

SECTION A (30 marks) – 30 Multiple choice questions. (1 mark each)
 Answer questions on the answer sheet provided with the examination paper.

(1) P and Q are the points $(3, -2)$ and $(-5, 4)$ respectively. The length of the interval PQ is

- (A) $\sqrt{8}$ units (B) 5 units (C) 10 units (D) 12 units

(2) $\sqrt{3} + \sqrt{3} =$

- (A) 3 (B) $2\sqrt{3}$ (C) $\sqrt{6}$ (D) 6

(3) $a \times b \times a \times b \times a \times b = ?$

- (A) $3ab$ (B) $3a \times 3b$ (C) ab^3 (D) $(ab)^3$

(4) Factorise $x^2 - (y+4)^2$

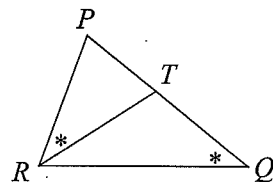
- (A) $(x-y+4)(x+y+4)$ (B) $(x-y+4)(x+y-4)$
 (C) $(x-y-4)(x+y+4)$ (D) $(x-y-4)(x-y+4)$

(5) $p^2 - 5p + 6$ expressed as a product of factors is

- (A) $(p-3)(p-2)$ (B) $(p-3)(p+2)$
 (C) $(p+3)(p-2)$ (D) $(p+3)(p+2)$

(6) $\triangle PQR$ is similar to $\triangle PRT$ with $\angle PQR = \angle PRT$.

Then $\frac{QR}{RT} =$



- (A) $\frac{PQ}{PT}$ (B) $\frac{PT}{PR}$ (C) $\frac{RT}{PT}$ (D) $\frac{PR}{PT}$

(7) In the diagram, the bearing of A from B is

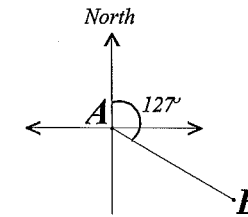


Diagram not to scale

- (A) $N37^\circ W$ (B) $N53^\circ W$ (C) $S37^\circ E$ (D) $S53^\circ E$

(8) Make y the subject of $2y = 1 - xy$

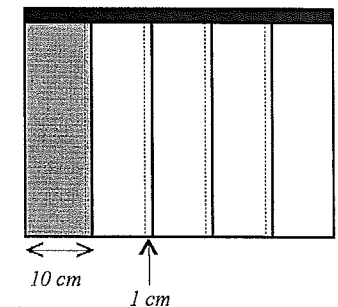
- (A) $y = \frac{1-x}{3}$ (B) $y = \frac{1}{2+x}$ (C) $y = \frac{1-xy}{2}$ (D) $y = \frac{1-2y}{x}$

(9) $8^{\frac{1}{3}} =$

- (A) $\frac{1}{8^3}$ (B) $\frac{1}{2}$ (C) 2 (D) $2\frac{2}{3}$

(10) Vertical blinds 10 centimetres wide overlap by 1 centimetre when they are closed. (see diagram)

Which expression represents the width, in centimetres, covered by n blinds when they are closed?

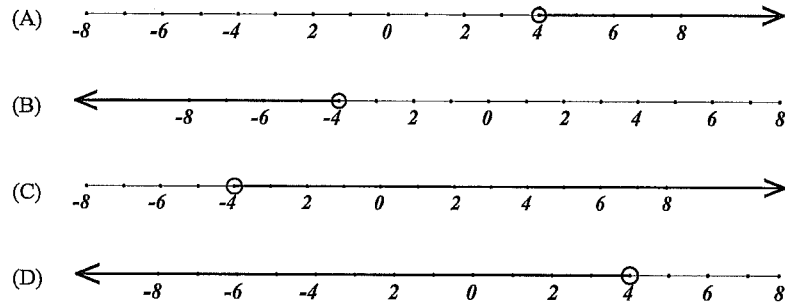


- (A) $9n$ (B) $9n+1$ (C) $10n-9$ (D) $10n-1$

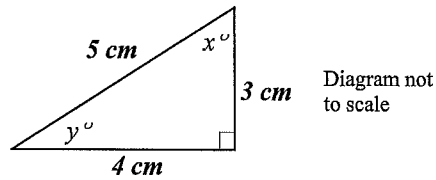
(11) If $x - 2y = 7$, what is the value of y when $x = 3$?

- (A) 5 (B) 2 (C) -2 (D) -5

(12) Which one of the following represents the solution set of the inequality $11 - 4x > 3 - 2x$?



