Name:	
Class:	

JAMES RUSE AHS GESTA NON VERBA

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION 2012

MATHEMATICS

General Instructions:

- Reading Time: 5 minutes.
- Working Time: 3 hours.
- Write in black or blue pen.
- Board approved calculators & templates may be used
- A Standard integral Sheet is provided.
- In every question, show all necessary working
- Marks may not be awarded for careless or badly arranged working.

Total Marks 100

Section I: 10 marks

- Attempt Question 1 – 10.
- Answer on the Multiple Choice answer sheet provided.
- Allow about 15 minutes for this section.

Section II: 90 Marks

- Attempt Question 11 - 16
- Answer on blank paper unless otherwise instructed. Start a new page for each new question.
- Allow about 2 hours & 45 minutes for this section.

The answers to all questions are to be returned in separate *stapled* bundles clearly labeled Question 11, Question 12, etc. Each question must show your Candidate Number.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a \neq 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Section I Multiple Choice (10 marks)

Attempt Question 1 – 10 (1 mark each)
Allow approximately 15 minutes for this section.

Question 1

An infinite geometric series has a first term of 8 and a limiting sum of 12. What is the common ratio?

A) 1/6 B) 5/3 C) 1/2 D) 1/3

Question 2

What is the greatest value taken by the function $f(x) = 4 - 2\cos x$ for $x \geq 0$?

A) 2 B) 4 C) 6 D) 8

Question 3

What is the value of $\int_2^6 \frac{1}{x+2}\,dx$?

A) ln 2 B) ln 4 C) ln 6 D) ln 8

Question 4

The table below shows the values of a function *f(x)* for five values of *x*.

x	2	2.5	3	3.5	4
$f(x)$	4	1	-2	3	8

What value is an estimate for $\int_2^4 f(x)\,dx$ using Simpson's Rule with these five values ?

A) 4 B) 6 C) 8 D) 12

Question 5

It is known that the number *N(t)* of ants in a certain nest at time $t \geq 0$ is given by $N(t) = \frac{K}{1+e^t}$
where K is constant and t is measured in months.
At time $t = 0$, *N(t)* is estimated at 2×10^5 ants. What is the value of K ?

A) 2×10^5 B) 2×10^{-5} C) 4×10^5 D) 4×10^{-5}

Question 6

Sixty tickets are sold in a raffle. There are two prizes. Lincoln buys 5 tickets. Which expression gives the probability that Lincoln wins both prizes ?

A) $\frac{5}{60} + \frac{4}{59}$ B) $\frac{5}{60} + \frac{4}{60}$ C) $\frac{5}{60} \times \frac{4}{59}$ D) $\frac{5}{60} \times \frac{4}{60}$

Question 7

What is the equation of the **normal** to the curve $y = x^2 - 4x$ at $(1, -3)$?

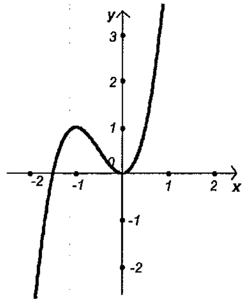
A) $x + 2y - 7 = 0$ B) $x - 2y - 7 = 0$

C) $2x - y - 5 = 0$ D) $2x + y + 5 = 0$

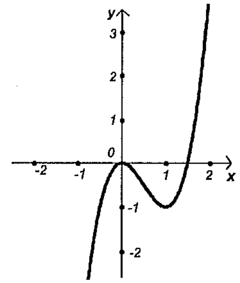
Question 8

Which of the following is the graph of $y = 2x^3 - 3x^2$?

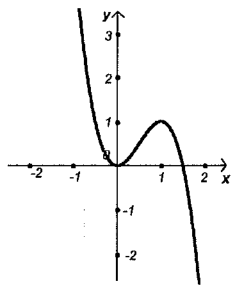
A)



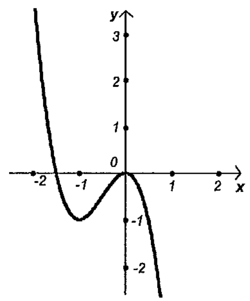
B)



C)

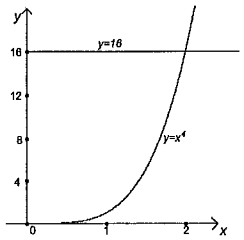


D)



Question 9

The region in the diagram is bounded by the curve $y = x^4$, the y-axis and the line $y = 16$.



Which of the following expressions is correct for the volume of the solid of revolution when this region is rotated about the y-axis ?

A) $\pi \int_0^2 x^8 dx$ B) $\pi \int_0^{16} x^8 dx$

C) $\pi \int_0^2 \sqrt{y}\, dy$ D) $\pi \int_0^{16} \sqrt{y}\, dy$

Question 10

What are the solutions to the equation $e^{6x} - 7e^{3x} + 6 = 0$?

A) $x = 1$ or $x = 6$ B) $x = 0$ or $x = \frac{\ln 6}{2}$

C) $x = 0$ or $x = \frac{\ln 6}{3}$ D) $x = 1$ or $x = \frac{\ln 6}{2}$

End of Section I

Section II Total Marks is 90

Attempt Questions 11 – 16.
Allow approximately 2 hours & 45 minutes for this section.

Answer all questions, starting each new question on a new sheet of paper with your **student ID number** in the top right hand corner and the question number on the left hand side of your paper.

All necessary working must be shown in each and every question.

