jellybeans are red



$$P(\text{Red, Red}) = \frac{5}{8} \times \frac{4}{7}$$

$$= \frac{5}{14}$$

(ii) the chosen jellybeans are different colours.

$$P(\text{different}) = P(R, G) + P(G, R)$$

$$= \frac{5}{8} \times \frac{3}{7} + \frac{3}{8} \times \frac{5}{7}$$

$$= \frac{15}{28}$$

(d) The tangent to the curve  $y = 4\sqrt{x}$  at a point  $P$  has a slope of 2. Find the co-ordinates of point  $P$ .

$$y = 4x^{\frac{1}{2}}$$

$$y' = 2x^{-\frac{1}{2}}$$

$$= \frac{2}{\sqrt{x}}$$

$$\therefore 2 = \frac{2}{\sqrt{x}}$$

$$\sqrt{x} = 1$$

$$x = 1$$

$$y = 4\sqrt{1}$$

$$y = 4$$

$$P \text{ is } (1, 4)$$

1

Question 5 (START A NEW PAGE)

Marks

(a)



Pattern #1

Pattern #2

Pattern #3

The above patterns are made using small sticks. Pattern #1 requires 6 sticks, pattern #2 requires 11 sticks and pattern #3 requires 16 sticks.

(i) Write a formula for the number of sticks,  $U_n$ , needed to construct pattern number  $n$

$$U_n = 6 + 5(n-1)$$

$$U_n = 5n + 1$$

(ii) What is the largest pattern that can be constructed from 200 sticks?

$$5n + 1 = 200$$

$$n = 39.8$$

$$\therefore \text{can build pattern number } 39$$

(iii) How many sticks would be needed to construct all the patterns from pattern #1 to pattern #20?

$$S_{20} = \frac{20}{2} \{2(6) + 5(19)\}$$

$$= 1070$$

$\therefore$  need 1070 sticks

(b) (i) Show that the curves  $y = 1 + \sqrt{x}$  and  $y = 7 - x$  meet at the point  $(4, 3)$ .

$$\text{when } x = 4, \quad y = 1 + \sqrt{x}$$

$$= 1 + \sqrt{4}$$

$$= 3$$

$$\text{when } x = 4, \quad y = 7 - x$$

$$= 7 - 4$$

$$= 3$$

$\therefore$  point  $(4, 3)$  lies on both curves

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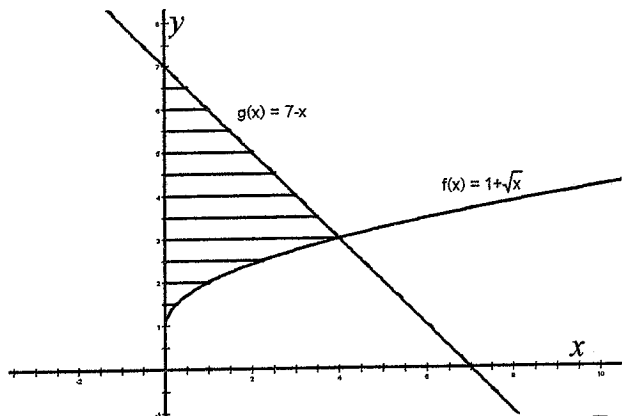
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- (ii) Sketch, on the same set of axes, the curves  $y=1+\sqrt{x}$  and  $y=7-x$  and shade the region bounded by these curves and the  $y$ -axis.



- (iii) Find the volume of the solid formed when the area bounded by the  $y=1+\sqrt{x}$ ,  $y=7-x$  and the  $y$ -axis is rotated one revolution about the  $y$ -axis.

Volume = Vol. of revolution of  $y=1+\sqrt{x}$  for  $1 \leq y \leq 3$  + Vol. of cone for  $3 \leq y \leq 7$

$$\text{Volume} = \pi \int_1^3 x^2 dy + \frac{1}{3} \pi (4)^2 (4) u^3$$

$$= \pi \int_1^3 (y-1)^4 dy + \frac{64\pi}{3} u^3$$

$$= \pi \left[ \frac{(y-1)^5}{5} \right]_1^3 + \frac{64\pi}{3} u^3$$

$$= \pi \left\{ \frac{32}{5} - 0 \right\} + \frac{64\pi}{3} u^3$$

$$\text{Volume} = \frac{416\pi}{15} u^3$$

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**Question 6 (START A NEW PAGE)**

