JAMES RUSE – YR 10 – HALF YEARLY – 2008

SECTION A (30 Marks) – 30 Multiple choice questions (1 mark each)
Answer questions on the answer sheet provided with the examination paper.

1. Expand and simplify: $(3k-1)-2(k-5)$

   (A) $k-6$ (B) $k+9$ (C) $k-4$ (D) $k-11$

2 $6x^0+7=$

   (A) $6x+7$ (B) 6 (C) 7 (D) 13

3. $\sqrt{25m^{16}}=$

   (A) $5m^4$ (B) $5m^8$ (C) $\pm 5m^4$ (D) $\pm 5m^8$

4 $12\times\left(\frac{2}{3}\right)^{-2}=$

   (A) $\frac{16}{3}$ (B) $\frac{27}{2}$ (C) 27 (D) 36

5. The solutions of $4m^2=m$ are

   (A) $m=0, \frac{1}{4}$ (B) $m=0, 1$ (C) $m=\frac{1}{2},-\frac{1}{2}$ (D) $m=2, -2$

6. Katie invests \$1000 in a share plan. It makes 20% in the first year but loses 20% in the second year. How much is Katie's investment worth after two years?

   (A) \$640 (B) \$960 (C) \$1000 (D) \$1440

7. There are 12 students in a class. The stem-and leaf plot shows the number of hours the students spent on homework each week.

stem	leaf
0	3 7 7 9 9
1	0 1 1 1 4
2	8 9

   What is the mean number of hours spent on homework each week, correct to one decimal place?

   (A) 4.9 (B) 12 (C) 13.0 (D) 12.4

8. Find the median for the number of hours spent on homework each week in Question 7.

   (A) 10 (B) 10.5 (C) 11 (D) 11.5

9. The ratio of boys to girls in a Maths class is 4:5. The number of girls increases from 15 to 18. What is the new ratio of boys to girls?

   (A) 3:2 (B) 2:3 (C) 4:5 (D) 5:4

10. The value of p in the following diagram is

    (A) 7° (B) 35° (C) 37 ° (D) 55°

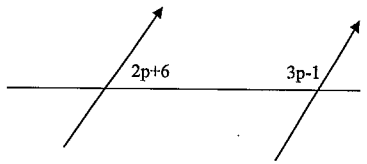


Diagram not to scale

11. Given that $\sin A=t$ and $90°<A<180°$, then $\cos A$ is

    (A) $-\sqrt{1-t^2}$ (B) $\sqrt{1-t^2}$ (C) $-\sqrt{1+t^2}$ (D) $\sqrt{1+t^2}$

12.

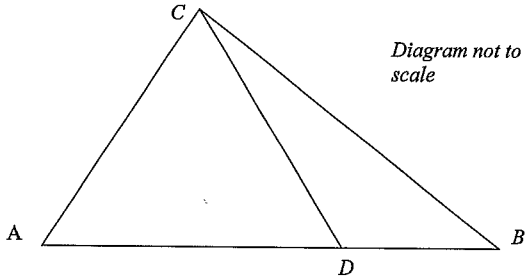


Diagram not to scale

ΔACD is equilateral. The area of ΔACD is half the area of ΔABC
What is the size of $\angle ACB$?

   (A) 75° (B) 80° (C) 90° (D) 100°

13. The centre and radius of the circle $x^2+y^2-4x+10y+14=0$ are

    (A) $(2,5), \sqrt{15}$ (B) $(-2,5), \sqrt{15}$ (C) $(2,-5), \sqrt{15}$ (D) $(2,5), 5$

14. The coordinates of the point that divides the interval from (-5, 6) to (4, -3) in the ratio 1:2 are

    (A) (-2, 3) (B) (1, 0) (C) (3, -2) (D) (0, 1)

15. The line $5x+12y=1$ is a tangent to a circle centred at (1, 1). The radius of that circle is

    (A) $\frac{16}{13}$ (B) $\frac{17}{13}$ (C) $\frac{18}{13}$ (D) 13

16. The equation of the straight line through the point of intersection of $x+3y-4=0$ and $3x-4y+1=0$, which also passes through the point (3, -1) may be given by $x+3y-4+k(3x-4y+1)=0$ where k is

(A) $-\frac{7}{2}$ (B) $-\frac{2}{7}$ (C) $\frac{2}{7}$ (D) $\frac{7}{2}$

17. Two sides of a triangular field are 60m and 50m and the included angle is 140 °. The area of the field correct to the nearest square metre.

