JAMES RUSE A.H.S

Year 10 Half Yearly Examination 2007

PART A : MULTI-CHOICE QUESTIONS (30 marks)

Instructions:

- On the multi-choice answer sheet, fill in the circle corresponding to the correct answer.
- HAND IN the answer sheet ensuring your NAME and MATHS CLASS are written at the top.

1. Evaluate $\frac{7.2+\sqrt{13.14}}{4.3\times 0.168}$, giving answer correct to 2 decimal places.

   A. 0.42 B. 6.24

   C. 12.22 D. 14.98

2. Solve for x: $x^2-14x-12=0$.

   A. $x = 7\pm\sqrt{61}$ B. $x=\frac{-7\pm\sqrt{244}}{2}$

   C. $x=\frac{-14\pm\sqrt{148}}{2}$ D. $x = 7\pm\sqrt{37}$

3. If $m=-3$, then $-4m^2 =$

   A. 36 B. -36

   C. 144 D. 144

4. Simplify $\frac{b+\sqrt{2b}}{\sqrt{b}}$

   A. $b+\sqrt{2}$ B. $\sqrt{3b}$

   C. $3\sqrt{b}$ D. $\sqrt{b}+\sqrt{2}$

5. The equation of the line perpendicular to $2x-3y=5$ and passing through the point $(-1,4)$ is:

   A. $2x+3y=-14$ B. $3x-2y=-5$

   C. $3x+2y=11$ D. $3x+2y=5$

6. ABCD is a rhombus with side length 13 cm. The circle of diameter 12 cm touches the sides of the rhombus as shown below.

   Calculate the shaded area.

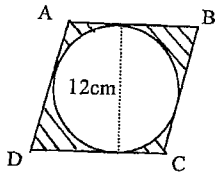


Diagram not to scale

   A. 55.9 cm^2 B. 35.1 cm^2

   C. 42.9 cm^2 D. 118.3 cm^2

7. In the diagram the radius of the larger circle is $2\frac{1}{2}$ times the radius of the smaller circle.
   The ratio of the unshaded area to the shaded area is:

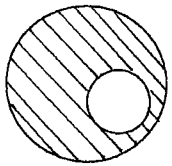


Diagram not to scale

   A. 2 : 5 B. 4 : 25

   C. $2\frac{1}{2}$: 1 D. 4 : 21

8. If a, a + d, a + 9d (d > 0) are the sides of a right-angled triangle, find the ratio of a : d.

   A. None of these B. 20 : 1

   C. 20 : 21 D. 8 : 1

9. If $9x^2-42xy+Fy^2=(3x+By)^2$ then:

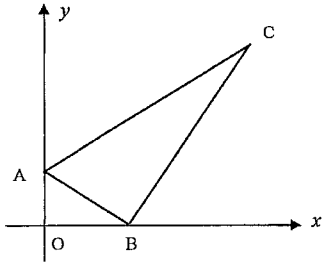
   A. $F=-49$ *and* $B=-7$ B. $F=49$ *and* $B=-7$

   C. $F=49$ *and* $B=7$ D. $F=-49$ *and* $B=7$

10. A(0, 2), B(3, 0) and C(5, 6) are points on the number plane as shown.

Find the area of Δ ABC, in units^2.

Diagram not to scale



A. 9

B. 4.5

C. 11

D. 12

11. Find the value of x.

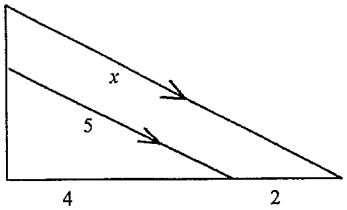


Diagram not to scale

A. 2.5

B. 7

C. 7.5

D. 10

12. In one week Susan worked 40 hours at her normal rate and 2 hours overtime at double time. If she earned $345.60 in that week, her hourly rate is:

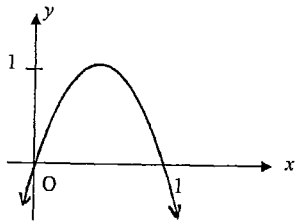
A. $7.20

B. $8.23

C. $7.85

D. $8.64

13.



Find the value of a given the graph of $y = ax(x+b)$.

A. -4

B. -1

C. $\frac{1}{2}$

D. $-\frac{1}{2}$

14.

TEST	KIM'S MARK	CLASS MEAN	CLASS STANDARD DEVIATION
TEST A	86	80	10
TEST B	82	65	15
TEST C	79	60	20
TEST D	70	60	10

On which test did Kim perform best in comparison with the rest of the class?