(13) For the diagram at the right, which of the following ratios is the largest?

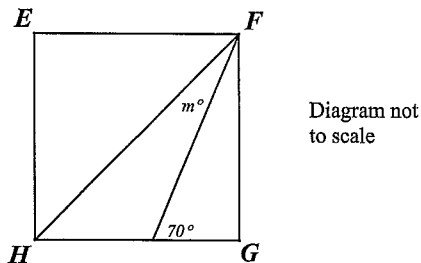


- (A) $\sin x^\circ$ (B) $\tan x^\circ$ (C) $\sin y^\circ$ (D) $\tan y^\circ$

(14) The solutions of the equation $m(2m - 1) = 0$ are

- (A) $m = 0$ and $m = \frac{1}{2}$ (B) $m = 0$ and $m = 2$
 (C) $m = 1$ and $m = 0$ (D) $m = 1$ and $m = \frac{1}{2}$

(15) $EFGH$ is a square. The value of m is



- (A) 20 (B) $22\frac{1}{2}$ (C) 25 (D) 35

(16) The equation of a circle with centre $(-1, 2)$ and radius 9 units is

- (A) $(x - 1)^2 + (y + 2)^2 = 9$ (B) $(x - 1)^2 + (y + 2)^2 = 81$
 (C) $(x + 1)^2 + (y - 2)^2 = 9$ (D) $(x + 1)^2 + (y - 2)^2 = 81$

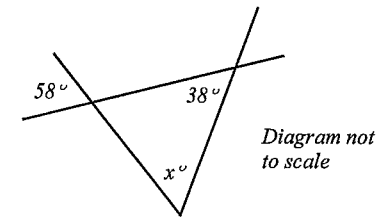
(17) A shipping container of mass T tonnes is filled with x car engines, each of mass y kilograms. The total mass, in tonnes, of the container and the engines is

- (A) $T + \frac{xy}{1000}$ (B) $\frac{T}{1000} + xy$ (C) $\frac{T + xy}{1000}$ (D) $1000T + xy$

(18) $\sqrt{36n^{16}} =$

- (A) $6n^4$ (B) $6n^8$ (C) $18n^4$ (D) $18n^8$

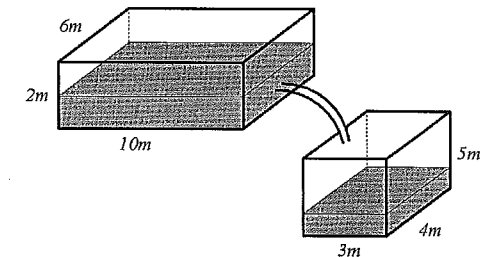
(19) In the diagram, the value of x is



- (A) 20 (B) 38 (C) 58 (D) 84

(20) Water is flowing from the top tank into the bottom tank at a constant rate. The depth of water in the top tank is falling at a rate of 10 centimetres per hour.

At what rate is the depth of water rising in the bottom tank?



- (A) 2 cm/h (B) 20 cm/h (C) 25 cm/h (D) 50 cm/h

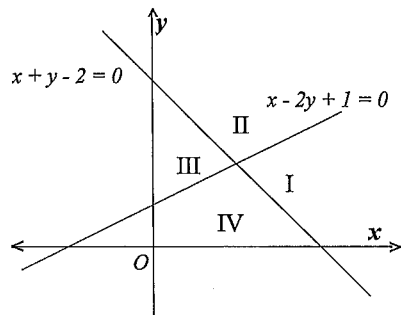
(21) The solutions of the equation $t^2 - 2t - 8 = 0$ are

- (A) $t = -4$ and $t = -2$ (B) $t = 4$ and $t = -2$
 (C) $t = 4$ and $t = 2$ (D) None of these

(22) If both of the inequalities

$$x - 2y + 1 \geq 0 \text{ and } x + y - 2 \leq 0$$

are graphed on the same diagram, which of the regions will be shaded?