(A) 496m^2 (B) 694m^2 (C) 964m^2 (D) 1928m^2

18. Given that x is inversely proportional to the square of y, when $x=2, y=4$, find the value of x when $y=-2$?

(A) 4 (B) -4 (C) 8 (D) -8

19. If the diameter of a sphere is increased by 15%, which of the following is true?
   a. Radius is increased by 15%.
   b. Surface area is increased by 0.0225.
   c. Volume is increased by 0.00375.
   d. Capacity is increased by 0.00375.

(A) None of the above (B) a only (C) a,b,c (D) All of the above

20. If $9^{x-2}=27$, then $x=$

(A) -3 (B) $-\frac{7}{2}$ (C) $\frac{7}{2}$ (D) 3

21. Four people need 3 days to paint an area of 2000m^2. How many days do 3 people need to paint an area of 500m^2 ?

(A) 1 (B) 2 (C) 3 (D) 4

22. Daniel sailed due south for 40km. Then he sailed due east for 30km for a reef. What is the true bearing of the reef from Daniel's starting point?

(A) S37°E (B) 143° (C) 053° (D) S53°E

23. Which of the following lines is perpendicular to the line $2x+3y=10$?

(A) $4x+6y=20$ (B) $-2x-3y=10$ (C) $2x=3y$ (D) $3x=2y$

24. Solve for x: $\frac{3}{2x-3}=\frac{5}{2+x}$

(A) -4 (B) -3 (C) 3 (D) 4

25. A shop bought an MP3 player for \$60 and sold it for \$80. Which of the following statement(s) must be true?

   a) The profit was 20% of the selling price.
   b) The profit was 25% of the cost price.
   c) The cost price was 25% less than the selling price.
   d) The selling price was $33\frac{1}{3}$% more than the cost price.

(A) b, c, d (B) c, d (C) a, c (D) b, d

26. The shaded area below is best described by
   (A) $3x+5y\leq 15, 2x+y\leq 6$ and $y\geq 0$.
   (B) $3x+5y\leq 15, 2x+y\geq 6$ and $y\geq 0$.
   (C) $3x+5y\geq 15, 2x+y\geq 6$ and $y\geq 0$.
   (D) $3x+5y\geq 15, 2x+y\leq 6$ and $y\geq 0$.

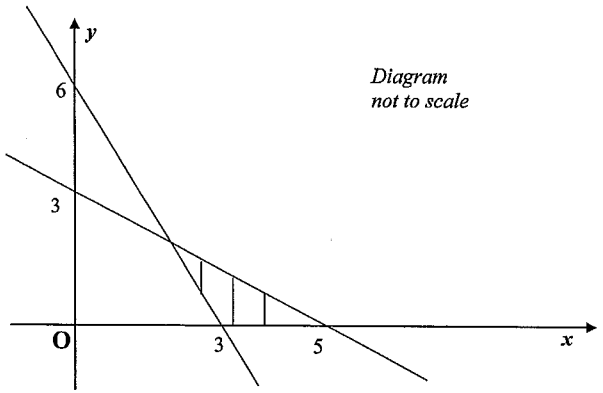


Diagram not to scale

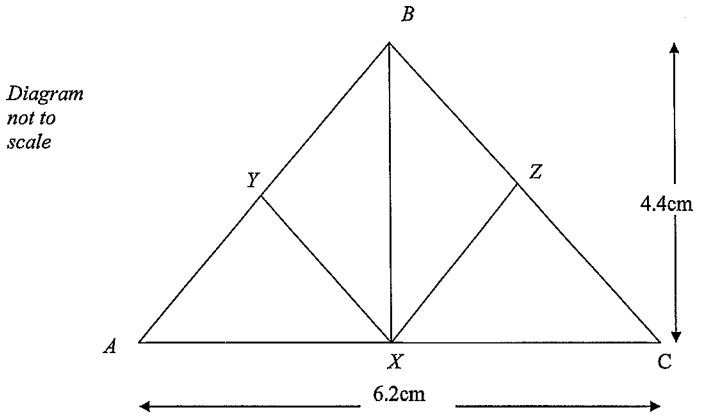
27. The remainder when $3x^3-7x^2+2x+4$ is divided by $(x-3)$ is:

