YEARLY EXAMINATION

YEAR 9 2006

MATHEMATICS

Time Allowed – 85 minutes

INSTRUCTIONS:

- All questions may be attempted
- Start each section on a new page
- Write your name at the top of each page
- Department of Education approved calculators are permitted
- Show all necessary working
- Marks may not be awarded for untidy or carelessly arranged work
- No grid paper is to be used unless provided with the examination paper
- **Teachers: Please collect each section separately.**

QUESTION 1.(16 Marks) — **Marks**

(a) Expand and simplify : $\left(2\sqrt{3}+5\right)^2$ — 2

(b) Simplify : $\dfrac{a^2-x^2}{x^2-a^2}$ — 1

(c) Simplify : $x^3\sqrt{x^8}$ — 2

(d) Find the remainder when $P(x)=x^7-8x^2+5$ is divided by $x+1$. — 1

(e) Fully factorise : x^4-16 — 2

(f) Simplify : $\dfrac{x^3+8}{x+2}$ — 2

(g) Solve : $0.5^x=16$ — 2

(h) Rationalise the denominator of : $\dfrac{3\sqrt{2}-1}{\sqrt{2}-1}$ — 2

(i) Find the value of $\sin\alpha$ when $\cos\alpha=\dfrac{3}{8}$. — 2

QUESTION 2.(16 Marks)

(a) The points $A(-1,7), B(1,9)$ and $C(-3,11)$ lie on the x-y plane.

(i) Show that ΔABC is an isosceles triangle and not an equilateral triangle. — 3

(ii) Find the co-ordinates of point M, the midpoint AB. — 2

(iii) Find the equation of AB in general form. — 2

(iv) Show that the equation of the perpendicular bisector of AB is given by : $x+y-8=0$ — 2

(v) Show that the point C lies on the perpendicular bisector of AB. — 1

(vi) Find the co-ordinates of the point D, if $ABCD$ is a parallelogram. — 2

(b) (i) Solve the equation : $2x^2-8x+7=0$. — 2

(ii) Graph $y=2x^2-8x+7$ showing all intercepts. — 2

QUESTION 3.(16 Marks)

(a) Which of the following is a monic polynomial of degree 3 and coefficient x^2 equal to 4 ? — 1

(i) $4x^3-7x+3$ (ii) $4x^2-x^3$

(iii) $x^3+4x^2+\sqrt{x}$ (iv) x^3+4x^2-7

(b) (i) Show that $x+1$ is a factor of $x^3+3x^2-97x-99$ — 1

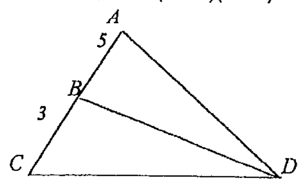
(ii) Hence solve $x^3+3x^2-97x-99=0$. — 3

(c) Solve by completing the square : $2x^2-10x-5=0$. — 3

(d) (i) Graph the polynomial $y=(x-3)(x+1)^2$ — 2

(ii) Hence solve $(x-3)(x+1)^2\geq 0$ — 2

(e)



Given $\angle ACD=\angle ADB$ then

(i) Prove $\Delta ACD\,|||\,\Delta ADB$ — 2

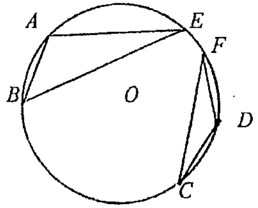
(ii) Find the length AD. — 2

QUESTION 4.(16 Marks) **Marks**

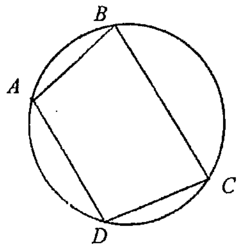
(a) Solve : $(3x+1)^2=9$ **2**

(b) Circle centre O with chords AB, CD, AE, BE, CF, FD and $AB=CD$. **3**

Prove $\angle AEB=\angle CFD$



(c) Circle with cyclic quadrilateral $ABCD$
With equal chords $AB=CD$.