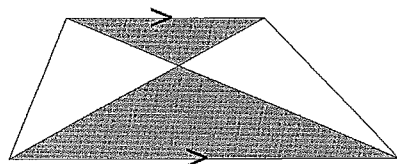
- (A) I, II and III only (B) I, II and IV only
 (C) I, III and IV only (D) II, III and IV only

(23) When a number N is divided by 5 the remainder is 3. Therefore, when $2N$ is divided by 5 the remainder is

- (A) 1 (B) 2 (C) 3 (D) 4

(24) In this figure,

- I. the shaded triangles must be similar
 II. the unshaded triangles must be similar



- (A) I only is true (B) II only is true
 (C) Both I and II are true (D) Neither I nor II is true

(25) $\frac{1}{3+\sqrt{2}}$ is equivalent to

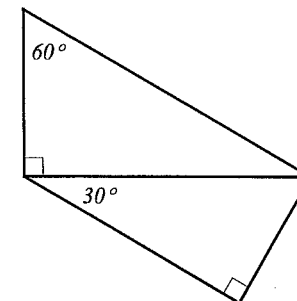
- (A) $3-\sqrt{2}$ (B) $\frac{1}{3}+\frac{1}{\sqrt{2}}$ (C) $\frac{3+\sqrt{2}}{7}$ (D) $\frac{3-\sqrt{2}}{7}$

(26) The solutions of the quadratic equation $x^2 + 5x - 3 = 0$ are

- (A) $x = \frac{5+\sqrt{37}}{2}$ and $x = \frac{5-\sqrt{37}}{2}$ (B) $x = \frac{-5+\sqrt{37}}{2}$ and $x = \frac{-5-\sqrt{37}}{2}$
 (C) $x = \frac{-5+\sqrt{13}}{2}$ and $x = \frac{-5-\sqrt{13}}{2}$ (D) $x = \frac{5+\sqrt{13}}{2}$ and $x = \frac{5-\sqrt{13}}{2}$

(27) In this figure

- I. the two triangles are similar
 II. the two triangles are congruent



- (A) I only is true (B) II only is true
 (C) Both I and II are true (D) Neither I nor II is true

(28) The interval joining the points $P(1, -2)$ and $Q(-3, 0)$ is divided internally by a point R in the ratio 5:4. Which of the following are the correct coordinates for R ?

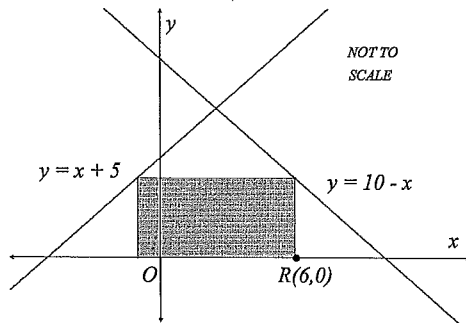
- (A) $(-\frac{11}{9}, -\frac{8}{9})$ (B) $(-\frac{5}{3}, -\frac{1}{3})$
 (C) $(-\frac{7}{9}, -\frac{10}{9})$ (D) $(-5\frac{1}{2}, -4)$

(29) If $ax + b > c$ for real numbers a, b, c and x , which of the following is true?

- (A) $x > \frac{c-b}{a}$ for all values of a, b and c
 (B) $x > \frac{c-b}{a}$ provided $a \neq 0$
 (C) $x > \frac{c-b}{a}$ for $a > 0$, and $x < \frac{c-b}{a}$ for $a < 0$
 (D) $x > \frac{c-b}{a}$ for $x > 0$, and $x < \frac{c-b}{a}$ for $x < 0$

(30) R is the point with coordinates $(6, 0)$.

The area of the shaded rectangle is



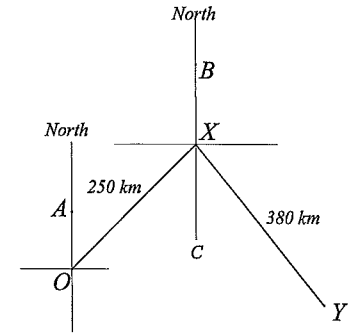
- (A) $20 u^2$ (B) $24 u^2$ (C) $28 u^2$ (D) $60 u^2$

SECTION B (40 marks) – Write your solutions to Q31-34 on your own paper showing all working.

Question 31(a) (START A NEW PAGE)

A plane flies 250 km from point O to point X on a bearing of $050^\circ T$. At point X the plane changes direction and flies 380 km on a bearing of $140^\circ T$ to point Y .

- (i) Copy the diagram onto your answer sheet and clearly indicate the size of $\angle AOX$ and $\angle BXY$.
 (ii) Prove that $\angle OXY = 90^\circ$.
 (iii) Find the bearing of point Y from point O . Give your answer correct to the nearest degree.



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Question 31(b)

The volume, $V \text{ cm}^3$, of a gas is directly proportional to the temperature, $T^\circ \text{ Kelvin}$, of the gas and inversely proportional to the pressure, P atmospheres, applied to the gas. A gas subject to a temperature of 20° K and a pressure of 5 atmospheres has a volume of 400 cm^3 .