A. TEST A

B. TEST B

C. TEST C

D. TEST D

15. What pair of values satisfies the equations:

$2x - 3y = 24$ and $x + 2y = 5$

A. $x = 33$, $y = -14$

B. $x = 1$, $y = -2$

C. $x = 9$, $y = -2$

D. $x = -9$, $y = -14$

16. Which of the following is a factor of $a^2 - a - ab + b$?

A. $a+1$

B. $b-1$

C. $a+b$

D. $a-b$

17. The x – coordinate of the points A and B are:

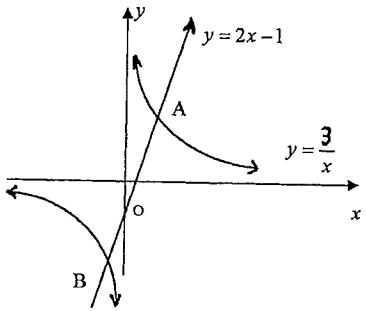


Diagram not to scale

A. $x=-\frac{3}{2}$ *and* $x=1$

B. $x=-1$ *and* $x=\frac{3}{2}$

C. $x=2$ *and* $x=-3$

D. $x=-2$ *and* $x=3$

18. Find the perpendicular distance from C to AB. All units in centimetres.

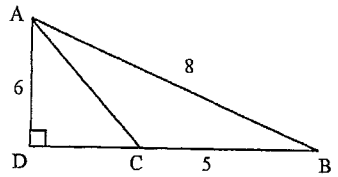


Diagram not to scale

A. 15 cm

B. 24 cm

C. $3\frac{3}{4}$ cm

D. 20 cm

19. The volume (V) of an object varies as the reciprocal of its density (D). An object with density 15 has a volume of 20.

Find the density of a certain objects if its volume is 25?

A. 15

B. 12

C. 10

D. 300

20. Find the length AB.

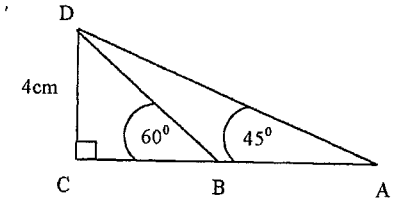


Diagram not to scale

A. $\frac{4\sqrt{3}-4}{\sqrt{3}}$ cm

B. $\frac{4}{\sqrt{3}}$ cm

C. 4 cm

D. $4-4\sqrt{3}$ cm

21. O is the centre of the circle. Find the value of m :

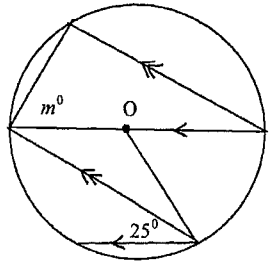


Diagram not to scale

A. 50

B. 65

C. 70

D. 25

22. A water trough has a cross-section in the form of a trapezium, as shown below.

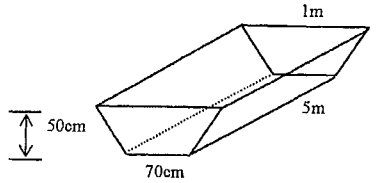


Diagram not to scale

Calculate the capacity of the trough, in litres.

A. 8 750 000 L

B. 2 125 000 L

C. 8 750 L

D. 2 125 L

23. A pendulum, of length 1 metre, swings through an angle of 60^0, about a point O.

Find the difference in the levels of A and B.

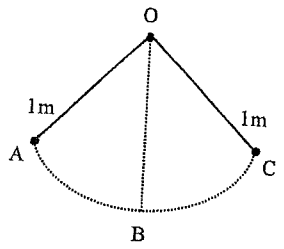


Diagram not to scale

A. $1-\frac{\sqrt{3}}{2}$ m

B. $\frac{\sqrt{3}}{2}$ m

C. $1-\frac{1}{\sqrt{3}}$ m

D. $\frac{1}{\sqrt{3}}$ m

24. A printing business has machinery worth \$60 000. For taxation purposes, the machinery depreciates by 15% of its value each year. What is the value of the machinery at the end of 5 years?

A. \$ 120 680.

B. \$ 15 000.

C. \$ 45 000.

D. \$ 26 622.

25. An equilateral triangle is inscribed in a circle as shown below. The perimeter of the triangle is 24 cm.

What is the diameter of the circle ?