(i) Using 4(b) or otherwise
Prove $BC\,//AD$ **3**

(ii) Prove $\angle BAD=\angle CDA$ **2**

(iii) Prove $\Delta ABD\equiv\Delta CDA$ **2**

(d) A polynomial has a remainders of 3 and -2 when divided by $x+1$ and $x-2$ respectively. **4**
Find the remainder when the polynomial is divided by x^2-x-2.

QUESTION 5.(16 Marks)

(a) Solve $\sqrt{4x+11}=8x-6$ **3**

(b) A factory has two machines X an Y which make widgets.
Machine X can make 100 widgits per hour at a cost of \$ 2.50 each and
machine Y can make 200 widgits per hour at a cost of \$ 2.00 each.
At least 1000 widgits are needed every day, and every widget is sold for \$ 3.00.
Each machine must be in operation for at least one hour per day.
The total hours of machine X and machine Y must be less than or equal to 8 hours per day.
Let machine X make x widgets per day and machine Y make y widgets per day.

(i) On a daily basis show that the inequalities for the manufacture of widget are :
$100\le x\le 700$, $200\le y\le 1400$, $x+y\ge 1000$ and $400\le 2x+y\le 1600$. **2**

(ii) Graph all the inequalities in terms of x and y using a scale of 1cm = 400 widgits.
Clearly shade the region for the manufacture of widgets. **2**

(iii) Find the co-ordinates of each vertice of the region. **4**

(iv) When all widgets are sold find the profit in terms of x and y. **1**

(v) How many widgets from each machine give the minimum profit ? **1**

(c) The equation of the tangent to a polynomial $y=P(x)$ at $x=a$ is given by the equation **3**
$y=R(x)$, where $R(x)$ is the remainder when $P(x)$is divided by $(x-a)^2$.

(i) Find the equation of the tangent to the polynomial $y=2x^3-3x^2+4x-5$ at $x=1$.

(ii) Hence state the value of the gradient of the tangent at $x=1$.

End Of Exam

[illegible] Year [illegible]

(a) $(2\sqrt{3}+5)^2 = 12 + 20\sqrt{3} + 25$
$= 37 + 20\sqrt{3}$

(b) $\frac{a^2-x^2}{x^2-a^2} = -1$

(c) $x^3\sqrt{x^8} = x^3 x^4$
$= x^7$

(d) $R(x) = P(-1)$
$= (-1)^7 - 8(-1)^2 + 5$
$= -1 - 8 + 5$
$= -4$

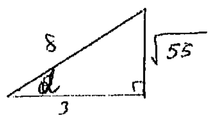
(e) $x^4 - 16 = (x^2-4)(x^2+4)$
$= (x-2)(x+2)(x^2+4)$

(f) $\frac{x^3+8}{x+2} = \frac{(x+2)(x^2-2x+4)}{x+2}$
$= x^2 - 2x + 4$

g) $2^{-x} = 16$
$2^{-x} = 2^4$
$x = -4$

h) $\frac{(3\sqrt{2}-1)}{\sqrt{2}-1}\frac{(\sqrt{2}+1)}{\sqrt{2}+1}$
$= \frac{6+3\sqrt{2}-\sqrt{2}-1}{2-1}$
$= 5 + 2\sqrt{2}$

i)



$\sin\alpha = \frac{\sqrt{55}}{8}$

2(a)
(i) $AB = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$
$= \sqrt{(-2)^2+(-2)^2}$
$= \sqrt{8}$ units.
$AC = \sqrt{2^2+4^2}$
$= \sqrt{20}$ units
$BC = \sqrt{4^2+2^2}$
$= \sqrt{20}$ units
∴ isosceles only $AC = BC = 2\sqrt{5}$ units