(A) -137 (B) -28 (C) 28 (D) 137

28. $\frac{\sqrt{3}}{2\sqrt{7}-\sqrt{3}}$ is equivalent to

(A) $\frac{\sqrt{3}}{2\sqrt{7}+\sqrt{3}}$ (B) $\frac{2\sqrt{21}-3}{25}$ (C) $\frac{2\sqrt{21}+3}{25}$ (D) $\frac{2\sqrt{21}+3}{31}$

29. ΔABC is isosceles with $AB = CB$. Y and Z are the midpoints of AB and CB respectively. X is a point on AC such that $YX = ZX$.



The length of AB (in cm) is closest to.

(A) 5.3 (B) 5.4 (C) 7.6 (D) 57.8

30. $\angle BYX$ in Question 29 is closest to

(A) 35° (B) 45° (C) 71° (D) 110°

END OF SECTION A

SECTION B (40 Marks) – Write your solutions to Q31-34 on your own paper, showing all working, and starting each question on a separate page.

Question 31 (10 Marks) (START A NEW PAGE) **Marks**

a) Solve the equation $2\cos x^\circ + \sqrt{3} = 0$ for $0 \le x \le 360$. 2

b) $A(-1, 2)$, $B(3, -1)$ and $P(6, 3)$ are points on a Cartesian plane.

i) Plot these points on a suitable diagram and show that the equation of the line AB is given by $3x + 4y - 5 = 0$. 2

ii) Find the distance of P from the line AB. 1

iii) Find the area of triangle PAB. 2

iv) Determine the coordinates of C, the point which divides AB externally in the ratio 3:1. 2

v) Write down the area of triangle PBC. 1

Question 32 (10 Marks) (START A NEW PAGE) **Marks**

a) Factorise $x^3 + 27y^3$. 2

b) In triangle PQR, $PQ = 15cm$, $PR = 10cm$ and $\angle PQR = 40^\circ$.

i) Show that $\sin R = 0.9642$ to 4 decimal places. 1

ii) Illustrate on a sketch the two possible configurations of triangle PQR. (Sizes of angles should be marked to the nearest degree but the sketch need not be exactly to scale). 2

iii) Find the area of the smaller triangle PQR. 2

c) i) Write down the general equation (involving a constant k), of a line which passes through P, the point of intersection of the lines $3x - 2y + 7 = 0$ and $y = 4x + 1$. 1

ii) Hence find the equation of the line through P which is parallel to the line $y = 2x - 3$. 2

Questions 33 and 34 are on the next page

Question 33 (10 Marks) **(START A NEW PAGE)** **Marks**

a) Make b the subject of $a = \dfrac{2b}{(b-1)} + 1$ 2

b) Draw the graph of $y = 3\sin 2x^\circ$ for $-180 \le x \le 180$. 3

c) The period T of a simple pendulum is the time taken for one complete small oscillation. T varies directly with the square root of its length L. When the length of the pendulum is 98cm, the period is exactly 2 seconds

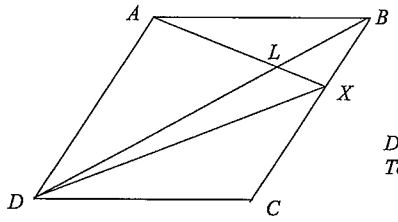
i) What will the period become if the pendulum is lengthened to 1m. ? 2

ii) What length should the pendulum be if its period is to be halved ? 1

iii) A grandfather clock is regulated by this pendulum. The pendulum is lengthened to 1m (as in part (i)) and the clock started at noon. Calculate the actual time, to the nearest second, when the clock first shows 1 o'clock. 2

Question 34 (10 Marks) **(START A NEW PAGE)** **Marks**

a)



Drawing Not To Scale

$ABCD$ is a rhombus and X is a point on BC. The line AX cuts the diagonal BD at the point L.

i) Prove that the triangles ADL and CDL are congruent. 3

ii) Prove that $\angle LCD = \angle LXB$. 2

b) The points L, M and N are respectively (3, 0), (7, -3) and (13, 5).

i) Prove that the triangle LMN is right angled 2

ii) Find the equation of the circumcircle of triangle LMN. 3
(The circumcircle of a triangle goes through the three vertices.)