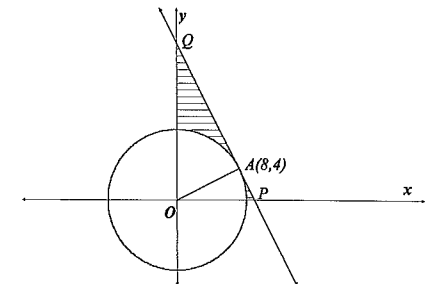
- (i) Write a formula relating the variables V, T and P .
 (ii) If the temperature is increased to 35° K and the pressure is decreased to 2 atmospheres, find the new volume of the gas.

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Question 32(a) (START A NEW PAGE)

PQ is a tangent to the circle at the point $A(8, 4)$.

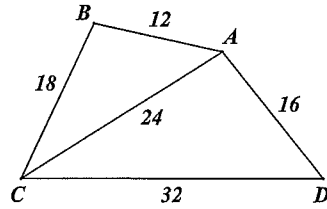
- (i) Find the equation of the line PQ .
 (ii) Find the coordinates of the points P and Q .
 (iii) Find the exact value of the shaded area, which is the area bounded by the tangent PQ , the coordinate axes and the circle.



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Question 32(b)

Copy the diagram onto your answer sheet and prove that $\triangle ABC$ and $\triangle ACD$ are similar.



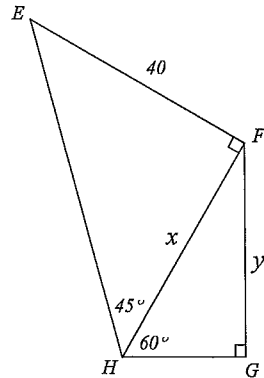
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Question 33(a) (START A NEW PAGE)

In $\triangle EFH$ and $\triangle FGH$, $EF = 40$, $\angle EHF = 45^\circ$ and $\angle FHG = 60^\circ$.

(i) Find the exact length of FH .

(ii) Hence find the exact length of FG .



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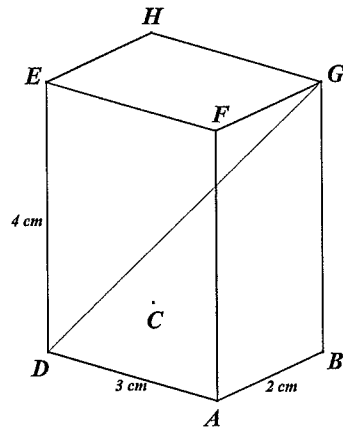
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Question 33(b)

$ABCDEFGH$ is a rectangular prism with sides of lengths 2 cm , 3 cm and 4 cm .

(i) Find the exact length of diagonal GD .

(ii) Find the size of $\angle GDA$ correct to the nearest minute.



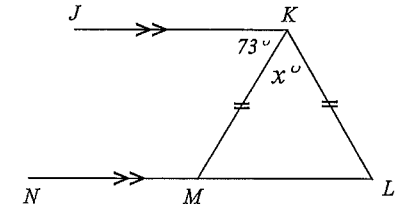
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Question 34(a) (START A NEW PAGE)

The points N , M and L are collinear. $JK \parallel NL$, $KM = KL$ and $\angle JKM = 73^\circ$.

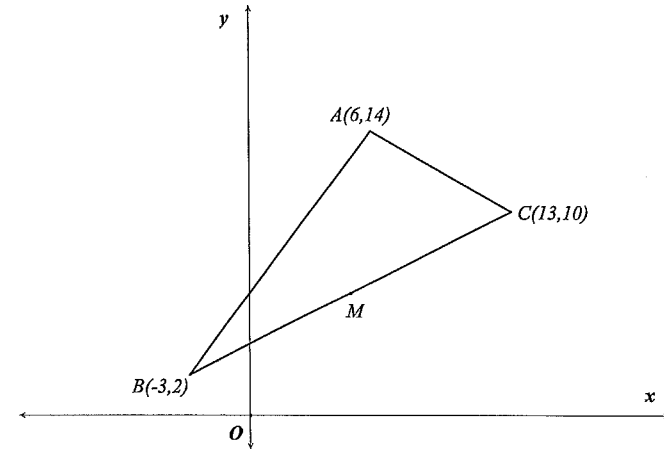
Copy the diagram onto your answer sheet and find the value of x , giving reasons.



3

Question 34(b)

$\triangle ABC$ has vertices $A(6,14)$, $B(-3,2)$ and $C(13,10)$. M is the midpoint of side BC .



(i) Find the equation of the line BC .