(ii) $M_{AB} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
$= (0, 8)$

(iii) $m_{AB} = \frac{y_1-y_2}{x_1-x_2}$
$= \frac{9-7}{1+1}$
$= 1$
$y - y_1 = m(x-x_1)$
$y - 7 = 1(x+1)$
$x - y + 8 = 0$

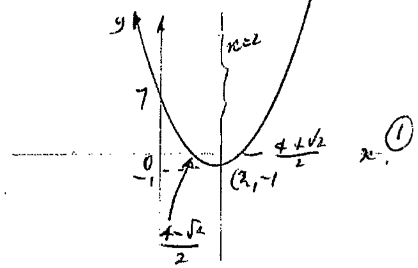
(iv) $m_2 = \frac{-1}{m_1}$
$= -1$
∴ $y - y_1 = m(x-x_1)$
$y - 8 = -1(x-0)$
$x + y - 8 = 0$

(v) $LHS = x + y - 8$
$= -3 + 11 - 8$
$= 0$
∴ LHS = RHS
∴ C lies on line.

(vi) $M_{AC} = M_{BD}$
$(-2, 9) \equiv \left(\frac{x+1}{2}, \frac{y+9}{2}\right)$
∴ $x = -5$
$y = 9$ ⇒ $D(-5, 9)$

2(c)(i) $2x^2 - 8x + 7 = 0$
$x = \frac{8 \pm \sqrt{64-56}}{4}$
$= \frac{8 \pm \sqrt{8}}{4}$
$= \frac{4 \pm \sqrt{2}}{2}$ (2)

Axis $x = \frac{-b}{2a}$
$x = 2$ ⇒ $y = -1$
vertex $(2, -1)$ (1)



(1)

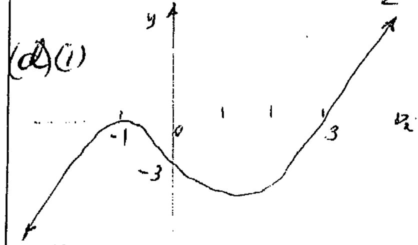
3(a) (iv)

(b)(i) $R = P(-1)$
$= (-1)^3 + 3(-1)^2 - 97(-1) - 99$
$= -1 + 3 + 97 - 99$
$= 0$
∴ $x+1$ is factor (1)

(ii) $P(x) = (x+1)(x^2+2x-99)$
$= (x+1)(x+11)(x-9)$
∴ $P(x) = 0$
$x = -1$ or -11 or 9 (3)

$2x^2 - 10x = 5$
$x^2 - 5x = \frac{5}{2}$
$x^2 - 5x + \left(\frac{5}{2}\right)^2 = \frac{5}{2} + \frac{25}{4}$
$\left(x - \frac{5}{2}\right)^2 = \frac{35}{4}$
$x - \frac{5}{2} = \pm\frac{\sqrt{35}}{2}$
$x = \frac{5 \pm \sqrt{35}}{2}$

(d)(i)



(d)(ii) $x \geq 3$ or $x = -1$

(e) In ΔACD and ΔADB
$\angle A$ is common
$\angle ACD = \angle ADB$ (data)
∴ $\Delta ACD \parallel\!\mid \Delta ADB$ (equiangular)
∴ $\frac{AD}{AB} = \frac{AC}{AD}$ [corresponding sides of similar triangles are in the same ratio
$AD^2 = AB \cdot AC$
$= 5 \cdot 8$
$AD = \sqrt{40}$
$= 2\sqrt{10}$ units.