Question 11 begins on the next page

Question 11 (15 Marks) Start a new piece of paper Marks

a) Simplify $\frac{3x}{x+2} - \frac{5x+19}{x^2+5x+6}$ **2**

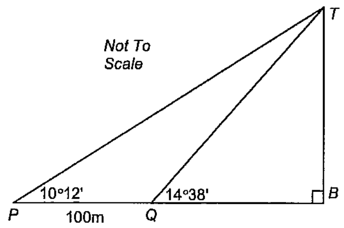
b) Differentiate i) $y = ln\left(\frac{x-1}{x^2}\right)$ ii) $y = x^2 \cos 4x$ **4**

c) Integrate i) $\int 3x + e^{4x}\, dx$ ii) $\int \frac{x^2-1}{x}\, dx$ **4**

d) The angle of elevation of the top of tree BT when viewed from point P is 10°12'.

After walking 100m directly towards the tree one arrives at Q where the angle of elevation is 14°38'. **3**

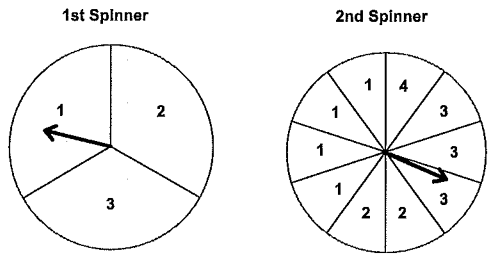
Copy the diagram and find the height of the tree to the nearest centimetre.



e) Make a careful sketch of the curve $y = 1 + 3\sin\frac{x}{2}$ over the domain $0 \le x \le 2\pi$. **2**

Question 12 (15 Marks) Start a new piece of paper Marks

a) The diagram below shows two spinners.



Each of the three outcomes on the first spinner are equally likely. On the second spinner there are ten equally likely sectors for the arrow to land on with four possible outcomes.

In a game, both spinners are spun simultaneously. The player's score is the **sum** of the two numbers that the spinners land on. (eg A score of 4 in the above diagram). A player wins if their score is an odd number greater than 4.

i) What is the probability of scoring 7 ? **1**
ii) What is the probability that a player will win the first game? **3**
iii) What is the probability that a player will win the first three games? **1**

Question 12 is continued on the next page

Question 12 (continued)

b) The fourth term of an Arithmetic Sequence is (-12) and the tenth term is 21. Calculate the value of the nineteenth term. **2**

c) The point $Q(-2,1)$ lies on the line k which has equation $9x - 2y + 20 = 0$.
The point $R(4,-2)$ lies on the line l which has equation $3x + y - 10 = 0$.

i) Find the coordinates of P, the point on the y-axis where k and l intersect. **2**

ii) The line m joins Q and R. Show that the equation of m is $x + 2y = 0$. **2**

iii) Show, by shading on a sketch, the region defined by the three inequalities:

$9x - 2y + 20 \geq 0, \quad 3x + y - 10 \leq 0, \quad x + 2y \geq 0.$ **1**

iv) Find, as a surd, the perpendicular distance from P to m and hence, or otherwise, find the exact area of the triangle bounded by the lines k, l and m. **3**

Question 13 (15 Marks) Start a new piece of paper **Marks**

a) i) By completion of the square, or otherwise, show that the vertex of the parabola $x^2 - 10x + 15 = 2y$ is at $(5, -5)$ **1**

ii) Write down the focus of this parabola. **2**

iii) Show that the parabola $y = 4x - x^2$ also passes through the point $(5, -5)$. **1**

iv) Find the other point of intersection of these two parabolas. **2**

v) Hence find the area enclosed between the two parabolas. **3**

b) The velocity, $\dot{x}$, in m/s of a particle moving in a straight line is given by $\dot{x} = 1 - 2\sin t$ for $0 \leq t \leq 2\pi$, where t is the time in seconds. The particle is initially at $x = 2$.

i) At what time(s) is the acceleration zero? **1**

ii) What is the maximum velocity of the particle during this period. (You should demonstrate that this is a maximum and not a minimum.) **2**

iii) Find the first time that the particle changes direction during this period. **1**

iv) Hence, or otherwise, find the exact distance travelled by the particle between $t = 0$ and the time when the particle first changes direction. **2**

Question 14 (15 Marks) Start a new piece of paper **Marks**

a) A sheep is grazing in a large paddock which is bounded on one side by a long straight fence. The sheep is tethered to a stake by a rope 20m in length. If the stake is placed 10m from the fence, find the area to the nearest square metre over which the sheep can graze. **3**

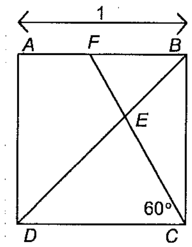
b) The quadratic equation $x^2 - 3x - 13$ has roots α and β.

i) Write down the values of $\alpha + \beta$ and $\alpha\beta$. **1**

ii) What is the exact value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$? **2**

iii) What is the value of $6\beta - 2\beta^2$? **1**

c) A square $ABCD$ of side length 1 unit is shown below. The point F is drawn on AB such that $\angle DCF = 60°$. The diagonal DB intersects CF at E.



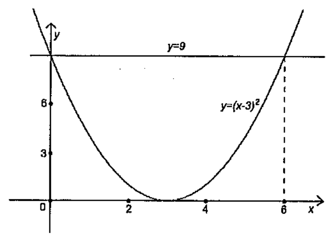
i) Show that $\Delta DEC \;|||\; \Delta BEF$ **3**

ii) Show that $FB = {}^{1}/_{\sqrt{3}}$ **1**

iii) Hence, or otherwise, find the ratio AreaΔDEC : AreaΔBEF **1**

d) The region bounded by the graph $y = (x-3)^2$ and the line $y = 9$ is rotated about the x-axis to form a solid of revolution.