(ii) Find the perpendicular distance from point A to the line BC .

(iii) Find the coordinates of point M , the midpoint of side BC .

(iv) If the acute angle between the lines AM and BC is θ , find the exact value of $\sin \theta$.

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THIS IS THE END OF THE EXAMINATION PAPER

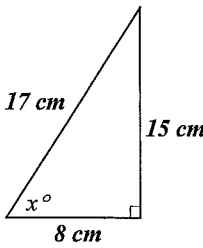
EXTRA MULTI CHOICE QUESTIONS

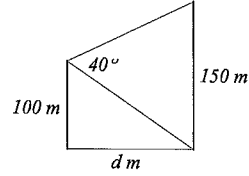
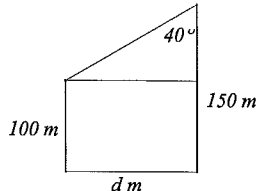
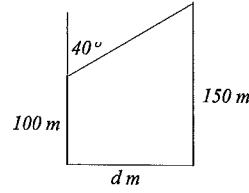
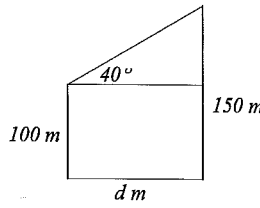
(31) If $f(x) = 3^x$, which of the following is equivalent to $f(-x)$?

- (A) $f(x)$
- (B) $\frac{1}{f(x)}$
- (C) $-f(x)$
- (D) $\frac{3}{f(x)}$

(32) Which of the following simplifications is false?

- (A) $\frac{x(x-1)}{x-1} = x$
- (B) $\frac{x^2-1}{x+1} = x-1$
- (C) $\frac{x^2+3}{x+3} = x+1$
- (D) $\frac{5x}{x(x+1)} = \frac{5}{x+1}$

	(33) For the figure at the right, $\sin x^\circ = ?$	
(A)	$\frac{15}{17}$	
(B)	$\frac{8}{17}$	
(C)	$\frac{15}{8}$	
(D)	$\frac{17}{15}$	

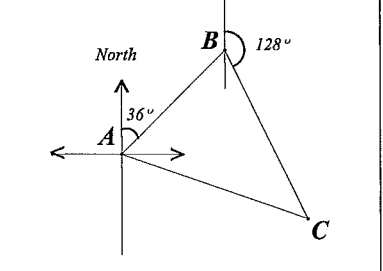
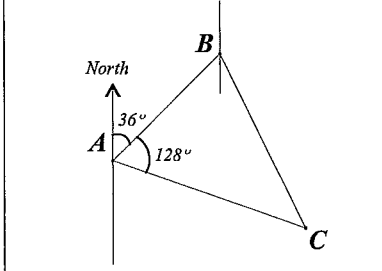
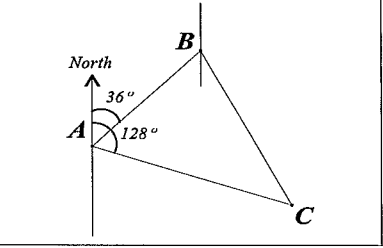
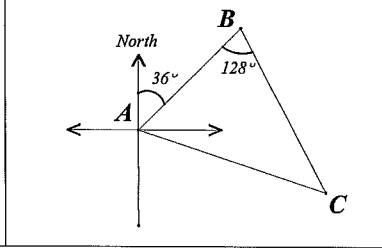
	(34) For the following problem, which is the <u>correct</u> diagram? "A radio mast 100 metres high and a TV mast 150 metres high stand on level ground. From the top of the radio mast, the angle of elevation of the top of the TV mast is 40° . How far apart are the masts?"		
(A)		(B)	
(C)		(D)	

	(35) Which of the following points lies on the line $3x - y = 8$?
	I. $P(-2, 2)$ II. $Q(2, -2)$
(A)	I only
(B)	II only
(C)	Both I and II
(D)	Neither I nor II

	(36) $\frac{k}{3} + \frac{k}{8} = ?$
(A)	$\frac{k}{11}$
(B)	$\frac{2k}{11}$
(C)	$\frac{11k}{24}$
(D)	$\frac{k}{12}$

(37)	If $\frac{a}{b} = \frac{c}{d}$, which one of the following sentences is true?
(A)	$a - c = b - d$
(B)	$ac = bd$
(C)	$\frac{2a - b}{b} = \frac{2c - d}{d}$
(D)	$\frac{a + b}{b + c} = \frac{b + c}{c + d}$

