

J.R.A.H.S. TRIAL HSC EXAMINATION 3/4 UNIT MATHEMATICS 1995

QUESTION 1 (Start a new page)

- (a) Express  $0.373737\dots$  as a proper fraction in simplest form.
- (b) Find the size of the acute angle between the lines  $2x + y = 5$  and  $3x - y = 1$ .
- (c) Find all solutions to :  $\frac{1}{x-2} \leq 4$ .
- (d) Differentiate with respect to  $x$ :  $y = \tan^{-1} 2x$ .
- (e) Solve  $2 \cos^2 x + 3 \sin x - 3 = 0$ , for  $0 \leq x \leq 2\pi$ .

QUESTION 2 (Start a new page)

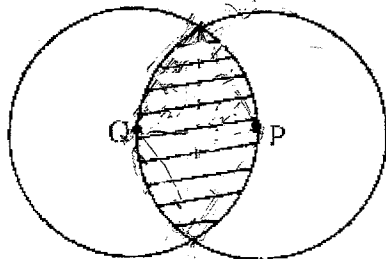
- (a) Prove the identity:  $\frac{\sin 2\theta}{1 + \cos 2\theta} = \tan \theta$ .
- (b) Given that  $\frac{dy}{dx} = \frac{1}{1+x^2}$ , and  $x=1$  when  $y=0$ , find  $y$  when  $x = \sqrt{3}$ .
- (c) Evaluate :  $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{3-x^2}}$
- (d) Prove by mathematical induction that  $3^{4n} - 1$  is divisible by 80 for all positive integers  $n$ .

QUESTION 3 (Start a new page)

- (a) Let the equation of motion of an object moving  $x$  metres along a straight line after  $t$  seconds be:  $x(t) = 4\sin 3t - 5\cos 3t$  ( $t \geq 0$ ). Show that its motion is Simple Harmonic, and find its period of motion.

- (b) Evaluate :  $\int_0^{\frac{\pi}{8}} \cos^2 2x dx$

- (c) In the diagram shown, the two circles are of radius 1 metre and pass through centres  $O$  and  $P$ . Find the area of their intersection (to two decimal places).



QUESTION 4 (Start a new page)

- (a) If 5% of monkeys are colourblind, what is the probability that a random sample of 20 monkeys should contain at least two colourblind monkeys? (Answer to three decimal places.)
- (b) A person invests \$1000 at the beginning of each year in a superannuation fund. If interest is paid at 9% per annum, find:
- the value of the investment at the end of 30 years.
  - how many years would elapse for the investment to be worth \$50,000.
- (c) Neatly sketch  $y = 3\cos^{-1}\pi x$ , and state its domain and range.

QUESTION 5 (Start a new page)

- (a) The acceleration of a particle is given by  $\frac{d^2x}{dt^2} = 16(1+x)$ , where  $x$  cm. is the displacement from the origin. When  $t = 0$ ,  $x = 0$  and  $v = 4$  cm./sec.
- Derive an expression for its velocity in terms of its displacement.
  - Deduce that its displacement function is  $x(t) = e^{4t} - 1$ .
- (b) Evaluate  $\cot 2\theta$  if  $\cot^2\theta - \cot\theta = 1$ .
- (c) ABCD and AEFG are two squares of different areas, and  $GD \perp BE$ . M is the mid point of DE.
- Give a reason why DE is the diameter of the circle with points A, D and E on its circumference.
  - Prove that BDEG is a cyclic quadrilateral.
  - Prove that  $AM \perp BG$ .

Copy the diagram below onto your answer sheet.