Find the volume of the solid so formed. **3**



Question 15 (15 Marks) **Start a new piece of paper** **Marks**

a) Following an accident, water started leaking out of a tank. If the volume of water in the tank was $V(t)$ litres, then t days after the accident, the rate of change of V was given by $\frac{dV}{dt} = 20t - 300$ litres per day.
When the tank stopped leaking, it still had 4750 litres in it.

i) For how many days was the tank leaking? **1**

ii) Find a formula for V in terms of t. **3**

iii) How much water was in the tank when it started leaking? **1**

b) Allcare Home Loans has a special package for first home buyers. The main details of the package, as shown in their brochure, are summarised in the table.

Stage	Term	Special Features	Interest Rate
Introductory Stage	0-2 years (2 years)	No monthly repayments	6% pa compounded monthly
Secondary Stage	2-10 years (8 years)	Monthly repayments start. At the end of this period, the amount owing must be reduced to the original size of the loan.	9% pa compounded monthly
Final Stage	Variable (but not exceeding 20 years)	The borrower determines the size of the monthly repayment, provided that the loan is repaid within 20 years from the start of this stage.	12% pa compounded monthly

Alice and Bernard have accepted the terms of the above plan and they have borrowed \$500,000 to finance their first house.

i) Show that the amount owing at the end of the Introductory Stage, to the nearest dollar, is \$563,580. **1**

ii) The principal for the Secondary Stage will be \$563,580. Assume that the first monthly repayment, M, is paid after one month into the Secondary period. Find M, to the nearest cent if the amount owing at the end of the Secondary Stage is to be \$500,000. **4**

iii) At the start of the Final Stage, Alice and Bernard have decided that they can afford to repay \$6500 per month.

α) Determine how many full payments of \$6500 it will take for the loan to have been repaid in full. **3**

β) The last monthly repayment of \$6500 is more than required. How much should be refunded to Alice and Bernard? **2**

Question 16 (15 Marks) **Start a new piece of paper** **Marks**

a) Given the function $y = \frac{10}{3+2\cos x}$ in the domain $0 \le x \le 2\pi$

i) Find the location and nature of all the stationary points in the domain. **3**

ii) Graph the function in the given domain. **2**

b) A mining company simultaneously established three new mining towns, A, B and C. Each had an initial population of 500 and it was planned that they each would grow by 50 inhabitants per year for the first ten years.

i) Only town A grew as planned. Write down an expression for the intended population of town A, t years after its opening ($t \le 10$ and t is an integer). **2**

ii) For various reasons, towns B and C did not grow as planned. Their populations are better modelled by:

Town B: $\frac{dP_B}{dt} = -0.3P_B$

Town C: $P_C = 100\left(5 + t - {t^2}/{4}\right)$

α) Show that the expression $P_B = 500e^{-0.3t}$ satisfies the equation describing town B. **1**

β) Calculate, to the nearest integer, the population of town B after 6 years. **1**

γ) Find when the population of town C reached its maximum and what was that maximum value? **3**

δ) The mining company has determined that any town is unviable if the population goes below 50. Which will be the first town to close? (Justify your answer) **3**

END OF EXAM

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MATHEMATICS: Question .... MULTIPLE CHOICE

Suggested Solutions	Marks	Marker's Comments
$S_\infty = 12$		
1. $\frac{8}{1-r} = 12$		
$\therefore r = \frac{1}{3}$ (D)		
2. $-1 \le \cos x \le 1$		
$\Rightarrow 4 - 2\cos x \le 6$ (C)		
3. $\left[\ln(x+2)\right]_2^6 = \ln 8/4 = \ln 2$ (A)		
4. $A \doteq \frac{0.5}{3}\{4 + 4\times(1) + (-2)\} +$		
$\frac{0.5}{3}\{-2 + 4\times 3 + 8\}$		
$= 4$ (A)		
5. $2\times 10^5 = \frac{k}{1+e^0}$		
$2\times 10^5 = \frac{k}{2}$ $\therefore k = 4\times 10^5$ (C)		
6. $\frac{5}{60} \times \frac{4}{59}$ (C)		
7. $y' = x^2 - 4x$		
$y' = 2x - 4$		
at $x = 1$ $m_T = -2$		
$\therefore m_N = \frac{1}{2}$ (B)		
$y + 3 = \frac{1}{2}(x-1)$		
$x - 2y - 7 = 0$		
8. $y = 2x^3 - 3x^2$		
$= x^2(2x - 3)$		
$\Rightarrow$ double root at $x = 0$ & a single root at $x = 3/2$		
When $x = 1$ $y = -1$ $\Rightarrow$ (B)		
9. $V = \int_0^{16} \pi x^2\, dy$		
$= \pi \int_0^{16} \sqrt{y}\, dy$ (D)		
10. $e^{6x} - 7e^{3x} + 6 = 0$ let $u = e^{3x}$		
$u^2 - 7u + 6 = 0$		
$(u-6)(u-1) = 0$		
$e^{3x} = 6$ or $e^{3x} = 1$		
$3x = \ln 6$ $3x = \ln 1$		
$x = \frac{\ln 6}{3}$ $x = 0$ (C)		

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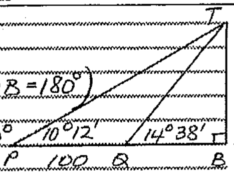
MATHEMATICS: Question 11 — 1