(39)	<p>Given the following four statements:</p> <p>(I) All isosceles triangles are similar (II) All rhombuses are similar (III) All equilateral triangles are similar (IV) All squares are similar</p> <p>Which of the following is true</p>
(A)	(I) and (II)
(B)	(I) and (III)
(C)	(II) and (IV)
(D)	(III) and (IV)

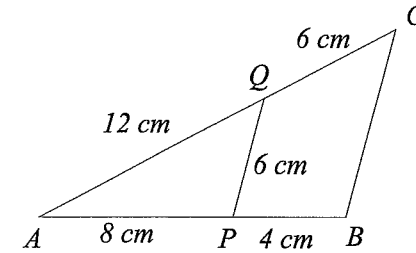
(40)	Which of the sketches below is the correct sketch for the following problem? "A plane leaves from point A and flies on a bearing of $036^\circ T$ to point B . The plane then flies to point C on a bearing of $128^\circ T$."		
(A)		(B)	
(C)		(D)	

(44)	Points P and R lie on the line $x + 3y = 3$, point Q lies on the curve $y = 2^x$ and S lies on the x -axis. If $PQRS$ is a rectangle, what are the coordinates of point S ?	<p style="text-align: center;">DIAGRAM IS NOT TO SCALE</p>
(A)	$S(-1\frac{2}{3}, 0)$	
(B)	$S(-3, 0)$	
(C)	$S(-15, 0)$	
(D)	$S(-21, 0)$	

EXTRA FREE RESPONSE QUESTIONS

Question 31(b)

In $\triangle ABC$ and $\triangle APQ$, $AQ = 12\text{ cm}$, $QC = 6\text{ cm}$, $AP = 8\text{ cm}$, $PB = 4\text{ cm}$ and $PQ = 6\text{ cm}$



- (i) Copy the diagram onto your answer sheet and prove that $\triangle ABC$ and $\triangle APQ$ are similar.
- (ii) Find the length of side BC .

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Year 10 Half-Yearly Mathematics Examination 2006

SECTION A (30 marks) – 30 Multiple choice questions. (1 mark each)

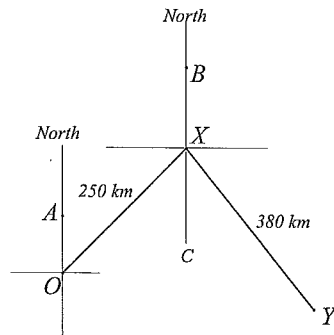
Answer questions on the answer sheet provided with the examination paper.

1	C	2	B	3	D	4	C
5	A	6	D	7	B	8	B
9	C	10	B	11	C	12	D
13	B	14	A	15	C	16	D
17	A	18	B	19	D	20	D
21	B	22	C	23	A	24	A
25	D	26	B	27	A	28	A
29	C	30	C				

SECTION B (40 marks) – Write your solutions to Q31-34 on your own paper showing all working.

Question 31(a) (START A NEW PAGE)

A plane flies 250 km from point O to point X on a bearing of $050^\circ T$. At point X the plane changes direction and flies 380 km on a bearing of $140^\circ T$ to point Y .



(i) Copy the diagram onto your answer sheet and clearly indicate the size of $\angle AOX$ and $\angle BXY$.

(ii) Prove that $\angle OXY = 90^\circ$.

$$\angle OXC = 50^\circ \left(\begin{array}{l} \text{alternate angles are equal as} \\ AO \parallel XC \end{array} \right)$$

$$\angle CXY + 140^\circ = 180^\circ \left(\begin{array}{l} \text{angle sum of straight} \\ \text{angle } BXC = 180^\circ \end{array} \right)$$

$$\angle CXY = 40^\circ$$

$$\begin{aligned} \angle OXY &= 50^\circ + 40^\circ \\ &= 90^\circ \end{aligned}$$

(iii) Find the bearing of point Y from point O . Give your answer correct to the nearest degree.

$$\tan \hat{XOY} = \frac{380}{250}$$

$$\hat{XOY} = 56.7^\circ$$

$$= 57^\circ \text{ (to nearest degree)}$$

$$\hat{AOY} = 50^\circ + 57^\circ$$

$$= 107^\circ$$

$$\therefore \text{bearing is } 107^\circ T \text{ or } S73^\circ E$$

Question 31(b)

The volume, $V \text{ cm}^3$, of a gas is directly proportional to the temperature, $T^\circ \text{ Kelvin}$, of the gas and inversely proportional to the pressure, P atmospheres, applied to the gas. A gas subject to a temperature of 20° K and a pressure of 5 atmospheres has a volume of 400 cm^3 .

