JAMES RUSE – **Year 9 Half Yearly Examination 2008**

SECTION A (14 Marks)

Marks

(1) Write an expression for the cost (in dollars) of N books valued at 75 cent each. 1

(2) If $R=\dfrac{ab}{a+b}$, find the value of R, correct to 1 decimal place, when $a=4$ and $b=\sqrt{6}$. 1

(3) The volume of a cylinder is given by the formula $V=\pi r^2 h$. Rewrite the formula with h as the subject. 1

(4) Factorise $12-4c-c^2$. 1

(5) Simplify $5\sqrt{24}$. 1

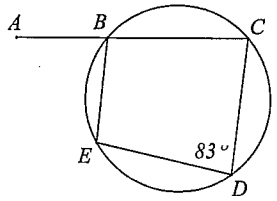
(6) Copy and complete the following theorem. "A cyclic quadrilateral has its 4 vertices ....." 1

(7) Evaluate $8^{\frac{4}{3}}$. 1

(8) Write the basic numeral for 3^0+3^{-2}. 1

(9) Solve for x: $2^x=\dfrac{1}{8}$. 1

(10) Find the size of $\angle ABE$ giving reasons. 1



(11) Solve for y: $5\sqrt{y}=3$. 1

(12) Expand $-3(5-2t)$. 1

(13) Factorise t^3+1000. 1

(14) Three unbiased 10 cent coins are tossed onto a table. Find the probability that at least one of the uppermost faces shows a head. 1

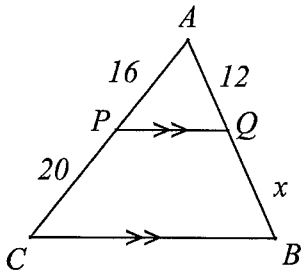
SECTION B (14 Marks)

Marks

(1) Rationalise the denominator of the fraction $\dfrac{3}{4-\sqrt{6}}$. 2

(2) Solve for p: $1-3p=11+2p$. 2

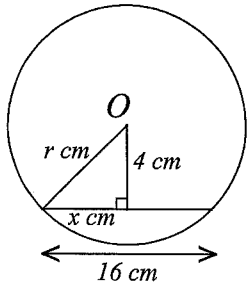
(3) Find the value of x giving reasons. 2



(4) Solve for k: $(2k-1)^2=25$ 2

(5) Make x the subject of the formula $H=\dfrac{1}{2}\left(\dfrac{1}{x}+\dfrac{1}{y}\right)$. (Write your answer in simplest form) 3

(6) A 16cm chord is drawn in a circle. If the shortest distance between the chord and the centre of the circle is 4cm, find the exact length of the radius of the circle giving reasons. 3



SECTION C (15 Marks)

Marks

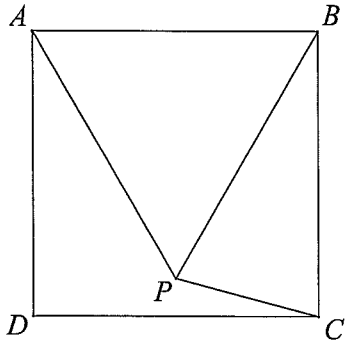
(1) Draw a number line and using a scale of 3cm = 1 unit mark the numbers 0, 1, 2, 3 and 4 on your number line.
On the number line you have drawn, construct the position of the number $\sqrt{5}$. Use a protractor, ruler and a pair of compasses. Show all constructions. — 2

(2) Fully factorise $6x-3y-4x^2+y^2$. — 3

(3) Solve for t: $\frac{1}{1-2t}=\frac{4}{t}$. — 3

(4) Simplify $\frac{2x}{x^2-4}-\frac{1}{x-2}$, $(x\neq\pm 2)$. — 3

(5) $ABCD$ is a square and APB is an equilateral triangle. Copy the diagram onto your answer page and find the size of $\angle BPC$ giving reasons.