Suggested Solutions	Marks	Marker's Comments
(a) $\frac{3x}{x+2} - \frac{5x+19}{(x+2)(x+3)}$		($\frac{1}{2}$ off for each error)
$= \frac{3x(x+3) - 5x - 19}{(x+2)(x+3)}$	1	Students still have difficulty expanding with negative signs.
$= \frac{3x^2 + 4x - 19}{(x+2)(x+3)}$	1	$\boxed{2}$
(b)(i) $y = \ln\left(\frac{x-1}{x^2}\right)$		
$= \ln(x-1) - \ln x^2$		($\frac{1}{2}$ off for each error)
$= \ln(x-1) - 2\ln x$		
$y' = \frac{1}{x-1} - \frac{2}{x}$		Students used quotient rule instead of using log laws.
$= \frac{2-x}{x(x-1)}$	2	$\boxed{2}$
(ii) $y = x^2\cos 4x$		
$\frac{dy}{dx} = 2x.\cos 4x + x^2(-4\sin 4x)$ (1) (1)		$\boxed{2}$
$= 2x(\cos 4x - 2x\sin 4x)$	2	
(c) (i) $\int 3x + e^{4x}\, dx$		($-\frac{1}{2}$ if no constant)
$= \frac{3x^2}{2} + \frac{e^{4x}}{4} + C$	2	$\boxed{2}$
(ii) $\int \frac{x^2 - 1}{x}\, dx$		
$= \int \frac{x^2}{x} - \frac{1}{x}\, dx$		($-\frac{1}{2}$ for each error)
$= \int x - \frac{1}{x}\, dx$		
$= \frac{x^2}{2} - \ln x + C$	2	$\boxed{2}$
		Students failing to make good use of S.I. sheet.

MATHEMATICS: Question 11 — 2

Suggested Solutions | **Marks** | **Marker's Comments**

Method 1



(½) $\angle PQT + 14°38' = 180°$
(sum of straight angle $PQB = 180°$)
$\therefore \angle PQT = 165°22'$

(½) $\angle PTQ + 10°12' + 165°22' = 180°$
(angle sum of $\Delta PTQ = 180°$)

(½) — diagram.

(½) $\dfrac{PT}{\sin 165°22'} = \dfrac{100}{\sin 4°26'}$

$\sin 10°12' = \dfrac{BT}{PT}$

(½) $\therefore BT = PT \sin 10°12'$
$= \dfrac{100 \sin 165°22' \cdot \sin 10°12'}{\sin 4°26'}$
$= 57.8756\ldots$

(½) Thus, the height of the tree is
57.88 m (to nearest cm)

Marks: 3

(−½ if not rounded off correctly)

3

Method 2:

$\angle PTB + 90° + 10°12' = 180°$
(½) (angle sum of $\Delta PTB = 180°$)
$\therefore \angle PTB = 79°48'$

Let $h = BT$ and $x = QB$

(½) $\tan 79°48' = \dfrac{100 + x}{h}$ — ①

$\angle QTB + 14°38' + 90° = 180°$
(½) (angle sum of ΔQTB is $180°$)
$\therefore \angle QTB = 75°22'$

$\tan 75°22' = \dfrac{x}{h}$ — ②

(½) From ② $x = h \tan 75°22'$
subst into ①

$\tan 79°48' = \dfrac{100 + h\tan 75°22'}{h}$

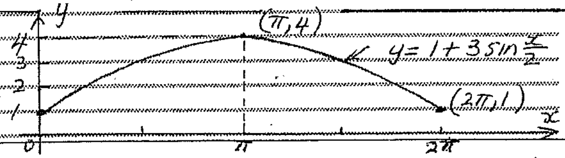
$h(\tan 79°48' - \tan 75°22') = 100$

$h = \dfrac{100}{\tan 79°48' - \tan 75°22'}$

(½) $= 57.8756\ldots$

$\therefore$ Height of the tree is 57.88 (nearest cm).

(e)



Marks: 2

½ each for the max. pt & end pts.
½ equation
−½ incorrect curvature.

2

MATHEMATICS: Question 12

Suggested Solutions | **Marks** | **Marker's Comments**

(a)(i) $P(\text{score of } 7) = P(3 \text{ and } 4)$
$= \frac{1}{3} \times \frac{1}{10}$
$= \frac{1}{30}$

Marks: 1

(ii) $P(\text{win 1st game}) = P(5) \text{ or } P(7)$
$= P(1 \,\&\, 4) + P(2 \,\&\, 3) +$
$P(3 \,\&\, 2) + P(3 \,\&\, 4)$
$= \left(\frac{1}{3} \times \frac{1}{10}\right) + \left(\frac{1}{3} \times \frac{3}{10}\right) + \left(\frac{1}{3} \times \frac{2}{10}\right) + \frac{1}{30}$
$= \frac{7}{30}$

1 mark
1 mks
1 mark

* If they got an answer of $\frac{1}{5}$ ∴ 2mks max.
* If they got $\frac{1}{5}$ but had the first line ∴ 2½ mks.

(iii) $P(\text{win 1st 3 games}) = \frac{7}{30} \times \frac{7}{30} \times \frac{7}{30}$
$= \frac{343}{27000}$

Marks: 1

(b) $a + 3d = -12$ ... ①
$a + 9d = 21$ ... ②

Marks: ½

② − ①
$6d = 33$
$d = \frac{33}{6} = 5\frac{1}{2}$ — ½
sub into ①
$a + 3(5\frac{1}{2}) = -12$
$a = -28\frac{1}{2}$ — ½

$T_n = a + (n-1)d$
$= -28\frac{1}{2} + 18 \times 5\frac{1}{2}$
$= 70\frac{1}{2}$ — ½

* A lot of students did NOT know the formula

* If they left off the "−" sign for ① lost 1mk.