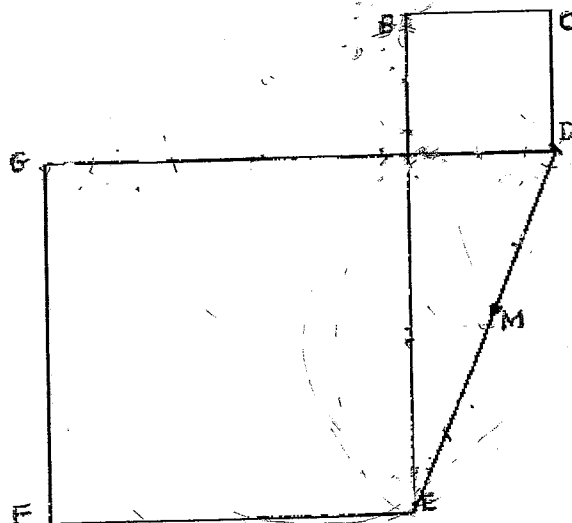
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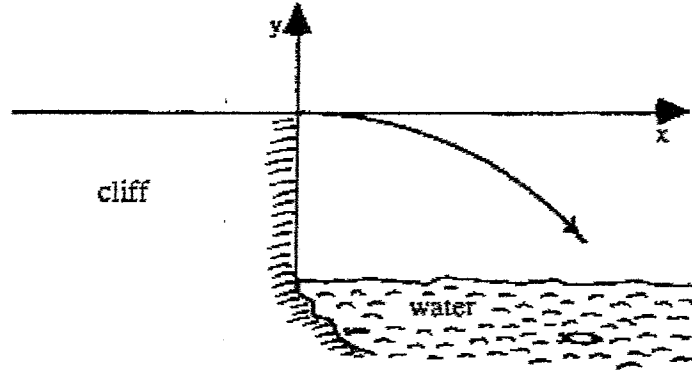
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  - (iii) Prove that  $AM \perp BG$ .

---Copy the diagram below onto your answer sheet.



**QUESTION 6 ( Start a new page )**

- (a) An object is projected horizontally from the top edge of a vertical cliff 40 metres above sea level with a velocity of 40m/s. ( Take  $g = 10 \text{ m/s}^2$  )



- (i) Using the top edge of the cliff as origin, prove that the parametric equations of the path of the object are:

$$x = 40t \qquad y = -5t^2$$

- (ii) Calculate when and where the object hits the water.  
 (iii) Find the velocity ( magnitude and direction ) of the object the instant it hits the water.

- (b) The inside of a vessel used for water has the shape of a solid of revolution obtained by the rotation of the parabola  $9y = 8x^2$  about the  $y$  - axis. The depth of the vessel is 8 cm.

(i) Prove that a volume of water  $h$  cm. from its bottom is  $\frac{9}{16} \pi h^2$ .

- (ii) If water is poured into the vessel at a rate of  $20 \text{ cm}^3/\text{sec.}$ , find the rate at which the level of water is rising when the vessel is half full.

**QUESTION 7 ( Start a new page )**

- (a) Two parametric points  $P(2p, p^2)$  and  $Q(2q, q^2)$  lie on the parabola  $x^2 = 4y$ , and the line through PQ is parallel to the line  $y = mx$ .

- (i) Show that  $p + q = 2m$ .  
 (ii) Derive the equation of the normal to the parabola at the point P.  
 (iii) Find the co-ordinates of N, the point of intersection of the normals from P and Q.  
 (iv) Determine the locus of N as the line PQ moves parallel to the line  $y = mx$ . State any restrictions on the locus of N.

- (b)  $A_n$  and  $B_n$  are two series given by :

$$A_n = 1^2 + 5^2 + 9^2 + 13^2 + \dots + (4n - 3)^2$$

$$B_n = 3^2 + 7^2 + 11^2 + 15^2 + \dots \qquad \text{for } n = 1, 2, 3, \dots$$

- (i) Find the  $n$ th term of  $B_n$ .  
 (ii) If  $S_{2n} = A_n - B_n$ , prove that  $S_{2n} = -8n^3$ .  
 (iii) Hence, or otherwise, evaluate :  
 $101^2 - 103^2 + 105^2 - 107^2 + \dots + 1993^2 - 1995^2$ .