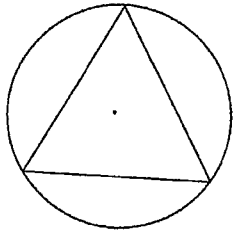


Diagram not to scale

A. $\frac{16\sqrt{3}}{3}$ cm

B. $\frac{8\sqrt{3}}{3}$ cm

C. 16 cm

D. Insufficient information

26. Calculate the shaded area (in square units).

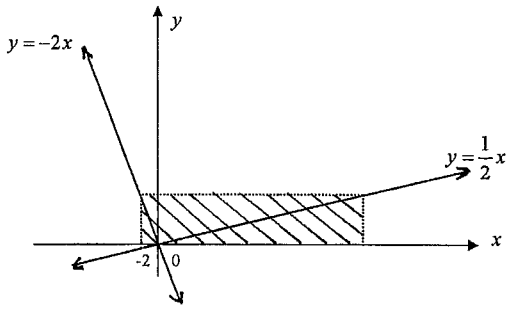


Diagram not to scale

A. 16

B. 24

C. 32

D. 40

27. PQ is a chord of a circle, centre O, and R is a point on the major arc. If $\angle OPR = 5°$, $\angle OQP = 40°$, find the value of x.

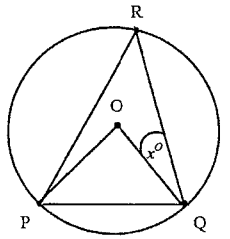


Diagram not to scale

A. 45

B. 5

C. 40

D. 100

28. Two dice are thrown onto a table and the number on the upper most faces examined. What is the probability that one number will be a factor of the other number?

A. $\frac{1}{5}$

B. $\frac{1}{9}$

C. $\frac{11}{18}$

D. $\frac{1}{6}$

29. A triangle PQR is inscribed in a rectangle ABCD which has dimensions 12 cm by 8 cm.
PB = QC = DR = x cm. The area (A) of the triangle PQR is given by the expression $A = x^2 - 10x + 48$.

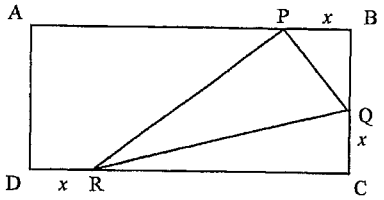


Diagram not to scale

What is the smallest value that the area of the triangle may have?

A. 18 cm^2 B. 23 cm^2

C. 32 cm^2 D. 48 cm^2

30. The mean of a set of 16 scores is 12. The standard deviation of the scores is 2.4. An additional score of 3 is added to the set. Consider the statements:

I The mean decreases.
II The standard deviation decreases.

Which is true?

A. I only B. II only

C. Both I and II D. Neither I nor II

END OF MULI-CHOICE QUESTIONS

GO ONTO PART B – Short Answer Questions

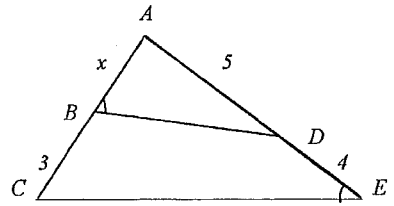
PART B **Marks**

Question 31 (Start A New Page)

(a) The quantity Q varies directly to the square of the quantity T and inversely to the quantity M. **3**
If initially Q is 500 , T is 350 and M is 10000, find the value of Q (one decimal place) when T is 500 and M is 12000.

(b) Given $T = 2\pi\sqrt{\frac{L}{g}}$ find g as subject. **2**

(c)



Not to scale

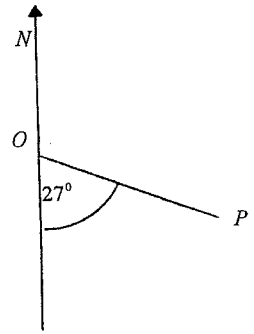
In the diagram above $\angle ABD = \angle AEC$.
(i) Prove $\Delta ABD \,|||\, \Delta AEC$ **3**
(ii) Find the exact length of AB **2**

Question 32 (Start A New Page)

(a) Find the centre and radius of the circle with the equation : **3**
$x^2 + y^2 - 8x - 12y - 3 = 0$

(b) The line with equation $4x + 5y - 7 = 0$ is a tangent to a circle of radius r. **2**
Find the radius r if the equation of the circle is given by : $x^2 + y^2 = r^2$.

(c)



Not to scale

(i) State the true bearing of P from O. **1**

(ii) Point P is 70 km from point O.
Find the easterly distance (2 decimal places) of the point P from O. **2**

(d) A rectangle has sides of length 8 cm and 5 cm. **2**
Find the acute angle (nearest degree) between the diagonals.