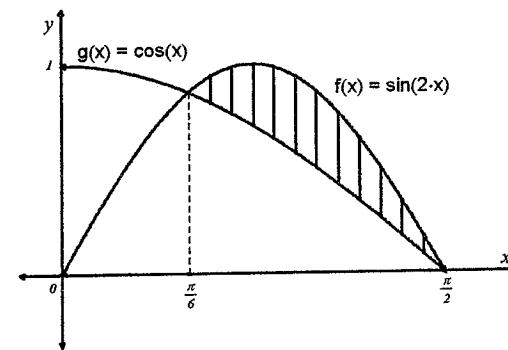
- (a) (i) Show that the curves  $y = \sin 2x$  and  $y = \cos x$  meet at the point with abscissa  $x = \frac{\pi}{6}$ .

$$\text{when } x = \frac{\pi}{6}, y = \sin\left(\frac{2\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$\text{when } x = \frac{\pi}{6}, y = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$\therefore$  point  $\left(\frac{\pi}{6}, \frac{\sqrt{3}}{2}\right)$  lies on both curves

- (ii) Sketch, on the same set of axes, the curves  $y = \sin 2x$  and  $y = \cos x$  in the domain  $0 \leq x \leq \frac{\pi}{2}$  and shade the region bounded only by these curves.



- (iii) Find the area of the shaded region described in (ii).

$$A = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin 2x - \cos x) dx$$

$$= \left[ -\frac{1}{2} \cos 2x - \sin x \right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= \left( -\frac{1}{2} \cos \pi - \sin \frac{\pi}{2} \right) - \left( -\frac{1}{2} \cos \frac{\pi}{3} - \sin \frac{\pi}{6} \right)$$

$$= \left( \frac{1}{2} - 1 \right) - \left( -\frac{1}{4} - \frac{1}{2} \right)$$

$$= \frac{1}{4}$$

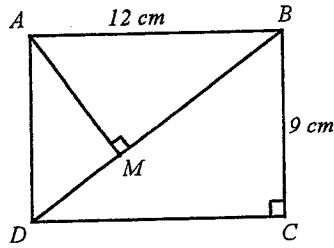
$$\text{Area} = \frac{1}{4} u^2$$

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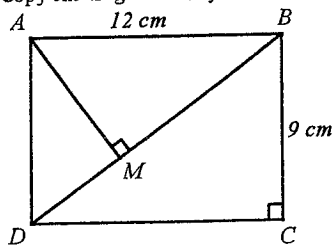
(b)



(Diagram not drawn to scale)

ABCD is a rectangle with  $AB = 12\text{ cm}$ ,  $BC = 9\text{ cm}$  and  $AM$  perpendicular to  $BD$

- (i) Copy the diagram onto your answer sheet and find the length of  $BD$



(Diagram not drawn to scale)

$$\begin{aligned} BD^2 &= DC^2 + CB^2 \quad (\text{Pythagoras' Theorem}) \\ &= 12^2 + 9^2 \\ &= 225 \\ BD &= 15\text{ cm} \end{aligned}$$

- (ii) Prove that  $\triangle ABM$  is similar to  $\triangle BDC$ .

$AB \parallel DC$  (opposite sides of a rectangle are parallel)  
 In  $\triangle ABM$  and  $\triangle BDC$   
 $\angle ABM = \angle BDC$  (alternate angles are equal as  $AB \parallel DC$ )  
 $\angle AMB = \angle DCB$  (both  $90^\circ$ )  
 $\triangle ABM \parallel \triangle BDC$  (equiangular)

- (iii) Hence find the length of  $BM$ . (Give reasons)

$$\begin{aligned} \frac{BM}{12} &= \frac{12}{15} \quad (\text{ratios of corresponding sides are equal in similar triangles}) \\ BM &= 9.6\text{ cm} \end{aligned}$$

Question 7 (START A NEW PAGE)

Marks

- (a) In an experiment, the amount of crystals,  $x$  grams, that dissolved in a solution after  $t$  minutes was given by:

$$x = 200(1 - e^{-kt})$$

- (i) After 3 minutes it was found that 50 grams of crystals had dissolved, hence find the value of  $k$

when  $t = 3$ ,  $x = 50$

$$50 = 200(1 - e^{-3k})$$

$$0.25 = 1 - e^{-3k}$$

$$e^{-3k} = 0.75$$

$$-3k = \ln(0.75)$$

$$k = -\frac{\ln(0.75)}{3} \quad (\approx 0.09589)$$

- (ii) At what rate were the crystals dissolving after 5 minutes? Give your answer to the nearest gram/minute

$$x = 200(1 - e^{-kt})$$

$$\frac{dx}{dt} = 200ke^{-kt} \quad \text{where } k = -\frac{\ln(0.75)}{3}$$

when  $t = 5$

$$\frac{dx}{dt} = 200 \left( -\frac{\ln(0.75)}{3} \right) e^{\left( \frac{\ln(0.75)}{3} \right) 5}$$