* wrong formula = 0 mks

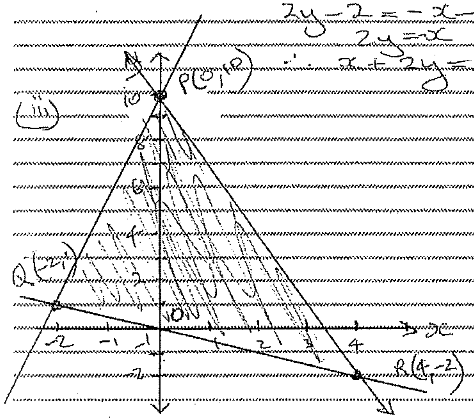
(c)(i) $9x - 2y + 20 = 0$ ... ①
$3x + y - 10 = 0$
$\therefore 6x + 2y - 20 = 0$ ... ②

① + ②
$15x = 0$
$x = 0$
sub into ①
$0 - 2y + 20 = 0$
$2y = 20$
$y = 10$
$\therefore P$ is $(0, 10)$

2 mks

2/2

MATHEMATICS: Question 12

Suggested Solutions	Marks	Marker's Comments
(c)(ii) $M_{QR} = \frac{y_2-y_1}{x_2-x_1} = \frac{1-{-2}}{-2-4} = \frac{3}{-6} = -\frac{1}{2}$	1	It was a proof so you had to show working to justify the marks!!
eqn of QR: $y-y_1=m(x-x_1)$ $y-1=-\frac{1}{2}(x+2)$ $2y-2=-x-2$ $2y=-x$ $\therefore x+2y=0$	1	
(iii) 	1	*All students needed to do was plot P, Q and R but.......
(iv) $d=\frac{\lvert ax+by+c\rvert}{\sqrt{a^2+b^2}}$ $x+2y-0=0$ $P(0,10)$ $=\left\lvert\frac{0\times1+2\times10+0}{\sqrt{1^2+2^2}}\right\rvert$ $=\frac{20}{\sqrt{5}}$ $=4\sqrt{5}$ units	1mk	*students forget how to simplify surds!!!
$QR=\sqrt{(4--2)^2+(-2-1)^2}$ $=\sqrt{36+9}$ $=\sqrt{45}$ $=3\sqrt{5}$	1	$\sqrt{9}=3$
Area $=\frac{1}{2}bh$ $=\frac{1}{2}\times3\sqrt{5}\times4\sqrt{5}$ $=30$ units^2	1	

2U MATHS TRIAL SOLUTIONS, 2012

MATHEMATICS: Question 13

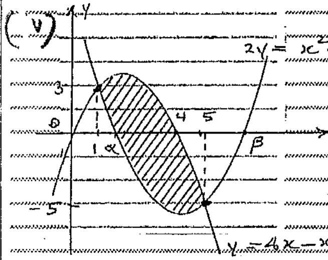
Suggested Solutions	Marks	Marker's Comments
(a)(i) $x^2-10x+15=2y$ METHOD I: $x^2-10x+25=2y-15+25$ $(x-5)^2=2y+10$ $(x-5)^2=2(y+5)$ As it is of the form $(x-h)^2=4a(y-k)$ $\therefore$ Vertex $V=(h,k)=(5,-5)$ ed. METHOD II: $y=\frac{1}{2}x^2-5x+\frac{15}{2}$ Axis of symmetry $x=-\frac{b}{2a}=\frac{-(-5)}{2\times\frac{1}{2}}=5$ at $x=5$ $25-10\times5+15=2y$ $2y=-10$ $y=-5$ $\therefore$ T. point $=V(5,-5)$	$\frac{1}{2}$ $\frac{1}{2}$	$\boxed{1}$
(ii) As $4a=2$ $\therefore a=\frac{1}{2}$ $\therefore$ focus $S=(5,-5+\frac{1}{2})=(5,-4\frac{1}{2})$	1 1	$\boxed{2}$
(iii) Verify $(5,-5)$ satisfies $y=4x-x^2$ LHS: $y=-5$ RHS: $4x-x^2=4\times5-5^2$ $=20-25$ $=-5$ $\therefore$ LHS = RHS $\therefore (5,-5)$ also lies on $y=4x-x^2$	$\frac{1}{2}$ each	$\boxed{1}$
(iv) $y=4x-x^2$ — (1) $2y=x^2-10x+15$ — (2) $\therefore 2(4x-x^2)=x^2-10x+15$ $8x-2x^2=x^2-10x+15$ $0=3x^2-18x+15$ $(\div3)$ $x^2-6x+5=0$ $(x-1)(x-5)=0$ $\therefore x=1$ or 5 in eqn (1) $y=3$ or -5 $\therefore$ Other point is $(1,3)$	$\frac{1}{2}$ $\frac{1}{2}$ 1	$\boxed{2}$

MATHEMATICS: Question 13

Suggested Solutions | **Marks** | **Marker's Comments**

$\alpha = 5-\sqrt{10}$ $\quad \beta = 5+\sqrt{10}$

(v)



$$\text{AREA} = \int_1^5 y_u - y_L \, dx$$

$$= \int_1^5 4x - x^2 - \left(\tfrac{1}{2}x^2 - 5x + \tfrac{15}{2}\right) dx$$

$$= \int_1^5 4x - x^2 - \tfrac{1}{2}x^2 + 5x - \tfrac{15}{2}\, dx$$

$$= \int_1^5 9x - \tfrac{3}{2}x^2 - \tfrac{15}{2}\, dx$$

Marks: $\boxed{3}$

$$\therefore \text{Area} = \left[\tfrac{9}{2}x^2 - \tfrac{1}{2}x^3 - \tfrac{15}{2}x\right]_1^5$$

$$= \left[\left(\tfrac{9\times 25}{2} - \tfrac{125}{2} - \tfrac{75}{2}\right) - \left(\tfrac{9}{2} - \tfrac{1}{2} - \tfrac{15}{2}\right)\right]$$

Area = 16 sq units

Comment: $\frac{25}{2} - \left(-\frac{7}{2}\right)$ $\quad \frac{32}{2}$

(b) (i) $\dot{x} = v = 1 - 2\sin t \quad 0 \le t \le 2\pi$

Data $t=0 \quad x=2 \quad v=1 \quad \ddot{x} = -2$

$\ddot{x} = -2\cos t = 0$ — $\frac{1}{2}$

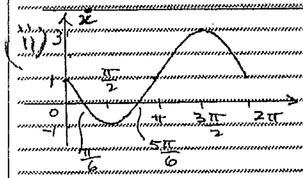
$\therefore \cos t = 0$

$t = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ — $\frac{1}{2}$

$\therefore$ accel. is zero at $\frac{\pi}{2}$ or $\frac{3\pi}{2}$ seconds.