Question 33 (Start A New Page) **Marks**

(a) Find the co-ordinates of the point P which divides $A(3,-2)$ and $B(-1,5)$ externally in the ratio 5:3. **2**

(b) Find the equation of the straight line which passes through $T(1,2)$ and the intersection point of the lines $2x-3y-6=0$ and $5x+7y-3=0$. **3**

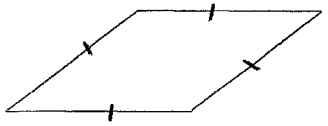
(c) The points $A(1,5), B(-1,2)$ and $C(2,-3)$ represent the triangle ABC. **3**

(i) Show that the equation of the perpendicular bisector BC is $3x-5y-4=0$

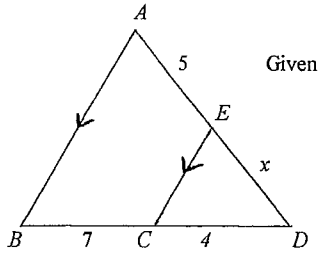
(ii) If the equation of the perpendicular bisector of AB is $4x+6y-21=0$, find the co-ordinates of the centre of the circle which passes through all the vertices of triangle ABC. **2**

Question 34 (Start A New Page)

(a) A rhombus has diagonals of length 84 cm and 13 cm. **2**
Find the perimeter of the rhombus, giving reasons.



(b) **2**

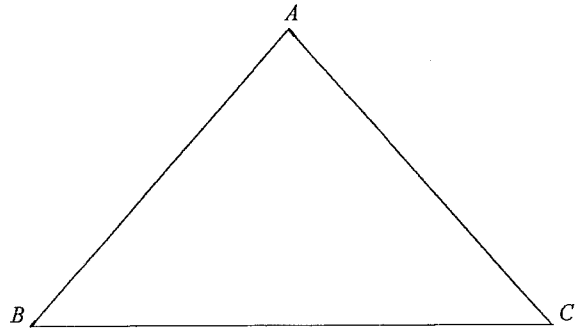


Given $AB//CE$, find the length of ED, giving reasons.

Diagram not to scale.

Question 34 continued.

(c) **All constructions are to be drawn on the exam paper.**
Detach from the exam paper and submit with your answer to question 34.



(i) Measure to the nearest degree $\angle ABC$. 1

(ii) Using a pair of compasses and ruler bisect $\angle ABC$. 2

(iii) Using a pair of compasses and ruler construct the perpendicular bisector of AC. 2

(iv) Measure the length of the median from vertex B in ΔABC. 1

Note: Detach and hand in with Question 34.

End Of Exam.

JRAHS MATHEMATICS 2007
YEAR 10- SOLUTIONS TO PART A

1. D	6. C	11. C	16. D	21. B	26. D
2. A	7. D	12. C	17. B	22. D	27. A
3. B	8. B	13. A	18. C	23. A	28. C
4. D	9. B	14. B	19. B	24. D	29. B
5. D	10. C	15. C	20. A	25. A	30. A

Question 31

(a) $Q \propto \frac{T^2}{M}$

$Q = \frac{kT^2}{M}$ (1)

$Q=500 \quad T=350 \quad Q=10000$

$\therefore \frac{500}{10000} = k\frac{350^2}{10000}$

$k = \frac{5 \times 10000}{35^2}$

$= \frac{2000}{49}$ (1)

$Q = \frac{2000}{49} \times \frac{500^2}{12000}$

$= 850.3$. (1)

(b) $T = 2\pi\sqrt{\frac{l}{g}}$

$T^2 = 4\pi^2\frac{l}{g}$ (1)

$g = \frac{4\pi^2 l}{T^2}$ (1)

(c) In $\triangle ABD$ and $\triangle AEC$

$\angle A$ is common (1)

$\angle ABD = \angle AEC$ (given) (1)

$\therefore \triangle ABD \parallel\parallel \triangle AEC$ (equiangular) (1)

$\therefore \frac{AB}{AE} = \frac{AD}{AC}$ [corresponding sides of similar triangles are in the same ratio] (1)

$\frac{x}{9} = \frac{5}{x+3}$

$x^2+3x-45=0$

$x = \frac{-3 \pm \sqrt{9+180}}{2}$

$= \frac{-3+\sqrt{189}}{2}$ only $x>0$. (1)

Question 32

(a)(i) $x^2+y^2-8x-12y-3=0$

$x^2-8x+16+y^2-12y+36 = 3+16+36$

$(x-4)^2+(y-6)^2 = 55$ (1)

centre $(4,6)$ (1) radius $\sqrt{55}$ units (1)

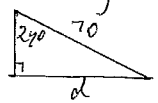
(ii) $D = \left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

$r = \left|\frac{4\times 0+5\times 0-7}{\sqrt{4^2+5^2}}\right|$ (1)

$= \frac{7}{\sqrt{41}}$ units. (1)

(c)(i) Bearing P from O 153°T (1)

(ii)



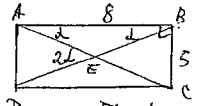
$\frac{d}{70} = \sin 22^\circ$ (1)

$d = 70\sin 22^\circ$

$= 31.78$

$\therefore$ P is 31.78 km East of O. (1)

(d)



$AE = BE$ [Equal diagonals rectangle bisect each other]

$\therefore \angle EAB = \angle EBA$ [equal angles opposite equal sides in $\triangle AEB$]

$= d$.