**END OF PAPER**

SOLUTIONS

1 (a)  $\frac{37}{49}$

(b)  $\frac{5}{4}$

(c)  $x < 2, x \geq \frac{9}{4}$

(d)  $\frac{2}{1+4x^2}$

(e)  $\frac{5}{6}, \frac{5\pi}{6}, \frac{\pi}{2}$

2 (a) -

(b)  $\frac{\pi}{2}$

(c)  $\frac{\pi}{2}$

(d) -

3 (a)  $\tau = \frac{2\pi}{3}$

(b)  $\frac{\pi}{16} + \frac{1}{8}$

(c)  $23$  (40)

(d)  $0.264$  (30)

(e) (i) \$148 575

(ii) 19 (nearest yr)

(f) D:  $\frac{1}{2} \leq x \leq \frac{1}{2}$

R:  $0 \leq y \leq 3\pi$

7 (a) (i)  $y = 4(1+x)$

(ii) -

(b)  $\frac{1}{2}$

(c) -

6 (a) (i) -

(ii)  $80\sqrt{2}, t = 2\sqrt{2} \text{ sec}$

(iii)  $35^\circ$  with speed  $20\sqrt{6} \text{ m/s}$

(b)  $\frac{20\sqrt{2}}{45} \text{ m/s}$

7 (a) (i) -

(ii)  $x+py = p^2+2p$

(iii)  $N[-7pq(p+q), p^2+q+pq+q^2]$

(iv)  $y = \frac{x}{2m} + 4m^2 + 2$

Restriction  $y > 3m^2 + 2$   
 $x > -2m^2$

(b) (i)  $T_{3n} = (4n-1)^2$

(ii) -

(iii) -

TRAMS - HSC - 9/4 unit.

(1995) (1) (0)

1. a)  $0.3737... = \cancel{0.3} + 0.0$

~~$0.303030... + 0.070707...$~~

$= \left[ \frac{3}{10} + .003 + .00003... \right] + \left[ .07 + .0007... \right]$

$a = 0.3$   
 $r = .001$

$a = 0.07$   
 $r = .0001$

$+ \frac{0.3}{1 - 0.001} + \frac{0.07}{1 - .01}$

$= \frac{10}{33} + \frac{7}{99}$

$= \frac{37}{99}$

b)  $2x + y = 5$

$3x - y = 1$

$m = -2$

$m = 3$

$\tan \theta = \left| \frac{3 + 2}{1 - 3(2)} \right|$

$\tan \theta = \frac{5}{7-5} = 1$

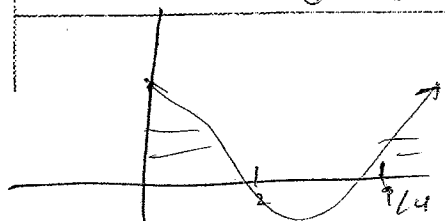
~~$\theta = 8.1^\circ$~~   $\theta = \pi/4$

c)  $\frac{(x-2)^2}{x-2} \leq 4(x-2)^2$

$0 \leq 4(x-2)^2 - (x-2)$

$0 \leq (x-2) [4(x-2) - 1]$

$0 \leq (x-2) [4x - 9]$



$x \leq 2$  or  $x \geq 9/4$

$$d) \quad y = \tan^{-1} 2x.$$

$$y' = \frac{2}{1 + 4x^2}.$$

$$e) \quad 2 \cos^2 x + 3 \sin x - 3 = 0$$

$$0 \leq x \leq 2\pi$$

$$= 2(1 - \sin^2 x) + 3 \sin x - 3 = 0.$$

$$2 - 2 \sin^2 x + 3 \sin x - 3 = 0.$$

$$0 = 2 \sin^2 x - 3 \sin x + 1$$

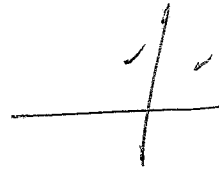
$$\begin{array}{r} 2 \sin x \\ \sin x \end{array} \quad \begin{array}{r} -1 \\ -1 \end{array}$$

$$0 = (2 \sin x - 1)(\sin x - 1)$$

$$\sin x = 1 \quad \left| \quad \sin x = \frac{1}{2}$$

$$x = \frac{\pi}{2}, \quad \left| \quad x = \frac{\pi}{6}, \quad \frac{5\pi}{6}$$