Comment: $v=1$; number line with 0, 2, x, (+). Marks: $\boxed{1}$

(ii)



$x = 1 - 2\sin t$

As $-1 \le \sin t \le 1$

so $-1 \le 1 - 2\sin t \le 3$ — 1

$\therefore$ max speed is 3 m/s at $t = \frac{3\pi}{2}$ — 1

Marks: $\boxed{2}$

OR use calculus using (b) (i) $t = \frac{\pi}{2}, \frac{3\pi}{2}$ and TEST

(iii) Particle at REST only $v = \dot{x} = 0$

$\therefore 1 - 2\sin t = 0$

$\sin t = \frac{1}{2}$

$t = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

First $t = \frac{\pi}{6}$ (seconds)

TEST if changes direction — see sketch (ii); $\ddot{x}\left(\frac{\pi}{6}\right) = -\sqrt{3} < 0$ $\therefore$ pushed back to left; table: $\frac{\pi}{6}^-$ $\quad \frac{\pi}{6}$ $\quad \frac{\pi}{3}$

Comment: $\frac{1}{2}$ for $\frac{\pi}{6}$ s; $\frac{1}{2}$ for TEST. Marks: $\boxed{1}$

(iv) distance travelled $= \int_0^{\pi/6} 1 - 2\sin t \, dt$ — [DOES NOT CHANGE direction IN $0 \le t \le \frac{\pi}{6}$ SEE (ii)

$= \left[t + 2\cos t\right]_0^{\pi/6}$

$= \left[\left(\frac{\pi}{6} + 2\times\frac{\sqrt{3}}{2}\right) - (0+2)\right]$

$= \left(\frac{\pi}{6} + \sqrt{3} - 2\right)$ m

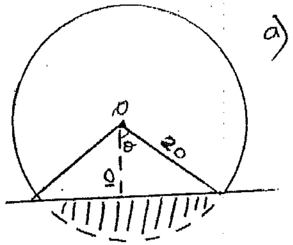
$0.2556\ldots$

Marks: $\boxed{2}$

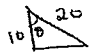
MATHEMATICS: Question 14

Suggested Solutions | **Marks** | **Marker's Comments**

a)



Step 1 Get θ.



$\cos\theta = \frac{10}{20}$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

Step 2 Find 2θ.

$\therefore 2\theta = \frac{2\pi}{3}$ ✓

Shaded Sector $= \frac{1}{2} r^2 (\theta - \sin\theta)$

$= \frac{20^2}{2}\left(\frac{2\pi}{3} - \sin\frac{2\pi}{3}\right)$

$= 200\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$ ✓

$\therefore$ Grazing area $= A_{\text{circle}} - A_{\text{segment}}$

$= \pi \times 20^2 - \left(200\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)\right)$

$= 400\pi - \frac{400\pi}{3} + 100\sqrt{3}$

$= 1010.96\ldots$ ✓

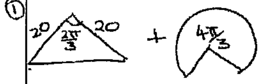
Area is 1011 m^2 (to nearest m^2)

accepted $\frac{800\pi}{3} + 100\sqrt{3}$.

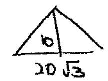
Marks: 3 Marks

Marker's Comments: Different Approaches

(1)



(2) $\frac{2}{3}\pi r^2 +$



If answer

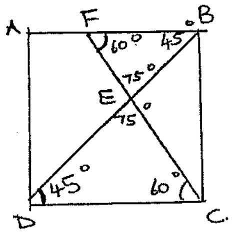
- 1251 m^2 means length of chord was incorrect. Took $10\sqrt{3}$ instead of $20\sqrt{3}$.
- 838 – means only area of major sector was calculated

b (i) $\alpha + \beta = 3$, $\alpha\beta = -13$ ✓ — 1 — $\frac{1}{2}$ mark each.

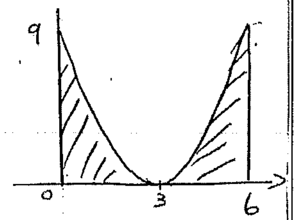
(ii) $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2+\beta^2}{(\alpha\beta)^2} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha^2\beta^2}$ ✓

$= \frac{9+26}{169} = \frac{35}{169}$ ✓ — 2

MATHEMATICS: Question 14 Pg 2

Suggested Solutions	Marks	Marker's Comments
(iii) $\alpha+\beta=3 \therefore \alpha=3-\beta$ $\alpha\beta = -13$ $(3-\beta)\beta=-13$ $3\beta-\beta^2=-13$ $\therefore 6\beta-2\beta^2=-26$ ✓	1	β is a root $\therefore \beta^2-3\beta-13=0$ $\therefore \beta^2-3\beta=13$ $\therefore 6\beta-2\beta^2=2\times(-13)$ $=-26$.
c) i) In ΔFBE and ΔEDC,  $\angle BFE=\angle ECD=60^\circ$ (alternate angles are equal, $AB \parallel CD$ opp. sides of square). ✓ $\angle FEB=\angle DEC$ (Vertically opposite angles are equal) ✓ $\therefore \Delta DEC \parallel\!\mid \Delta BEF$ (Equiangular) ✓	3.	
ii) $\angle FCB=30^\circ$ ($\angle DCB$ is 90°) $FB = BC\tan 30^\circ$ ✓ $=1\left(\frac{1}{\sqrt3}\right)$ $=\frac{1}{\sqrt3}$ iii) $DC=1$ and $FB=\frac{1}{\sqrt3}$ $\therefore \frac{DC}{FB}=\frac{1}{1/\sqrt3}=\sqrt3$ $\therefore$ ratio of areas $=(\sqrt3)^2:1^2$ $=3:1$ ✓		Different approaches $\frac{\text{Area } DEC}{\text{Area } BEF}$ using $A=\frac12 ab\sin C$.