$\angle AED = 2d$ [Exterior angle triangle is equal to sum two interior opposite angles]

$\angle AED = 2\tan^{-1}\frac{5}{8}$ - (1)

$= 64^\circ$ - (1)

opticonal

(a) 5 : -3

$A(3,-2)$ $B(-1,5)$

$P = \left[\frac{-5-9}{2}, \frac{25+6}{2}\right]$ ① ①

$= [-7, 15\frac{1}{2}]$

(b) $2x-3y-6+k[5x+7y-3]=0$ ①

$(1,2)$ satisfies ∴

$2-6-6+k[5+14-3]=0$

$k = \frac{10}{16}$

$= \frac{5}{8}$ ①

∴ Equation Line

$2x-3y-6+\frac{5}{8}[5x+7y-3]=0$

$16x-24y-48+25x+35y-15=0$

$41x+11y-63=0$ ①

(c) (i) $m_{BC} = -\frac{5}{3}$ $M_{BC}\left(\frac{1}{2}, -\frac{1}{2}\right)$ ①

$m_{\perp BC} = \frac{3}{5}$ $m_1 \times m_2 = -1$ ①

Eqn bisector

$y-y_1 = m(x-x_1)$

$y+\frac{1}{2} = \frac{3}{5}\left[x-\frac{1}{2}\right]$

$10y+5 = 6x-3$

$6x-10y-8=0$ ①

$3x-5y-4=0$

$4x+11y=21$ ①

$3x-5y=4$ ②

$12x+18y=63$

$12x-20y=16$

$38y=47$

$y = \frac{47}{38}$

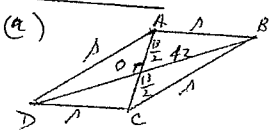
Substitute ② $3x-5\times\frac{47}{38}=4$

$x = 3\frac{15}{38}$

Centre $\left[3\frac{15}{38}, 1\frac{9}{38}\right]$

Question 34

(a)



$OA = \frac{13}{2}$, $BO = 42$ } Diagonals of rhombus bisect each other at 90°

$A^2 = 42^2 + \left(\frac{13}{2}\right)^2$ (Pythagoras)

$p = 42.5$ cm ①

∴ Perimeter $= 4A$

$= 170$ cm

(b) $\frac{x}{5} = \frac{4}{7}$ [A line drawn parallel to one side of a triangle divides the other two sides in the same ratio] ①

$x = 2\frac{6}{7}$

∴ $ER = 2\frac{6}{7}$ units ①

(c) (i) $\angle ABC = 49°$

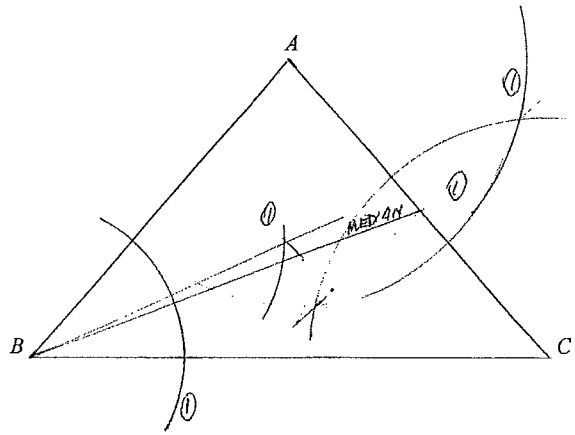
(ii) 1 mark arcs
1 accuracy to find bisector

(iii) 1 mark arcs
1 accuracy to find bisector

(iv) 81mm ± 1mm 1 mark

Question 34 continued.

(c) **All constructions are to be drawn on the exam paper.**
Detach from the exam paper and submit with your answer to question 34.



(i) Measure to the nearest degree $\angle ABC$. 1

(ii) Using a pair of compasses and ruler bisect $\angle ABC$. 2

(iii) Using a pair of compasses and ruler construct the perpendicular bisector of AC. 2

(iv) Measure the length of the median from vertex B in ΔABC. 1

Note: Detach and hand in with Question 34.

End Of Exam.