(i) Write a formula relating the variables V , T and P .

$$V = \frac{kT}{P}$$

when $T = 20$, $P = 5$ and $V = 400$

$$400 = \frac{20k}{5}$$

$$k = 100$$

$$V = \frac{100T}{P}$$

(ii) If the temperature is increased to 35° K and the pressure is decreased to 2 atmospheres, find the new volume of the gas.

$$V = \frac{100T}{P}$$

when $T = 35$, $P = 2$

$$V = \frac{100 \times 35}{2}$$

$$V = 1750$$

$$\text{Volume} = 1750 \text{ cm}^3$$

Question 32(a) (START A NEW PAGE)

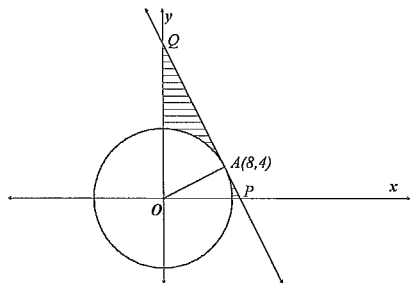
PQ is a tangent to the circle at the point $A(8, 4)$.

(i) Find the equation of the line PQ .

$$\begin{aligned} \text{slope } OA &= \frac{4-0}{8-0} \\ &= \frac{1}{2} \end{aligned}$$

$$\therefore \text{slope } PQ = -2$$

$$\begin{aligned} \text{Eqn. } PQ : y - 4 &= -2(x - 8) \\ 2x + y - 20 &= 0 \end{aligned}$$



(ii) Find the coordinates of the points P and Q .

$$P(10, 0) \text{ and } Q(0, 20)$$

(iii) Find the exact value of the shaded area, which is the area bounded by the tangent PQ , the coordinate axes and the circle.

$$\text{Area} = \text{area of triangle} - \frac{\text{area of circle}}{4}$$

$$\begin{aligned} \text{radius of circle : } r &= \sqrt{4^2 + 8^2} \\ &= \sqrt{80} \end{aligned}$$

$$A = \frac{1}{2}(10)(20) - \frac{1}{4}\pi(\sqrt{80})^2$$

$$= 100 - 20\pi$$

$$\text{Area} = 20(5 - \pi)$$

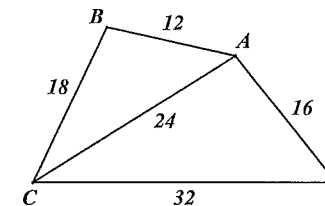
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Question 32(b)

Copy the diagram onto your answer sheet and prove that $\triangle ABC$ and $\triangle DAC$ are similar.



In $\triangle ABC$ and $\triangle DAC$

$$\begin{aligned} \frac{AC}{CD} &= \frac{24}{32} \\ &= \frac{3}{4} \end{aligned}$$

$$\begin{aligned} \frac{BC}{AC} &= \frac{18}{24} \\ &= \frac{3}{4} \end{aligned}$$

$$\begin{aligned} \frac{BA}{AD} &= \frac{12}{16} \\ &= \frac{3}{4} \end{aligned}$$

$\therefore \triangle ABC \parallel \triangle DAC$ (3 pairs of sides in same ratio)

4

Question 33(a) (START A NEW PAGE)

In $\triangle EFH$ and $\triangle FGH$, $EF = 40$, $\angle EHF = 45^\circ$ and $\angle FHG = 60^\circ$.

(i) Find the exact length of FH .

$$\frac{40}{x} = \tan 45^\circ$$

$$\frac{40}{x} = 1$$

$$x = 40$$

Or show/prove that triangle is isosceles $\Rightarrow x = 40$

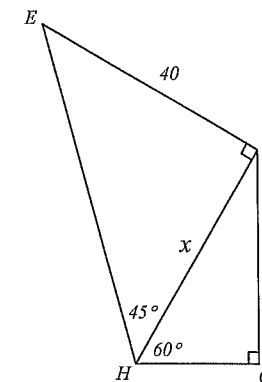
(ii) Hence find the exact length of FG .

$$\frac{y}{40} = \sin 60^\circ$$

$$y = 40 \sin 60$$

$$= 40 \times \frac{\sqrt{3}}{2}$$

$$y = 20\sqrt{3}$$



2

3

Question 33(b)

$ABCDEFGH$ is a rectangular prism with sides of lengths 2 cm , 3 cm and 4 cm .