SECTION D (15 Marks)

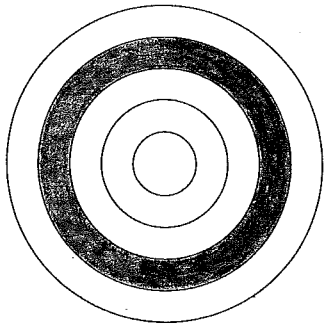
Marks

(1) Simplify $\frac{27-8p^3}{2p^2-5p+3}$. — 3

(2) Solve for q: $1-q=\sqrt{7-q}$. — 3

(3) If $x+\frac{1}{x}=4$ find the value of $x^2+\frac{1}{x^2}$. — 2

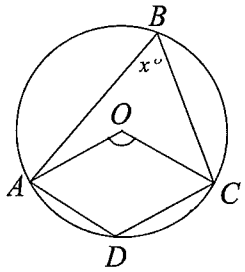
(4) A dart board is shown on the right. The dart board consists of 5 concentric circles with radii 2cm, 4cm, 6cm, 8cm and 10cm. A dart is thrown at the dart board hitting it at a random point. What is the probability that the dart hits the shaded region? — 3



(5) O is the centre of a circle that passes through the points A, B, C and D.

(i) If $\angle ABC = x°$, write an expression in terms of x for the size of $\angle AOC$ giving reasons. — 1

(ii) If $AOCD$ is a rhombus, find the value of x giving reasons. — 3



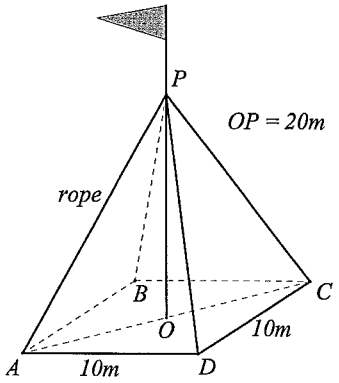
SECTION E (12 Marks)

Marks

(1) Simplify $\dfrac{4x+8}{x^2-9} \div \dfrac{x^2-4}{x^2-5x+6}$, $(x \neq \pm 2, \pm 3)$. 3

(2) If $m+n\sqrt{2} = \left(2\sqrt{3}-\sqrt{6}\right)^2$, find the value of m and n where m and n are rational numbers 3

(3) A flagpole is to be erected at the center of a square whose side lengths are 10m. The flagpole is held upright (i.e. perpendicular to the ground) by 4 ropes attached to the flagpole at a point 20m above the ground and to the ground at the corners of the square (see diagram). It is decided that the total length of rope needed for each support should be calculated as the shortest distance from the flagpole to the ground plus 5% (for tying the rope to the flagpole and the ground). Calculate the total length of rope needed to support the flagpole. (Give your answer to the nearest centimetre) 3



(4) On a table are two containers and each holds 5 discs with a single number on each disc. The blue container has 5 blue discs numbered 2, 3, 5, 7 and 11 while the green container has 5 green discs numbered 4, 6, 8, 9 and 10. A disc is chosen at random from each container and their sum is recorded.

(i) draw a diagram to illustrate the sample space. 1

Hence find the probability that

(ii) the sum of the two discs is prime. 1

(iii) the sum of the two discs is even given that at least one of the chosen discs shows a number greater than 6. 1

⌘ ✦ THIS IS THE END OF THE EXAMINATION PAPER ✦ ⌘

Year 9 Half Yearly Examination 2008 - SOLUTIONS

SECTION A (14 Marks)

Marks

(1) Write an expression for the cost (in dollars) of N books valued at 75 cent each.\ — 1

$0.75N$ or $\frac{3}{4}N$

(2) If $R=\dfrac{ab}{a+b}$, find the value of R, correct to 1 decimal place, when $a=4$ and $b=\sqrt{6}$. — 1

$$R=\frac{4\sqrt{6}}{4+\sqrt{6}}$$
$$=1.5 \text{ (to 1 d.p.)}$$

(3) The volume of a cylinder is given by the formula $V=\pi r^2 h$. Rewrite the formula with h as the subject. — 1

$$h=\frac{V}{\pi r^2}$$

(4) Factorise $12-4c-c^2$. — 1

$12-4c-c^2=(6+c)(2-c)$

(5) Simplify $5\sqrt{24}$. — 1

$5\sqrt{24}=5\times 2\sqrt{6}$
$=10\sqrt{6}$

(6) Copy and complete the following theorem. "A cyclic quadrilateral has its 4 vertices ....." — 1

A cyclic quadrilateral has its 4 vertices on the circumference of a circle.