$$x = \frac{\pi}{2}, \quad \frac{\pi}{6}, \quad \frac{5\pi}{6}$$



$$2) \quad \frac{\sin 2\theta}{1 + \cos 2\theta} = \tan \theta$$

$$\text{LHS: } \frac{2 \sin \theta \cos \theta}{1 + 2 \cos^2 \theta - 1} = \tan \theta.$$

$$b) \quad (1, 0)$$

$$y = \int \frac{1}{1+x^2} dx$$

$$= (\tan^{-1} x) + c$$

$$0 = \tan^{-1} 1 + c$$

$$c = -\pi/4$$

$$y = \tan^{-1} x - \pi/4.$$

$$y = \tan^{-1} \sqrt{3} - \pi/4$$

$$= \frac{\pi}{3} - \frac{\pi}{4}$$

$$= \frac{4\pi - 3\pi}{12} = \frac{\pi}{12}$$



2

2. c)  $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{3-x^2}}$   $a = \sqrt{3}$

$$= \left( \sin^{-1} \frac{x}{\sqrt{3}} \right)_0^{\sqrt{3}}$$

$$= \sin^{-1} 1 - \sin^{-1} 0$$

$$= \frac{\pi}{2}$$

d)  $P(n): 3^{4n} - 1 = 80M$  ( $M$  is an integer)

Consider  $P(1)$ :

$$3^4 - 1 = 80 = 80(1)$$

$\therefore P(1)$  is true

Assume  $P(k)$  is true;

$$3^{4k} - 1 = 80M \quad \text{--- (A) } (N = M)$$

Consider  $P(k+1)$ :

$$3^{4k+4} - 1 = 80N \quad (N = \text{integer})$$

By (A);  $3^4 (3^{4k} - 1) + 3^4 - 1$

$$= 3^4 (80M) + 3^4 - 1$$

$$= 6480M + 80$$

$$= 80(81M + 1)$$

$$= 80N \quad (81M + 1 \text{ is an integer})$$



3. a)  $x(t) = 4\sin 3t - 5\cos 3t \quad (t \geq 0)$

$v = 12\cos 3t + 15\sin 3t$

$a = -36\sin 3t + 45\cos 3t$

$= -9(4\sin 3t + 5\cos 3t)$

$a = -9x \quad (\text{where } n = 3)$

$T = \frac{2\pi}{3} \text{ s}$

b)  $\int_0^{\pi/8} \cos^2 2x \, dx$

$= \frac{1}{2} \int_0^{\pi/8} (1 + \cos 4x) \, dx$

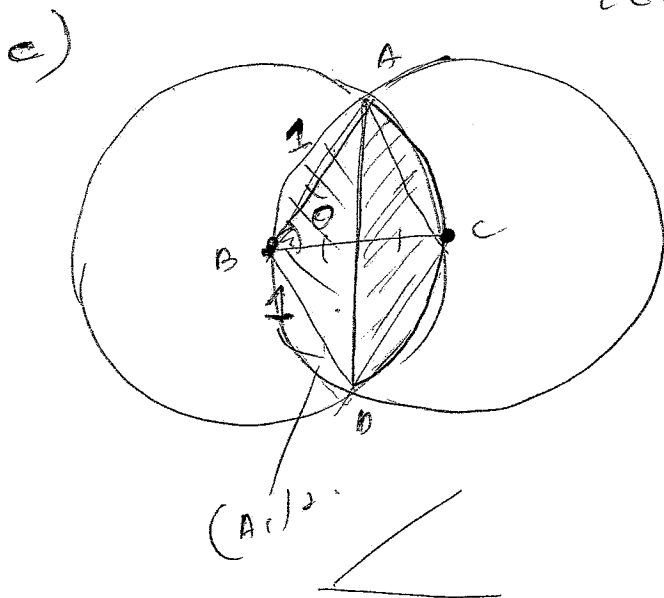
$= \frac{1}{2} \left[ x + \frac{1}{4} \sin 4x \right]_0^{\pi/8}$