$$\approx 30.9779566$$

rate = 31 grams/minute

- (b) Two particles  $A$  and  $B$  start from the origin at the same time and move along a straight line so that their velocities in m/s at any time  $t$  seconds are given by:

$$v_A = t^2 + 2 \quad \text{and} \quad v_B = 8 - 2t$$

- (i) Show that the two particles never move with the same acceleration

$$a_A = \frac{dv_A}{dt}$$

$$= 2t$$

$$a_B = \frac{dv_B}{dt}$$

$$= -2$$

$$\text{if } a_A = a_B$$

$$2t = -2$$

$$t = -1$$

$$\text{but } t \geq 0$$

$\therefore$  no solution

$\therefore$  particles never move with the same acceleration

- (ii) Write a formula for the position of particle A at time  $t$  seconds.

$$x_A = \frac{1}{3}t^3 + 2t + c$$

$$\text{when } t = 0, x_A = 0 \Rightarrow c = 0$$

$$x_A = \frac{1}{3}t^3 + 2t$$

- (iii) After leaving the origin, find when the two particles are again at the same position

$$x_B = 8t - t^2 + d$$

$$\text{when } t = 0, x_B = 0 \Rightarrow d = 0$$

$$x_B = 8t - t^2$$

$$\text{when } x_A = x_B$$

$$\frac{1}{3}t^3 + 2t = 8t - t^2$$

$$t^3 + 3t^2 - 18t = 0$$

$$t(t + 6)(t - 3) = 0$$

$$t = 3 \quad (t > 0)$$

$\therefore$  at same position after 3 seconds

3

**Question 8 (START A NEW PAGE)**

- (i) Given that  $f(x) = x^2\sqrt{10-x}$ , show that  $f'(x) = \frac{5x(8-x)}{2\sqrt{10-x}}$ .

$$f(x) = x^2(10-x)^{\frac{1}{2}}$$

$$= (2x)\left\{(10-x)^{\frac{1}{2}}\right\} + (x^2)\left\{\frac{1}{2}(10-x)^{-\frac{1}{2}} \times -1\right\}$$

$$= 2x\sqrt{10-x} - \frac{x^2}{2\sqrt{10-x}}$$

$$= \frac{4x(10-x) - x^2}{2\sqrt{10-x}}$$

$$= \frac{40x - 5x^2}{2\sqrt{10-x}}$$

$$= \frac{5x(8-x)}{2\sqrt{10-x}}$$

3

- (ii) State the domain of the function  $y = x^2\sqrt{10-x}$ .

$$\text{Domain: } 10 - x \geq 0$$

$$\therefore x \leq 10$$

1

- (iii) Find all the stationary points on the curve  $y = x^2\sqrt{10-x}$  and determine their nature.

$$\text{for stat. pts } \frac{dy}{dx} = 0$$

$$\therefore \frac{5x(8-x)}{2\sqrt{10-x}} = 0$$

$$x = 0 \text{ or } 8$$

$$x = 0, y = 0 \text{ and } x = 8, y = 8^2\sqrt{10-8}$$

$$= 64\sqrt{2}$$

stat. pts. are  $(0,0)$  and  $(8, 64\sqrt{2})$

Test nature

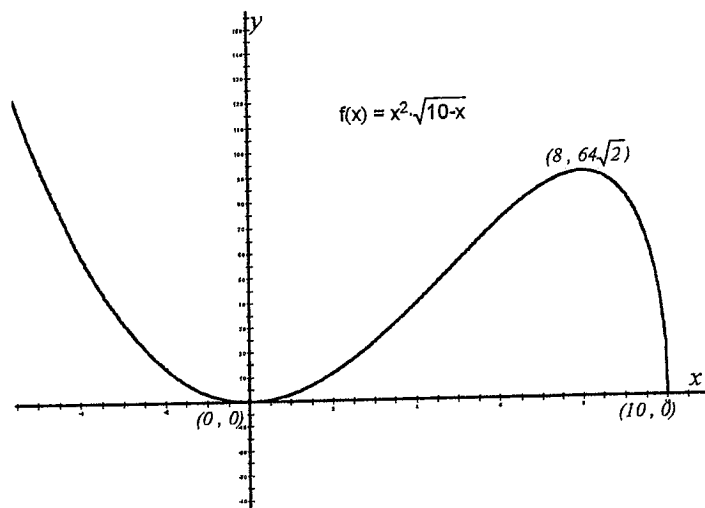
$x$	-1	0	1
$\frac{dy}{dx}$	$-\frac{45}{2\sqrt{11}}$	0	$\frac{35}{6}$
	< 0		> 0
	\ / local min		

$x$	7	8	9
$\frac{dy}{dx}$	$\frac{35}{2\sqrt{3}}$	0	$-\frac{45}{2}$
	> 0		< 0
	/ \ local max		

$\therefore$  local min. tp. at  $(0,0)$  and local max. tp. at  $(8, 64\sqrt{2})$

5

- (iv) Sketch the curve  $y = x^2\sqrt{10-x}$  indicating its intercepts with the co-ordinate axes and the position of all stationary points.