(7) Evaluate $8^{\frac{4}{3}}$. — 1

$8^{\frac{4}{3}}=16$

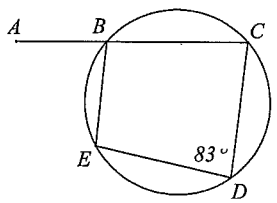
(8) Write the basic numeral for 3^0+3^{-2}. — 1

$3^0+3^{-2}=1\frac{1}{9}$

(9) Solve for x: $2^x=\dfrac{1}{8}$. — 1

$x=-3$

(10) Find the size of $\angle ABE$ giving reasons. — 1



$\angle ABE=83°$ (exterior angle of a cyclic quadrilateral equals its opposite interior angle)

(11) Solve for y: $5\sqrt{y}=3$. — 1

$5\sqrt{y}=3$
$25y=9$
$y=\frac{9}{25}$

(12) Expand $-3(5-2t)$. — 1

$-3(5-2t)=-15+6t$

(13) Factorise t^3+1000. — 1

$t^3+1000=(t+10)(t^2-10t+100)$

(14) Three unbiased 10 cent coins are tossed onto a table. Find the probability that at least one of the uppermost faces shows a head. — 1

Prob. $=\frac{7}{8}$

SECTION B (14 Marks)

Marks

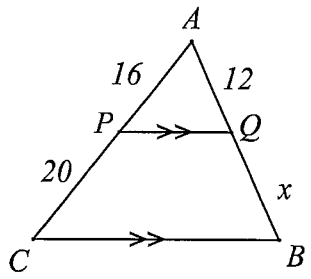
(1) Rationalise the denominator of the fraction $\frac{3}{4-\sqrt{6}}$. 2

$$\frac{3}{4-\sqrt{6}}=\frac{3}{4-\sqrt{6}}\times\frac{4+\sqrt{6}}{4+\sqrt{6}}$$
$$=\frac{3(4+\sqrt{6})}{16-6}$$
$$=\frac{3(4+\sqrt{6})}{10}$$

(2) Solve for p: $1-3p=11+2p$. 2

$1-3p=11+2p$
$-5p=10$
$p=-2$

(3) Find the value of x giving reasons. 2



$\frac{x}{12}=\frac{20}{16}$ (interval parallel to a side of a triangle divides other two sides in the same ratio)

$x=\frac{20\times12}{16}$
$=15$

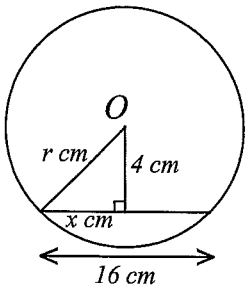
(4) Solve for k: $(2k-1)^2=25$ 2

$(2k-1)^2=25$
$2k-1=\pm5$
$2k=-4$ or 6
$k=-2$ or 3

(5) Make x the subject of the formula $H=\frac{1}{2}\left(\frac{1}{x}+\frac{1}{y}\right)$. (Write your answer in simplest form) 3

$H=\frac{1}{2}\left(\frac{1}{x}+\frac{1}{y}\right)$
$2H=\frac{1}{x}+\frac{1}{y}$
$\frac{1}{x}=2H-\frac{1}{y}$
$=\frac{2Hy-1}{y}$
$x=\frac{y}{2Hy-1}$

(6) A 16cm chord is drawn in a circle. If the shortest distance between the chord and the centre of the circle is 4cm, find the exact length of the radius of the circle giving reasons. 3

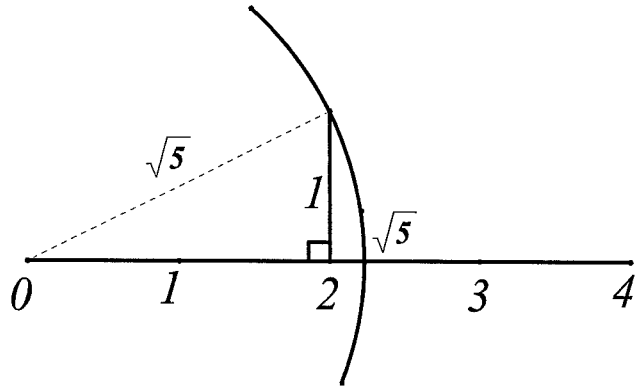


Shortest distance from point to line is perpendicular distance.
x = 8cm (radius perpendicular to chord bisects the chord)
$r^2=8^2+4^2$ (Pythagoras' Theorem)
$r^2=80$
$r=\sqrt{80}$
radius $=\sqrt{80}$ cm $(or\ 4\sqrt{5}cm)$

SECTION C (15 Marks)

Marks

(1) Draw a number line and using a scale of 3cm = 1 unit mark the numbers 0, 1, 2, 3 and 4 on your number line.
On the number line you have drawn, construct the position of the number $\sqrt{5}$. Use a protractor, ruler and a pair of compasses. Show all constructions. 2