$= \frac{1}{2} \left[ \frac{\pi}{8} + \frac{1}{4} \sin \frac{\pi}{2} \right] - 0$

$= \frac{\pi}{16} + \frac{\sin \pi/2}{8}$

$= \frac{\pi}{16} + \frac{1}{8} \checkmark$

$= \frac{\pi + 2}{16}$



$A = 2 \left( \frac{1}{2} r^2 (\theta - \sin \theta) \right)$   
 $= 2 \left( \frac{1}{2} (1)^2 (\theta - \sin \theta) \right)$

$= (\theta - \sin \theta)$

In  $\triangle ABC$ ;  $AB = AC = BC = 1$

$\therefore \triangle ABC$  is equilateral

$\therefore \angle ABC = 60^\circ$

$\angle CAB = 120^\circ = \frac{2\pi}{3}$

$A = \frac{2\pi}{3} - \sin 120$

$= \underline{\underline{1.23}} \checkmark$

4. a)  $p = 0.05$

$q = 0.95$

$(p+q)^{20}$

$1 - (P(0) + P(1))$

$= 1 - [20C_0 p^0 q^{20} + 20C_1 p q^{19}]$

$= 1 - [20C_0 (0.95)^{20} + 20C_1 (0.05)(0.95)^{19}]$

$= 0.264 \checkmark$

b) \$1000  $(1.09)$

i)  $A_n = 1000 (1.09)^n$

$A_1 = (1000(1.09) + 1000)(1.09)$

$= 1000(1.09 + 1.09^2)$

$A_{30} = 1000(1.09 + 1.09^2 + \dots + 1.09^{30})$

$a = 1.09$

$r = 1.09$

$n = 30$

$= 1000 \left[ \frac{(1.09) [(1.09)^{30} - 1]}{0.09} \right]$

$= \$148575.22 \checkmark$

ii)  $50000 = 1000 \frac{(1.09)^n - 1}{0.09}$

$5.1284 = (1.09)^n$

$\log 1.09 \quad 5.1284 = n$

$n = \frac{18.9}{0.09} \text{ yrs.}$

$\checkmark \approx 19 \text{ yrs.}$

$$5) \quad a = 16(1+x)$$

$$t=0; \quad x=0; \quad v=4$$

$$i) \quad 16(1+x) = \frac{dv}{dx} \left( \frac{1}{2} v^2 \right)$$

$$\frac{1}{2} v^2 = 16 \int (1+x) dx$$

$$\frac{1}{2} v^2 = 16 \left( x + \frac{x^2}{2} \right) + c$$

$$\frac{v^2}{2} = 16x + 8x^2 + c$$

$$x=0;$$

$$v=4$$

$$8 = c$$

$$\frac{v^2}{2} = 16x + 8x^2 + 8$$

$$v^2 = 32x + 16x^2 + 16$$

$$ii) \quad \frac{dx}{dt} = \sqrt{32x + 16x^2 + 16}$$

$$\frac{dt}{dx} = \frac{1}{\sqrt{32x + 16x^2 + 16}}^{-\frac{1}{2}}$$

$$t = \int \frac{1}{\sqrt{32x + 16x^2 + 16}}^{-\frac{1}{2}} dx$$

$$= \int \frac{1}{\sqrt{16(x^2 + 2x + 1)}} dx$$

$$= \int \frac{1}{\sqrt{16(x+1)^2}} dx$$

$$\frac{dx}{dt} = \frac{1}{4(x+1)} \quad \checkmark$$

$$\frac{dt}{dx} = \frac{1}{4(x+1)}$$

$$t = \frac{1}{4} \int \frac{1}{x+1} dx$$

$$t = \frac{1}{4} (\log_e(x+1)) + c$$

$$0 = \frac{1}{4} \ln 1 + c \quad \therefore c = 0$$

$$t = \frac{1}{4} \log_e(x+1)$$

$$4t = \log_e(x+1)$$