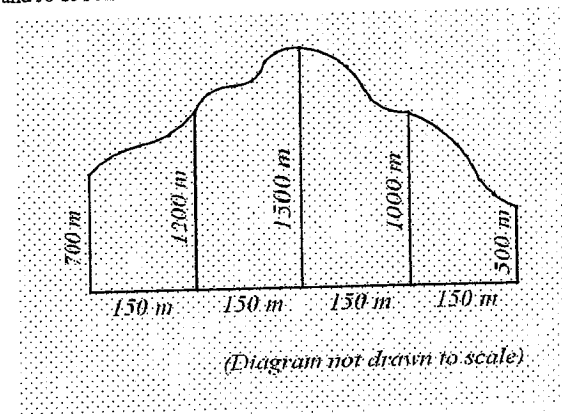
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**Question 9 (START A NEW PAGE)**

**Marks**

- (a) Prime land along a foreshore is to be reclaimed and developed as part of a housing estate. A plan of the land to be reclaimed is shown below

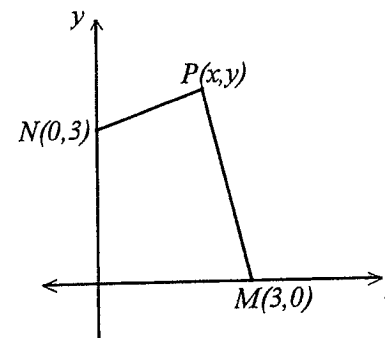
3



Use Simpson's rule with five function values to find the approximate area, expressed in hectares, of the land to be reclaimed

$$\begin{aligned} \text{area} &\approx \frac{300}{6}(700 + 4 \times 1200 + 1500) + \frac{300}{6}(1500 + 4 \times 1000 + 500) \text{ m}^2 \\ &\approx 650000 \text{ m}^2 \\ \text{area} &\approx 65 \text{ ha} \end{aligned}$$

- (b)



The point  $P(x,y)$  moves so that its distance from the point  $M(3,0)$  is always twice its distance from the point  $N(0,3)$ .

- (i) Show that the equation of the locus of the point  $P(x, y)$  is  $x^2 + 2x + y^2 - 8y + 9 = 0$ .

$$PM = 2PN$$

$$PM^2 = 4PN^2$$

$$(x-3)^2 + y^2 = 4\{x^2 + (y-3)^2\}$$

$$x^2 - 6x + 9 + y^2 = 4(x^2 + y^2 - 6y + 9)$$

$$x^2 - 6x + 9 + y^2 = 4x^2 + 4y^2 - 24y + 36$$

$$3x^2 + 6x + 3y^2 - 24y + 27 = 0$$

$$x^2 + 2x + y^2 - 8y + 9 = 0$$

- (ii) Give a geometric description of the locus.

$$x^2 + 2x + y^2 - 8y + 9 = 0$$

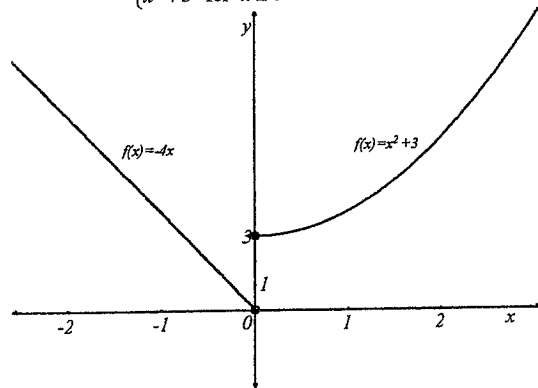
$$x^2 + 2x + y^2 - 8y = -9$$

$$x^2 + 2x + 1 + y^2 - 8y + 16 = -9 + 1 + 16$$

$$(x+1)^2 + (y-4)^2 = 8$$

locus is a circle with centre at  $(-1, 4)$  and radius  $= 2\sqrt{2}$

- (c) (i) Graph  $y = f(x)$  if  $f(x) = \begin{cases} -4x & \text{for } x < 0 \\ x^2 + 3 & \text{for } x \geq 0 \end{cases}$