(2) Fully factorise $6x-3y-4x^2+y^2$. 3

$$\begin{aligned}6x-3y-4x^2+y^2 &= 3(2x-y)-(4x^2-y^2)\\ &= 3(2x-y)-(2x-y)(2x+y)\\ &= (2x-y)[3-(2x+y)]\\ &= (2x-y)(3-2x-y)\end{aligned}$$

(3) Solve for t: $\dfrac{1}{1-2t}=\dfrac{4}{t}$. 3

$$\begin{aligned}&\frac{1}{1-2t}=\frac{4}{t}\\ &t=4-8t\\ &9t=4\\ &t=\tfrac{4}{9}\end{aligned}$$

(4) Simplify $\dfrac{2x}{x^2-4}-\dfrac{1}{x-2}$, $(x\neq\pm 2)$. 3

$$\begin{aligned}\frac{2x}{x^2-4}-\frac{1}{x-2} &= \frac{2x}{(x-2)(x+2)}-\frac{1}{x-2}\\ &= \frac{2x-(x+2)}{(x-2)(x+2)}\\ &= \frac{2x-x-2}{(x-2)(x+2)}\\ &= \frac{x-2}{(x-2)(x+2)}\\ &= \frac{1}{x+2}\end{aligned}$$

(5) $ABCD$ is a square and APB is an equilateral triangle. Copy the diagram onto your answer page and find the size of $\angle BPC$ giving reasons.

$\angle ABC = 90°$ (all angles at vertices in a square are 90°)

$\angle ABP = 60°$ (all angles at vertices in an equilateral triangle are 60°)

$\angle PBC = \angle ABC - \angle ABP$
$= 30°$

$BP = AB$ (all in an equilateral triangle are equal)

$BC = AB$ (all sides in a square are equal)

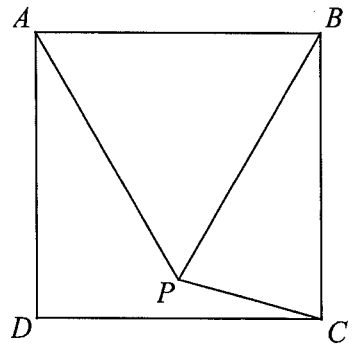
$BP = BC$ (both $= AB$)

$\angle BPC = \angle BCP$ (equal angles are opposite equal sides in ΔBPC)

$2\angle BPC + 30° = 180°$ (angle sum of $\Delta BPC = 180°$)

$2\angle BPC = 150°$

$\angle BPC = 75°$



SECTION D (15 Marks)

Marks

(1) Simplify $\dfrac{27-8p^3}{2p^2-5p+3}$. — 3

$$\frac{27-8p^3}{2p^2-5p+3}=\frac{(3-2p)(9+6p+4p^2)}{(2p-3)(p-1)}$$
$$=\frac{-(2p-3)(9+6p+4p^2)}{(2p-3)(p-1)}$$
$$=\frac{-(9+6p+4p^2)}{(p-1)}$$
$$=\frac{9+6p+4p^2}{1-p}$$

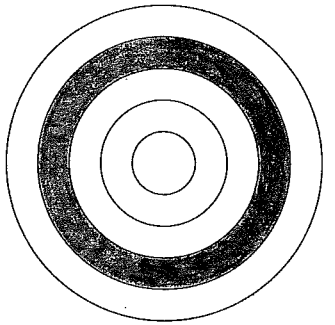
(2) Solve for q: $1-q=\sqrt{7-q}$. — 3

$1-q=\sqrt{7-q}$
$(1-q)^2=7-q$
$1-2q+q^2=7-q$
$q^2-q-6=0$
$(q-3)(q+2)=0$
$q=-2, 3$
but $1-q\geq 0$ and $7-q\geq 0$
$\therefore q\leq 1$
$q=-2$

(3) If $x+\dfrac{1}{x}=4$ find the value of $x^2+\dfrac{1}{x^2}$. — 2

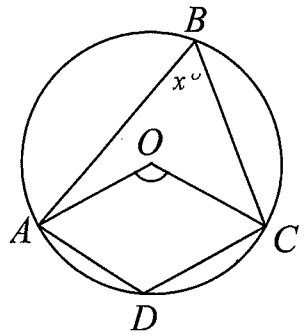
$\left(x+\dfrac{1}{x}\right)^2=x^2+2+\dfrac{1}{x^2}$
$4^2=x^2+\dfrac{1}{x^2}+2$
$x^2+\dfrac{1}{x^2}=14$

(4) A dart board is shown on the right. The dart board consists of 5 concentric circles with radii 2cm, 4cm, 6cm, 8cm and 10cm. A dart is thrown at the dart board hitting it at a random point. What is the probability that the dart hits the shaded region? — 3



Shaded area $=\pi(8)^2-\pi(6)^2\ cm^2$
$=28\pi\ cm^2$
Target area $=\pi(10)^2\ cm^2$
$=100\pi\ cm^2$
Probability $=\dfrac{28\pi}{100\pi}$
$=0.28$

(5) O is the centre of a circle that passes through the points A, B, C and D.