THIS IS THE END OF THE EXAMINATION

1) $3K - 1 - 2K + 10 = K + 9$ (B)

2) $6 + 7 = 13$ (D)

3) $5m^8$ (B)

4) $12 \times \frac{9}{4} = 27$ (C)

5) $m(4m - 1) = 0$
$m = 0,\ 1/4$ (A)

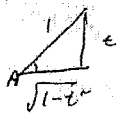
6) $1000 \times 1.2 \times 0.8 = 960$ (B)

7) 12.4 (D)

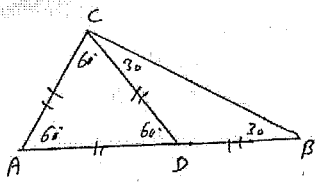
8) $\frac{10 + 11}{2} = 10.5$ (B)

9) $B : G = 4 : 5$
$\frac{B}{G} = \frac{4}{5}$
When $G = 15$ $B = 15 \times \frac{4}{5} = 12$
$B' : G' = 12 : 18 = 2 : 3$ (A)

10) $2p + 6 + 3p - 1 = 180$
$5p = 175$
$p = 35$ (B)

11)


$-\sqrt{1-t^2}$ (A)

12)


Area $\Delta ACO = \frac{1}{2}(AC)(AO)\sin A$
$= \frac{1}{2}(AO)^2 \sin A$
Area $\Delta ACB = \frac{1}{2}(AD)(AB)\sin A$ (since $AO = AD$)
$\therefore \frac{1}{2}(AD)^2 \sin A = \frac{1}{2}(\frac{1}{2}AD)(AB)\sin A$
$AD = \frac{1}{2}AB$
$\therefore DB = AD = CO$
$\therefore \angle OCB = 30°$
$\angle ACB = 60° + 30° = 90°$ (C)

13) $x^2 - 4x + 4 + y^2 + 10y + 25 = -14 + 4 + 25$
$(x - 2)^2 + (y + 5)^2 = 15$ (C)
centre $(2, -5)$ $r = \sqrt{15}$

14) $x = \frac{4 + 2(-5)}{3} = -2$
$y = \frac{-3 + 2(6)}{3} = 3$ (A)

15) $r = \frac{|5 + 12 - 1|}{\sqrt{5^2 + 12^2}} = \frac{16}{13}$ (A)

16) $x + 3y - 4 + k(3x - 4y + 1) = 0$
$3 + 3(1) - 4 + k(9 + 4 + 1) = 0$
$k = 2/7$ (C)

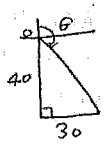
17) $A = \frac{1}{2} \times 60 \times 50 \sin 140°$
$= 964\ m^2$ (nearest m^2)
(C)

18) $xy^2 = k^2$
$2 \times 4^2 = x(-2)^2$
$x = 8$ (C)

19) B

20) $3^{2(x-2)} = 3^3$
$2x - 4 = 3$
$x = 7/2$ (C)

21) $x = \frac{500}{2000} \times \frac{4}{3} \times 3 = 1$ (A)

22)


$\theta = 90° + \tan^{-1}(\frac{40}{30})$
$= 143°$ (nearest degree)
(B)

23) $2x + 3y = 10$
$m = -2/3$
$m^\perp = 3/2$ (D)

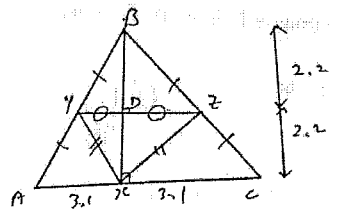
24) $3(x + 2) = 5(2x - 3)$
$6 + 3x = 10x - 15$
$21 = 7x$
$x = 3$ (C)