MATHEMATICS: Question 14 Pg 3

Suggested Solutions	Marks	Marker's Comments
d). Subtract rotation of shaded area from cylinder.  $\text{Volume}_{(\text{Total})} = V_{\text{cylinder}} - V_{(\text{shaded area})}$ $=\pi 9^2\times 6 - \pi\int_0^6 y^2\,dx$ $=486\pi - \pi\int_0^6 ((x-3)^2)^2\,dx$ $=486\pi - \pi\int_0^6 (x-3)^4\,dx$ $=486\pi - \pi\left[\frac{(x-3)^5}{5}\right]_0^6$ $=486\pi - \frac{2\pi 3^5}{5}$ $=486\pi - \frac{486\pi}{5}$ $=\frac{1944\pi}{5}$ $\therefore$ Volume is $\frac{1944\pi}{5}$ u^3 accepted 388.8π u^3. or 1221.45 u^3		✓ V_{cylinder} ✓ $V\,(x-3)^4$ ✓ $\pi\int_0^6$ with correct working. If • 486π – did not square $(x-3)^2$. Some did not find volume of the cylinder • Some expanded $(x-3)^4$ as $x^4-12x^3+54x^2-108x+81$

MATHEMATICS: Question 15

Suggested Solutions	Marks	Marker's Comments
(a)(i) Stops leaking when $\frac{dV}{dt} = 0$		
$20t - 300 = 0$		
$20t = 300$	1	
$t = 15$		
∴ After 15 days		
(ii) $V = 10t^2 - 300t + c$	1	generally very well done.
when $t = 15$, $V = 4750$		
$\therefore 4750 = 10(15)^2 - 300(15) + c$	1/2	
$4750 = 2250 - 4500 + c$		
$c = 7000$	1/2	
$\therefore V = 10t^2 - 300t + 7000$	1	
(iii) when $t = 0$, $V = 7000$	1	
∴ 7000L at the start.		
(b)(i) $A_{24} = 500000\left(1 + \frac{0.06}{12}\right)^{24}$		
$= 500000(1.005)^{24}$	1	
$= 563579.888$		
$= \$563580$ to the nearest $		
(ii) $A_1 = 563580(1.0075) - M$		
$A_2 = A_1 \times 1.0075 - M$		1/2
$= 563580(1.0075) - M(1.0075 + 1)$		
$A_3 = 563580(1.0075)^2 - M(1.0075^2 + 1.0075 + 1)$		
$A_n = 563580(1.0075)^n - M(1.0075^{n-1} + 1.0075^{n-2} + \dots + 1)$	1/2	
after 8yrs owe $500 000		
8yrs $= 8 \times 12 = 96$ months	1/2	
$\therefore A_{96} = 500000$		
$\therefore 500000 = 563580(1.0075)^{96} - M(1.0075^{95} + 1.0075^{94} + \dots + 1)$		1/2
$500000 = 563580(1.0075)^{96} - M\left[\frac{1(1.0075^{96} - 1)}{1.0075 - 1}\right]$		1/2

MATHEMATICS: Question……..

Suggested Solutions	Marks	Marker's Comments
$500000 = 1154731.026 - \frac{M(1.0075^{96} - 1)}{0.0075}$		1/2
$M = \frac{654731.026}{139.856}$	1/2	* If they sumed 95 terms instead of 96, they got $= \frac{654731.026}{137.8224951}$ $= 4750.538187$ → 3½ mks
$= 4681.459$		
∴ M is $4681.46 to the nearest cent	1/2	
(iii)(α) $P = 500000$ $r = 12\%$ p.a		
$= 1\%$ p.month		
$M = \$6500$		1/2
$A_n = 500000(1.01)^n - M(1 + 1.01 + 1.01^2 + \dots + 1.01^{n-1})$		
$= 500000(1.01)^n - 6500(1 + 1.01 + \dots + 1.01^{n-1})$		1/2
$= 500000(1.01)^n - 6500\left[\frac{1(1.01^n - 1)}{1.01 - 1}\right]$		
$A_n = 500000(1.01)^n - 650000[1.01^n - 1]$		
but $A_n = 0$		1/2
$\therefore 0 = 500000(1.01)^n - 650000(1.01^n - 1)$		
$0 = 500000(1.01)^n - 650000(1.01)^n + 650000$		
$150000(1.01)^n = 650000$		1/2
$1.01^n = 13/3$		
$\log_{1.01}\left(\frac{13}{3}\right) = n$		* If they used "n−1" in sum of G.P. they got 152 full payments.
$n = \frac{\ln(13/3)}{\ln(1.01)}$		½
$n = 147.365$		1/2
∴ 148 full payments are made.		

3/3

MATHEMATICS: Question........

Suggested Solutions | **Marks** | **Marker's Comments**

(b) $A_{148} = 500\,000(1.01)^{148} - 6500\left(\dfrac{1.01^{148}-1}{0.01}\right)$ ½

$= 500\,000(1.01)^{148} - 650000(1.01)^{148} + 650000$ ½

$= 2180\,385.696 - 2184\,501.404$ ½

$= -4115.708$

∴ Refund is \$4115.71 ½

* If you did A_{147}, you got 2360.68449

∴ 6500 − 2360.68449

refund = 4139.31551

= 4139.32

1 mk, as it doesn't include the interest on the last month!!

* refund of \$4123.27 ⇒ 0 mks

MATHEMATICS: Question...16 1.