(a) $(3x+1)^2 = 9$

$3x+1 = 3$ or $3x+1 = -3$

$3x = 2$ $\quad$ $3x = -4$

$x = \frac{2}{3}$ & $x = -1\frac{1}{3}$

) $\angle AOB = \angle COD$ [Equal chords subtend equal angles at the centre]

$\angle AOB = 2\angle AEB$ [Angle at the centre is twice the angle at the circumference standing on the same arc]

Similarly

$\angle COD = 2\angle CFD$

$2\angle AEB = 2\angle CFD$

$\angle AEB = \angle CFD$

) (i) Join AC

$\angle BCA = \angle CAD$ (by 4(b))

$BC \parallel AD$ [Alternate angles are equal]

) $\angle BAC = \angle BDC$ (Angles in the same segment are equal)

$\angle CAD = \angle BDA$ (by 4(b))

$\angle BAC + \angle CAD = \angle BDC + \angle BDA$

$\angle BAD = \angle CDA$

In $\triangle ABD$ and $\triangle CDA$

AD is common

$AB = CD$ (Data)

$\angle BAD = \angle CDA$ (by 4 c(ii))

$\therefore \triangle ABD \equiv \triangle CDA$ (SAS)

) $P(x) = (x+1)(x-2)Q(x) + ax + b$

$3 = -a + b$ $\quad$ $a = -\frac{5}{3}$ $\quad$ $b = \frac{4}{3}$

$-2 = 2a + b$ $\quad$ $\therefore R = -\frac{5x}{3} + \frac{4}{3}$

(a) $\sqrt{4x+4} = 8x-6$ $\quad$ $x \geq \frac{3}{4}$

$4x+4 = 64x^2 - 96x + 36$

$64x^2 - 100x + 25 = 0$

$(4x-5)(16x-5) = 0$

$x = \frac{5}{4}$ or $x = \frac{5}{16}$ but $x \geq \frac{3}{4}$

$\therefore x = 1\frac{1}{4}$ only

b(i) Min x is 1 hour ⇒ 100

Max x is 7 hours ⇒ 700

i.e. $\{100 \leq x \leq 700\}$

Min y is 1 hour ⇒ 200

Max y is 7 hours ⇒ 1400

$\{200 \leq y \leq 1400\}$

Total minimum $x + y \geq 1000$

1000 units widgets

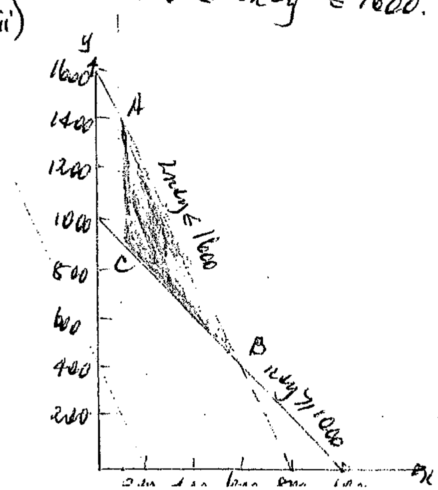
Total hours = 8 $\quad$ Min hours 2.

Hrs m/c X + hrs m/c Y

$2 \leq \frac{x}{100} + \frac{y}{200} \leq 8$

$400 \leq 2x + y \leq 1600.$

ii)



iii) for A $\quad$ x = 100 $\quad$ y = 1400

A(100, 1400)

for C $\quad$ x = 100 $\quad$ y = 900

C(100, 900)

for B $\quad$ x = 600 $\quad$ y = 400

B[600, 400]

(iv) $P = \$(0.5x + y)$

(v) At A $\quad P = \$1450$

B $\quad P = \$700$

C = $\quad P = \$950$

i.e. Minimum Profit

m/c X makes 600 widgets

m/c Y makes 400 widgets

(c) $(x-1)^2 = x^2 - 2x + 1$

$$
\begin{array}{r}
2x + 1 \\
x^2 - 2x + 1 \overline{)\, 2x^3 - 3x^2 + 4x - 5} \\
2x^3 - 4x^2 + 2x \\
\hline
x^2 + 2x - 5 \\
x^2 - 2x + 1 \\
\hline
4x - 6
\end{array}
$$

Eqn Tangent $y = 4x - 6$

i.e. hence gradient $m = 4$