25) PROFIT $= \$20$
$\frac{\text{PROFIT}}{SP} = \frac{20}{80} = 25\%$
$\frac{\text{PROFIT}}{CP} = \frac{20}{60} = 33\frac{1}{3}\%$
$\frac{SP - CP}{CP} = \frac{20}{60} = 33\frac{1}{3}\%$
$\frac{SP - CP}{SP} = \frac{20}{80} = 25\%$ (B)

26) B

27) $P(3) = 3(3)^3 - 7(3)^2 + 2(3) + 4$
$= 28$ (C)

28) $\frac{\sqrt{3}}{2\sqrt{7} - \sqrt{3}} \times \frac{2\sqrt{7} + \sqrt{3}}{2\sqrt{7} + \sqrt{3}} = \frac{2\sqrt{21} + 3}{28 - 3}$
(D)

29)


2 pairs of adj sides equal,
$\therefore BZXY$ is a kite
$\therefore BX \perp YZ$ and $YD = ZD$
Since Y, Z are mid pts of AB, BC
$\therefore AC \parallel YZ$
$\angle BXC = 90°$
$\therefore AX = CX = 3.1$
$AB = \sqrt{4.4^2 + 3.1^2} = 5.4$ (1 dp) (B)

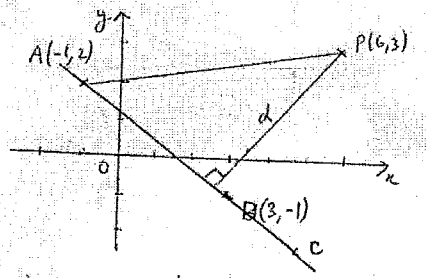
30) $YZ \parallel AC$
$\therefore BD = DX$
$\therefore BZXY$ is a rhombus
$\angle BYD = \angle XYD$
$\therefore \angle BYX = 2\tan^{-1}(\frac{2.2}{3.1}) = 71°$ (nearest degree)
(A) D

YR 10 HALF YEARLY 2003 ANSWERS

31 a) $\cos x° = -\frac{\sqrt{3}}{2}$

$x = 150$ or 210 ②

b)



i) $m_{AC} = \frac{2--1}{-1-3} = -\frac{3}{4}$.

$y - 2 = -\frac{3}{4}(x+1)$

$4y - 8 = -3x - 3$

$3x + 4y = 5$ ②

ii) $d = \left|\frac{3\times6 + 3\times4 - 5}{\sqrt{3^2+4^2}}\right| = 5$ units ①

iii) $AB = \sqrt{(2-1)^2 + (-1-3)^2} = 5$ units

$\therefore$ Area $\triangle ABP = \frac{1}{2}\times5\times5 = 12\frac{1}{2}$ units^2 ②

iv) (Use internal ratio 3:-1)

C is $\left(\frac{3\times3 + -1\times-1}{3+-1}, \frac{3\times-1 + -1\times2}{3+-1}\right)$

$= \left(5, -5\frac{1}{2}\right)$ ②

v) $BC = \frac{1}{2}AB$

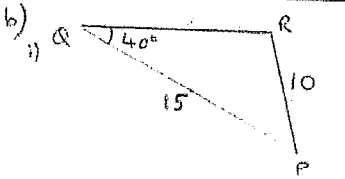
$\therefore$ Area $\triangle PBC = \frac{1}{2}$ Area $\triangle ABP$

$= 6\frac{1}{4}$ units^2 ①

32 a) $x^3 + 27y^3 = x^3 + (3y)^3$

$= (x+3y)(x^2 - 3xy + 9y^2)$

b) i)



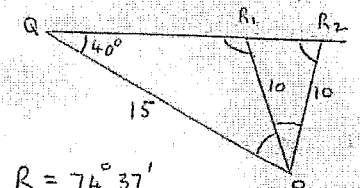
By Sine Rule

$\frac{\sin R}{15} = \frac{\sin 40°}{10}$

$\sin R = \frac{15\sin 40°}{10} = 0.9642$ ①

(to 4 dp)

ii)