$$e^{4t} = x+1$$

$$x = e^{4t} - 1$$

$$x=0; \quad t=0$$



$$8.6) \cot 2\theta$$

$$\cot^2 \theta - \cot \theta = 1.$$

$$= \cot(\theta + \theta)$$

$$= \frac{1}{\tan 2\theta}$$

$$= \frac{1 - \tan^2 \theta}{2 \tan \theta}$$

$$= \frac{1}{2 \tan \theta} - \frac{\tan \theta}{2 \tan \theta}$$

$$= \frac{1}{2} \cot \theta - \tan \theta$$

$$\cot 2\theta$$

$$= \frac{1}{\tan 2\theta}$$

$$= \frac{1 - \tan^2 \theta}{2 \tan \theta}$$

$$= \frac{1}{2 \tan \theta} - \frac{\tan \theta}{2}$$

$$= \frac{1}{2} \cot \theta - \frac{1}{2} \tan \theta$$

$$= \frac{\cot^2 \theta - 1}{2 \cot \theta}$$

$$= \frac{\cot \theta}{2 \cot \theta}$$

$$= \frac{1}{2}$$



$$5. c) \angle DAE = 90^\circ \quad (GD \perp BE)$$

$\therefore ED$  is diameter  $\angle C$  in semicircle  $= 90^\circ$

$$ii) \triangle BAC \equiv \triangle DAE \quad (SAS)$$

$$\therefore \angle CBA = \angle EDA \quad (\text{corresp. } \angle\text{'s in } \triangle\text{'s})$$

$\therefore BDEG$  is a cyclic quad ( $\angle\text{'s in same segment are equal}$ )

iii) (check).

$$6. a) \ddot{x} = 0$$

$$\dot{x} = 40$$

$$x = 40t$$

$$\ddot{y} = -10$$

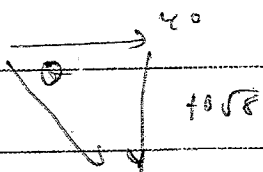
$$\dot{y} = -10t$$

$$y = -5t^2$$

$$ii) -40 = -10t^2$$

$$t = \sqrt{8} \text{ s.} \quad x = 40\sqrt{8}$$

$$iii) \text{ At } t = \sqrt{8}; \quad \dot{y} = -10\sqrt{8}; \quad \dot{x} = 40$$



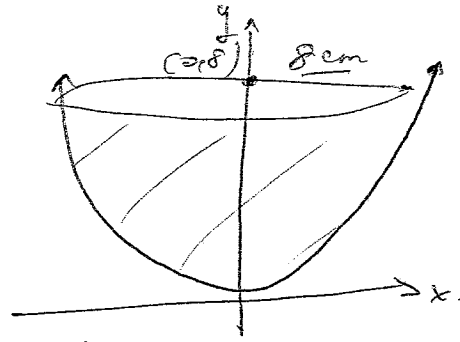
$$\tan \theta = \frac{10\sqrt{8}}{40}$$

$$\theta = 35.3^\circ \checkmark$$

$$v = \sqrt{40^2 + (10\sqrt{8})^2} = 48.9 \checkmark \text{ m.}$$

iii)

$$b) \quad y = \sqrt{8x^2}$$



$$i) \quad V = \pi \int_0^h x^2 dy.$$

$$= \pi \int_0^h \frac{9y}{8} dy.$$

$$= \frac{9\pi}{8} \left( \frac{y^2}{2} \right)_0^h$$

$$V = \frac{9\pi}{8} \left( \frac{h^2}{2} \right)$$

$$V = \frac{9\pi h^2}{16}$$

$$ii) \quad \frac{dV}{dt} = 20$$

$$\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$$

$$V = \frac{9\pi}{16} h^2$$

$$\frac{dV}{dh} = \frac{9\pi}{8} h^2 = \frac{9\pi \cdot 8^2}{8 \cdot 8^2} = 9\pi \quad (h=8)$$

$$= \frac{9\pi}{2}$$

$$\frac{dh}{dV} = \frac{2}{9\pi}$$

$$\frac{dh}{dt} = \frac{20}{9\pi} \quad 20 \times \frac{2}{9\pi} = \frac{40}{9\pi} \text{ cm/s.}$$