Suggested Solutions | **Marks** | **Marker's Comments**

(a) $y = \dfrac{10}{3+2\sin x}$ $\quad 0 \le x \le 2\pi$

$= 10(3+2\sin x)^{-1}$

$\dfrac{dy}{dx} = -10(3+2\sin x)^{-2} \times 2\cos x$

$\dfrac{dy}{dx} = \dfrac{-20\cos x}{(3+2\sin x)^2}$

For possible turning points $\dfrac{dy}{dx} = 0$ ½

ie $-20\cos x = 0$

$\cos x = 0$

$\therefore x = \dfrac{\pi}{2}$ or $\dfrac{3\pi}{2}$ ½

Testing the nature of the points $(\frac{\pi}{2}, 2)$ and $(\frac{3\pi}{2}, 10)$ 1 $(\frac{1}{2}+\frac{1}{2})$

x	0	$\pi/2$	π	$\frac{3\pi}{2}$	2π
$\frac{dy}{dx}$	$\frac{-20}{9}$	0	$\frac{20}{9}$	0	$\frac{-20}{9}$
	\	—	/	—	\

The function is continuous and differentiable throughout.

Since the gradient changes sign at $\frac{\pi}{2}$; $(\frac{\pi}{2}, 2)$ is a relative minimum ½

& since the gradient changes sign at $\frac{3\pi}{2}$; $(\frac{3\pi}{2}, 10)$ is a relative maximum ½

When $x = 0$, $y = \dfrac{10}{3+2\sin 0} = \dfrac{10}{3}$

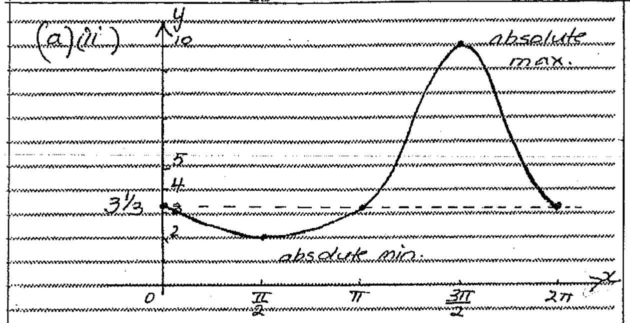
& $x = 2\pi$, $y = \dfrac{10}{3+2\sin 2\pi} = \dfrac{10}{3}$

Thus, $(\frac{\pi}{2}, 2)$ is an absolute minimum & $(\frac{3\pi}{2}, 10)$ is an absolute maximum.

Students used the quotient rule and many made errors with $\frac{d}{dx}(10) = 0$

Some students used the second derivative which was more tedious.

$\boxed{3}$

Suggested Solutions	Marks	Marker's Comments
(a)(ii) 	2	½ each for endpts & ½ each for max & min. [3]
b) (i) $P_A = 500 + 50t$	2	(Errors in making $P_A = 500 + (t-1)50 = 450 + 50t$) [2]
(ii) (α) Show $P_B = 500e^{-0.3t}$ satisfies $\frac{dP_B}{dt} = -0.3P_B$		
$P_B = 500e^{-0.3t}$		
$\frac{dP_B}{dt} = 500e^{-0.3t} \times (-0.3)$	½	
$= -0.3 . P_B$ since $P_B = 500e^{-0.3t}$	½	(Students failed to justify the final step) [1]
Thus, $P_B = 500e^{-0.3t}$ satisfies the equation		
(β) When $t = 6$, find P_B		
$P_B = 500e^{-0.3\times6}$ $= 500e^{-1.8}$ $= 82.649\ldots$	½	
$\therefore P_B = 83$ (to the nearest integer)	½	82?
(γ) $P_C = 100\left(5 + t - \frac{t^2}{4}\right)$ $= 500 + 100t - 25t^2$		Alternatively, $t = \frac{-(-4)}{2\times1} = 2$ (1mk) using $t = \frac{-b}{2a}$ because the max. lies on the axis of the parabola.
$\frac{dP_C}{dt} = 100 - 50t$	½	
We can determine a maximum when $\frac{dP_C}{dt} = 0$ and $\frac{d^2P_C}{dt^2} < 0$		
ie $100 - 50t = 0$ $\therefore t = 2$	½	

Suggested Solutions	Marks	Marker's Comments
$\frac{d^2P_C}{dt^2} = -50$	½	Alternatively, $a = -25$, $a < 0$ thus, curve is concave down and the vertex gives a maximum.
ie $\frac{d^2P_C}{dt^2} < 0$ and concave down when $t = 2$	1	
Thus, there is a maximum after 2 years		
$P_C(\text{max}) = 100\left(5 + 2 - \frac{2^2}{4}\right)$ $= 600$		
Maximum population of P_C is 600 people.	½	[4]
(δ) A town is deemed unviable if the population goes below 50.		
as $P_A = 500 + 50t$ it continues to increase for $t \le 10$, so does not drop below 50.	½	
$P_B = 500e^{-0.3t} = 50$ $e^{-0.3t} = \frac{1}{10}$	½	
$e^{0.3t} = 10$ $0.3t = \ln 10$ $t = \frac{10}{3}\ln 10$ $= 7.67\ldots$	½	
$P_C = 500 + 100t - 25t^2 = 50$ $t^2 - 4t - 18 = 0$	½	
$t = \frac{-(-4) \pm \sqrt{(-4)^2 - 4.1(-18)}}{2\times1}$ $= \frac{4 \pm \sqrt{88}}{2}$		
$t = \frac{4 + 2\sqrt{22}}{2}$, as $t > 0$ $\therefore t = 2 + \sqrt{22}$ $= 6.6904\ldots$	½	
The population of C will drop below 50 sooner than town B, thus town C will be the first to close	½	[3]