(i) If $\angle ABC=x°$, write an expression in terms of x for the size of $\angle AOC$ giving reasons. — 1

$\angle AOC=2x°$ (angle at centre of circle is twice the angle at the circumference standing on the same arc)

(ii) If $AOCD$ is a rhombus, find the value of x giving reasons. — 3

$\angle ADC=2x°$ (opposite angles of a rhombus are equal)
$x+2x=180$ (opposite angles of a cyclic quadrilateral are supplementary)
$x=60$

SECTION E (12 Marks)

Marks

(1) Simplify $\frac{4x+8}{x^2-9} \div \frac{x^2-4}{x^2-5x+6}$, $(x \neq \pm 2, \pm 3)$. 3

$$\frac{4x+8}{x^2-9} \div \frac{x^2-4}{x^2-5x+6} = \frac{4(x+2)}{(x-3)(x+3)} \times \frac{(x-3)(x-2)}{(x-2)(x+2)}$$

$$= \frac{4}{x+3}$$

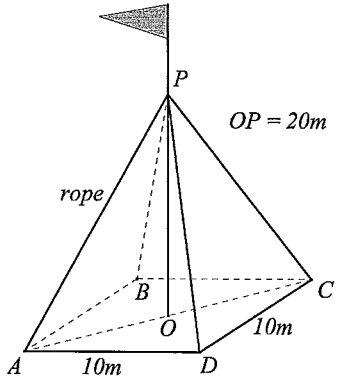
(2) If $m+n\sqrt{2} = \left(2\sqrt{3}-\sqrt{6}\right)^2$, find the value of m and n where m and n are rational numbers 3

$$\left(2\sqrt{3}-\sqrt{6}\right)^2 = 12-4\sqrt{18}+6$$

$$= 18-12\sqrt{2}$$

$m = 18$ and $n = -12$

(3) A flagpole is to be erected at the center of a square whose side lengths are 10m. The flagpole is held upright (i.e. perpendicular to the ground) by 4 ropes attached to the flagpole at a point 20m above the ground and to the ground at the corners of the square (see diagram). It is decided that the total length of rope needed for each support should be calculated as the shortest distance from the flagpole to the ground plus 5% (for tying the rope to the flagpole and the ground). Calculate the total length of rope needed to support the flagpole. (Give your answer to the nearest centimetre) 3



$\angle ADC = 90°$

$AC^2 = 10^2 + 10^2$ (Pythagoras' Theorem)

$= 200$

$AC = 10\sqrt{2}\ m$

$AO = 5\sqrt{2}\ m$

$AC^2 = \left(5\sqrt{2}\right)^2 + 20^2$ (Pythagoras' Theorem)

$= 450$

$AC = 15\sqrt{2}\ m$

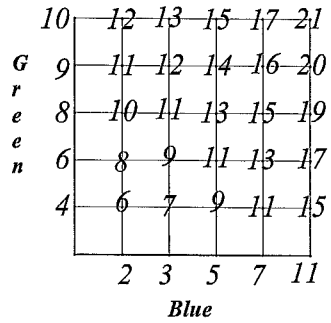
Length of rope $= 4 \times 15\sqrt{2} \times 1.05\ m$

$\approx 89.095\ m$

$= 89.10\ m$ (to nearest cm)

(4) On a table are two containers and each holds 5 discs with a single number on each disc. The blue container has 5 blue discs numbered 2, 3, 5, 7 and 11 while the green container has 5 green discs numbered 4, 6, 8, 9 and 10. A disc is chosen at random from each container and their sum is recorded.

(i) Draw a diagram to illustrate the sample space



Hence find the probability

(ii) that the sum of the two discs is prime.

$$\text{Prob.} = \frac{11}{25}$$

(iii) that the sum is even given that at least one of the chosen discs showed a number greater than 6.

$$\text{Prob.} = \frac{6}{19}$$

⌘ ✦ THIS IS THE END OF THE EXAMINATION PAPER ✦ ⌘