7

$$b) \quad A_n = 1^2 + 5^2 + 9^2 + 13^2 \dots + (4n-3)^2$$

$$B_n = 3^2 + 7^2 + 11^2 + 15^2 + \dots$$

$$ii) \quad B_n = (4n-1)^2 \quad \checkmark \quad \cancel{3^2}$$

$$iii) \quad S_n = A_n - B_n$$

$$A_n - B_n:$$

$$(1^2 + 5^2 + 9^2 \dots + (4n-3)^2) - (3^2 + 7^2 + 11^2 \dots + (4n-1)^2)$$

$$= 1^2 - 3^2 + 5^2 - 7^2 + 9^2 - 11^2 \dots + (4n-3)^2 - (4n-1)^2$$

$$= 1 - 9 + 25 - 49 + 81 - 121 \dots + (4n-3)^2 - (4n-1)^2$$

$$\cancel{16n^2 - 24n + 9} - \cancel{(16n^2 - 8n + 1)}$$

$$= (-16n + 8)$$

$n = ??$

$$\Rightarrow \begin{matrix} n=1 \\ [-8] + [-16(2)+8] + [-16(3)+8] + \dots + [-16n+8] \\ \underbrace{-8} \quad \quad \quad \underbrace{-24} \quad \quad \quad \underbrace{-40} \end{matrix}$$