(i) Find the exact length of diagonal GD .

$$D\hat{A}B = 90^\circ$$

$$BD^2 = 3^2 + 2^2 \text{ (Pythagoras' Theorem)}$$

$$BD = \sqrt{13}$$

$$D\hat{B}G = 90^\circ$$

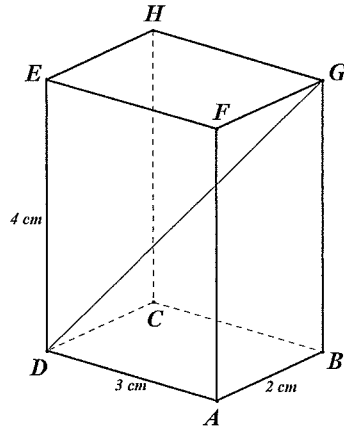
$$GD^2 = (\sqrt{13})^2 + 4^2 \text{ (Pythagoras' Theorem)}$$

$$BD = \sqrt{29}$$

(ii) Find the size of $\angle GDA$ correct to the nearest minute.

$$\cos G\hat{D}A = \frac{3}{\sqrt{29}}$$

$$G\hat{D}A = 56^\circ 9'$$

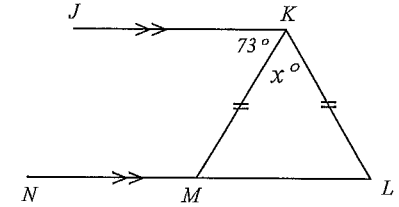


2
3

Question 34(a) (START A NEW PAGE)

The points N , M and L are collinear.
 $JK \parallel NL$, $KM = KL$ and $\angle JKM = 73^\circ$.

Copy the diagram onto your answer sheet and find the value of x , giving reasons.



$$K\hat{M}L = 73^\circ \begin{pmatrix} \text{alternate angles are equal} \\ \text{as } JK \parallel NL \end{pmatrix}$$

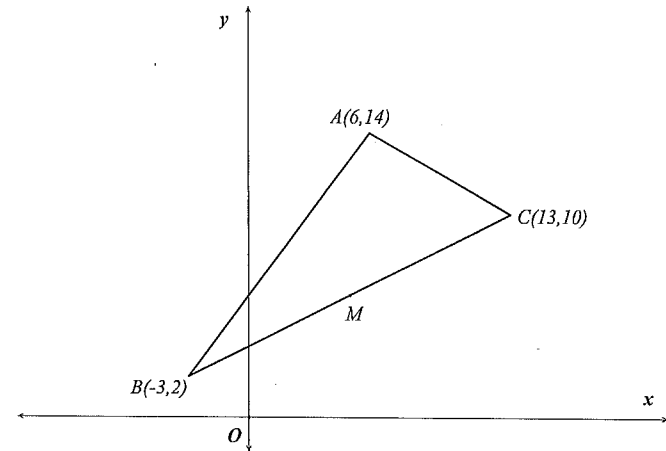
$$K\hat{L}M = 73^\circ \begin{pmatrix} \text{equal angles are opposite} \\ \text{equal sides} \end{pmatrix}$$

$$x + 146 = 180 \begin{pmatrix} \text{angle sum of} \\ \triangle KLM = 180^\circ \end{pmatrix}$$

$$x = 34$$

Question 34(b)

$\triangle ABC$ has vertices $A(6,14)$, $B(-3,2)$ and $C(13,10)$. M is the midpoint of side BC .



3

(i) Find the equation of the line BC .

$$m(BC) = \frac{10-2}{13+3}$$
$$= \frac{1}{2}$$

$$\text{Eqn. } BC: y-2 = \frac{1}{2}(x+3)$$
$$x-2y+7=0$$

2

(ii) Find the perpendicular distance from point A to the line BC .

$$\perp \text{ dist.} = \frac{|6-28+7|}{\sqrt{1+4}}$$
$$= \frac{15}{\sqrt{5}}$$
$$= 3\sqrt{5}$$

2

(iii) Find the coordinates of point M , the midpoint of side BC .

$$M\left(\frac{10}{2}, \frac{12}{2}\right) = M(5,6)$$

1

(iv) If the acute angle between the lines AM and BC is θ , find the exact value of $\sin \theta$.

$$AM = \sqrt{1^2 + 8^2}$$
$$= \sqrt{65}$$

$$\sin \theta = \frac{3\sqrt{5}}{\sqrt{65}}$$
$$= \frac{3}{\sqrt{13}}$$

$$\sin \theta = \frac{3\sqrt{13}}{13}$$

2

THIS IS THE END OF THE EXAMINATION PAPER