- (ii) Evaluate  $f(-2) + f(2)$ .

$$\begin{aligned} f(-2) + f(2) &= \{-4(-2)\} + \{2^2 + 3\} \\ &= 8 + 7 \\ &= 15 \end{aligned}$$

- (iii) Solve for  $a > 0$ :  $f(a) = f(-a)$ .

$$a^2 + 3 = 4a$$

$$a^2 - 4a + 3 = 0$$

$$(a-1)(a-3) = 0$$

$$a = 1 \text{ or } 3$$

2

### Question 10 (START A NEW PAGE)

Marks

- (a) Abigail decides to save for a car. She creates a savings account into which she deposits \$100 at the beginning of each week. The account pays interest at a rate of 0.16% per week calculated on the value of the account at the end of each week.

- (i) Show that Abigail's account is worth  $62600(1.0016^n - 1)$  dollars at the end of  $n$  weeks.

3

Let  $A_n$  dollars be the value of Abigail's account at the end of  $n$  weeks

$$A_1 = 100 \times 1.0016$$

$$A_2 = (A_1 + 100) \times 1.0016$$

$$= 100 \times 1.0016^2 + 100 \times 1.0016$$

$$= 100(1.0016^2 + 1.0016)$$

$$A_3 = (A_2 + 100) \times 1.0016$$

$$= 100(1.0016^3 + 1.0016^2 + 1.0016)$$

$$A_n = 100(1.0016^n + 1.0016^{n-1} + \dots + 1.0016^2 + 1.0016)$$

$$= 100 \times \frac{1.0016(1.0016^n - 1)}{1.0016 - 1}$$

$$= \frac{100.16(1.0016^n - 1)}{0.0016}$$

$$A_n = 62600(1.0016^n - 1)$$

- (ii) Find the value of Abigail's account at the end of 52 weeks. Give your answer correct to the nearest dollar.

1

$$A_n = 62600(1.0016^n - 1)$$

$$\text{when } n = 52$$

$$A_{52} = 62600(1.0016^{52} - 1)$$

$$\approx 5426.60$$

Value of account = \$5427 (to the nearest dollar)

1

2

- (iii) Find the number of weeks needed for Abigail's account to accumulate \$20000.  
(Give your answer to nearest week)

$$20000 = 62600(1.0016^n - 1)$$

$$1.0016^n - 1 = \frac{20000}{62600}$$

$$1.0016^n = \frac{413}{313}$$

$$n \ln(1.0016) = \ln\left(\frac{413}{313}\right)$$

$$n = \frac{\ln\left(\frac{413}{313}\right)}{\ln(1.0016)}$$

$$\approx 173.416$$

$$\therefore \text{no. week} = 173$$

(Note: 174 weeks is possibly a more correct answer since at 173 weeks the account would be slightly less than \$20000)

- (b) When a ship is travelling at a speed of  $v$  km/hr, its rate of consumption of fuel in tonnes per hour is given by  $125 + 0.004v^3$ .

- (i) Show that on a voyage of 5000 km at a speed of  $v$  km/hr the formula for the total fuel used,  $T$  tonnes, is given by:

$$T = \frac{625000}{v} + 20v^2$$

$$\text{time} = \frac{5000}{v}$$

Amt. of fuel = time  $\times$  consumption rate

$$T = \frac{5000}{v} \times (125 + 0.004v^3)$$

$$T = \frac{625000}{v} + 20v^2$$

3

- (ii) Hence find the speed for the greatest fuel economy and the amount of fuel used at this speed. (Justify your answer.)

$$T = \frac{625000}{v} + 20v^2$$

$$\frac{dT}{dv} = -\frac{625000}{v^2} + 40v$$

$$\text{for stat. pt. } \frac{dT}{dv} = 0$$

$$-\frac{625000}{v^2} + 40v = 0$$

$$40v = \frac{625000}{v^2}$$

$$v^3 = 15625$$

$$v = 25$$

$$\frac{d^2T}{dv^2} = \frac{1250000}{v^3} + 40$$

$$\text{when } v = 25, \frac{d^2T}{dv^2} = \frac{1250000}{25^3} + 40 > 0$$

$\therefore$  concave up,  $\therefore$  local min. tp.

Since the function is continuous for  $v > 0$  and there is only one stat. pt. then the local min. tp. is the absolute minimum.

$\therefore$  minimum speed = 25 km/hr and amount fuel used = 37500 tonnes

1



This is the END of the examination paper