$$= -8 - 24 - 40 - 46 \dots + [-16n+8]$$

$\therefore A_n - B_n$  is AP with  $a = -8$   
 $d = -16$   
 $n = n$

$$= \frac{n}{2} (2(-8) + (n-1)(-16))$$

$$= \frac{n}{2} (-16 - 16(n-1))$$

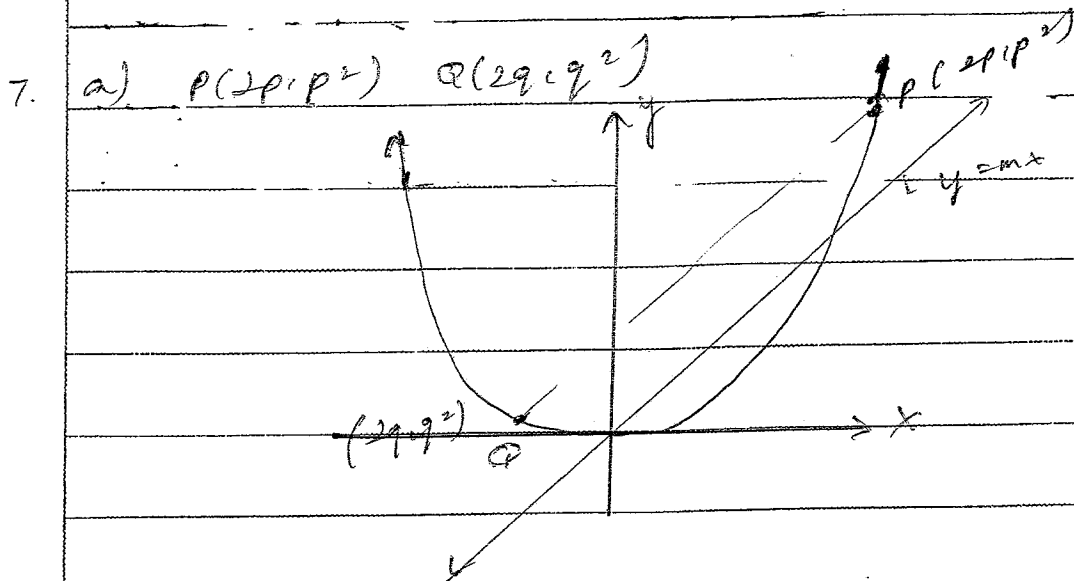
$$= \frac{n}{2} (-16 - 16n + 16)$$

$$= \underline{\underline{-8n^2}} \quad \checkmark \checkmark \checkmark$$

$$(4n-3) = 1001$$

$$4n = 1004$$

$$n = 251$$



i)  $m_{PQ} = m$

$$\frac{p^2 - q^2}{2p - 2q} = m$$

$$\frac{(p-q)(p+q)}{2(p-q)} = m$$

$$p+q = 2m.$$

ii)  $m = \frac{y}{x} = \frac{x^2/4}{x} = \frac{x}{4} = \frac{x}{2}$

$$m = -\frac{1}{p} \quad (2p, p^2)$$

$$y - p^2 = -\frac{1}{p}(x - 2p)$$

$$yp - p^3 = -(x - 2p)$$

$$\boxed{x + py = 2p + p^3}$$

iii)  $Q: x + qy = 2q + q^3$

$$N[-pq(p+q), p^2 + q^2 + pq + 2]$$

$$x = -pq(p+q) = \boxed{-2mpq = x}$$

$$y = p^2 + q^2 + pq + 2$$

$$y - 2 = (p+q)^2 - pq$$

$$y - 2 = (2m)^2 - pq$$

$$\boxed{y - 2 = 4m^2 - pq} \text{ (1)}$$

$$pq = \frac{x}{-2m} \text{ (2)}$$

$$y - 2 = 4m^2 - \frac{x}{-2m}$$

$$y - 2 = 4m^2 + \frac{x}{2m} \checkmark$$



$$b) A_n = 1^2 + 5^2 + 9^2 + 13^2 \dots + (4n-3)^2$$

$$B_n = 3^2 + 7^2 + 11^2 + 15^2 + \dots$$

$$ii) B_n = (4n-1)^2 \quad \text{✓} \quad \text{---} 3^2$$

$$iii) S_{2n} = A_n - B_n$$

$$A_n - B_n:$$

$$(1^2 + 5^2 + 9^2 \dots + (4n-3)^2) - (3^2 + 7^2 + 11^2 \dots + (4n-1)^2)$$

$$= 1^2 - 3^2 + 5^2 - 7^2 + 9^2 - 11^2 \dots + (4n-3)^2 - (4n-1)^2$$

$$= 1 - 9 + 25 - 49 + 81 - 121 \dots + (4n-3)^2 - (4n-1)^2$$

$$\underline{16n^2 - 24n + 9 - (16n^2 - 8n + 1)}$$

$$= [-16n + 8]$$

$n = ??$

$$\Rightarrow \begin{matrix} n=1 \\ [-16+8] + [-16(2)+8] + [-16(3)+8] + \dots + [-16n+8] \\ \quad \quad \quad -8 \quad \quad \quad -24 \quad \quad \quad -40 \end{matrix}$$

$$= -8 - 24 - 40 - 46 \dots + [-16n+8]$$

$\therefore A_n - B_n$  is AP with  $a = -8$   
 $d = -16$   
 $n = n$

$$= \frac{n}{2} (2(-8) + (n-1)(-16))$$

$$= \frac{n}{2} (-16 - 16(n-1))$$

$$= \frac{n}{2} (-16 - 16n + 16)$$

$$= \underline{\underline{-8n^2}} \quad \checkmark \checkmark \checkmark$$

$$\begin{aligned} (4n-3) &= 1001 \\ 4n &= 1004 \\ n &= 251 \end{aligned}$$



(see form)

$$\text{iii) } (101^2 + 103^2 + 105^2 - 107^2 \dots + 1993^2 - 1995^2)$$

$$= -8 \quad \underline{\underline{-8n^2}}$$

$$a = 101 \quad d = 4 \quad ; \quad (101 + (n-1)4)$$

$$(4n-3)^2 - (4n-1)^2$$

$$= -8(2n)^2$$



in  $\triangle PQR$ ;

$$\frac{PR}{\sin \angle QPR} = \frac{QR}{\sin \angle QRP} \quad \left. \right\} \quad \frac{PR}{\sin \angle QPR} = \frac{QR}{\sin \angle QRP}$$

$$QR = \frac{PR \sin \angle QPR}{\sin \angle QRP}$$

~~in  $\triangle QRS$~~ ; But  $\angle QPR = \angle QSR$ .

$\angle Q$