$R = 74° 37'$

or $105° 23'$ (2 pictures OK) ②

iii) Area $= \frac{1}{2}\,15.10.\sin 34° 37'$

$= 42.6$ sq units ②

c) i) $3x - 2y + 7 + k(y - 4x - 1) = 0$

(or equivalent) ①

ii) $y(k-2) + x(3-4k) + 7 - k = 0$

$y = x\frac{(4k-3)}{(k-2)} + \frac{k-7}{k-2}$

$m = 2 \quad \therefore \frac{4k-3}{k-2} = 2$

$4k - 3 = 2k - 4$

$2k = -1 \quad k = -\frac{1}{2}$

$\therefore$ Line is $y\left(-\frac{5}{2}\right) + x(5) + 7\frac{1}{2} = 0$

$10x - 5y + 15 = 0$

$2x - y + 3 = 0$

33 a) $a(b-1) = 2b + (b-1)$

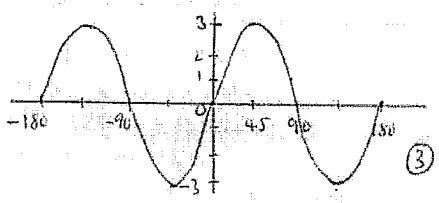
$ab - a = 3b - 1$

$ab - 3b = a - 1$

$b(a-3) = a - 1$

$b = \frac{a-1}{a-3}$ ②

b)



③

33 c) i) $T \propto \sqrt{L}$

$T = k\sqrt{L}$

$2 = k\sqrt{98}$

$\frac{2}{\sqrt{98}} = k$

When $L = 100$, $T = \frac{20}{\sqrt{98}}$

$= 2.02$ secs.

(to 2 dp)

ii) $1 = \frac{2}{\sqrt{98}}\sqrt{L_1}$ (L_1 new length)

$\sqrt{L_1} = \frac{\sqrt{98}}{2}$

$L_1 = \frac{98}{4} = 24.5$ cm.

iii) The clock will need 1800 periods to time 1 hour.

This now takes

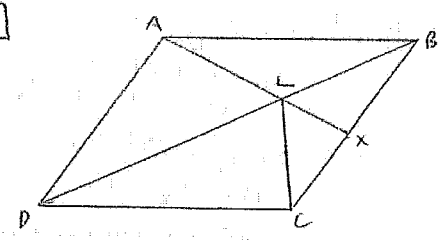
1800×2.0203 secs

$= 1$ hr and 37 seconds

(Time will be 1:00:37)

(allow 36 seconds if 2 dp used).

34 a)



i) In $\triangle$s ADL and CDL

$\angle ADL = \angle LDC$ (Diagonals of a rhombus bisect angles)

$AD = CD$ (Sides of a rhombus are equal)

LD is common

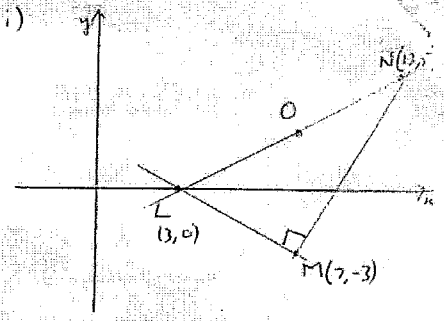
$\therefore \triangle ADL \equiv \triangle CDL$ (SAS)

ii) $\therefore \angle LCD = \angle DAL$ (Corresponding angles in congruent triangles)

But $\angle DAL = \angle LXB$ (AD // BC, alternate angles are equal)

$\therefore \angle LCD = \angle LXB$

b) i)



$m_{LM} = \frac{-3-0}{7-3} = -\frac{3}{4}$

$m_{MN} = \frac{5--3}{13-7} = \frac{8}{6} = \frac{4}{3}$.

$m_{LM}\,m_{MN} = -\frac{3}{4}\times\frac{4}{3} = -1$

$\therefore$ LM $\perp$ MN. ②

ii) The circumcircle will have centre O and radius LN/2 (since LN must be a diameter)

$\therefore$ O is $(8, 5\frac{1}{2})$

$LN = \sqrt{25 + 100} = \sqrt{125} = 5\sqrt{5}$

$\therefore$ Radius is $\frac{5\sqrt{5}}{2}$.

$\therefore$ Equation is

$(x-8)^2 + \left(y - 5\frac{1}{2}\right)^2 = \frac{125}{4}$ ③

This is far enough but this will simplify to

$x^2 + y^2 - 16x - 5y + 39 = 0$