JAMES RUSE - YR 9 HALF YRLY - 06

SECTION A (15 Marks)

	Question	Marks
1.	Simplify:	
	a) $3\sqrt{96}-\sqrt{150}$	2
	b) $2x+1-3(2-x)$	1
2.	Solve: $\frac{2x}{3}=\frac{1}{8}$	1
3.	Express $64^{-\frac{4}{3}}$ in the form $\frac{a}{b}$ where a and b are integers.	1
4.	Make G the subject in: $F=H+\frac{1}{G}$	2
5.	Factorise:	
	a) $8+27x^3$	2
	b) $-2t^2+11t-12$	2
	c) x^5-x	2
6.	Simplify: $\frac{x^3-125}{5-x}$	2

SECTION B (15 Marks) Start a new page

	Question	Marks
1.	A bag contains 20 marbles coloured red, white, blue or green. There is one more red than white, 4 more white than blue and one more blue than green.	
	a) By letting x be the number of green marbles, write an equation and solve it and show that there are 8 red marbles.	2
	b) Find the probability of drawing a red marble from the bag.	1
	c) If two marbles are selected one after the other, the first not replaced before the second is drawn, find the probability of selecting two red marbles.	1
2.	Make x the subject in $y=\frac{x+2}{x-3}$	2
3.	Given the following stem and leaf plot:	

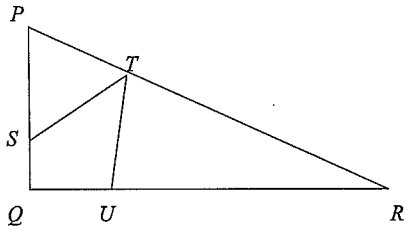
Stem	Leaf
3	8 6
4	2 5 6 5
5	8 6 3 4 7 7
6	7 5 9
7	6 5

	a) Find the median score.	1
	b) Find the interquartile range.	1
	c) Find the mean (to 1 decimal place).	1
	d) Find the standard deviation (to 1 decimal place).	1
	e) Draw a box and whisker plot that represents the given set of scores.	2
4.	Solve the equation: $\frac{4}{3x^2+16x+5}-\frac{2}{x^2+3x-10}=0$	3

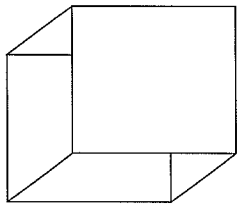
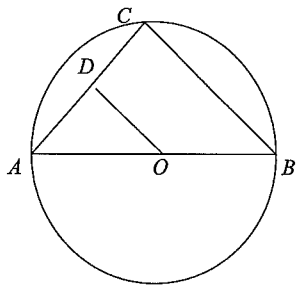
SECTION C (16 Marks) Start a new page

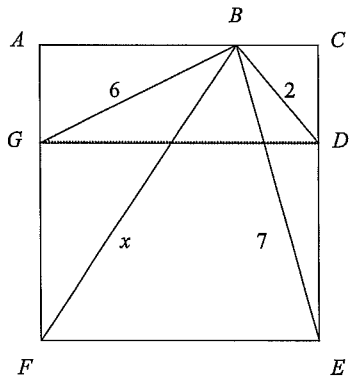
	Question	Marks
1.	a) Expand $\left(x+\frac{1}{x}\right)^2$	1
	b) Hence, if $x+\frac{1}{x}=3$, find the value of $x^2+\frac{1}{x^2}$	1
2.	Simplify: $\frac{3^{2n}-1}{3^{n+1}-3}$	2
3.	A deck of 12 cards contains 4 kings, 4 queens, and 4 jacks. My opponent is dealt 2 cards at random. He says he holds at least one king. What is the probability that he holds 2 kings in his hand?	2
4.	John in year 9 scored 81% in each of his Mathematics and Science exams. In which subject did he perform better? Give reasons for your answer.	2

Subject	$\bar{x}$	σ	x
Maths	72	4	81
Science	78	3	81

5.	If $a=\frac{1}{\sqrt{5}-2}$, show that $4+\frac{1}{a}=a$	2
6.	After 200 throws, a coin has shown heads exactly 110 times. By forming an equation and solving it, find how many further consecutive times must a head be thrown for heads to show exactly 70% of the total number of throws?	2
7.		
	Triangle PQR is right angled at Q. Triangles PST and TRU are isosceles as shown. Let $\angle STU=x^\circ$ and $\angle SPT=y^\circ$.	
	a) Show $\angle UTR=\left(45+\frac{1}{2}y\right)^0$, giving reasons for your answer.	2
	b) Find the value of x, giving reasons for your answer.	2

SECTION D (16 Marks) Start a new page

	Question	Marks
1.		
	The diagram represents a cube with diagonal $PU=\sqrt{54}$ cm. The cube has side length x cm.	3
	Find the exact volume of the cube.	
2.	a) Factorise $\sqrt{7x}-\sqrt{3x}$.	1
	b) Hence, solve the equation: $\sqrt{7x}-\sqrt{3x}=4$.	2
3.		
	Given AB is a diameter of the circle, centre O. $AD=DC$.	
	a) State why $\angle ADO=90^0$.	1
	b) Hence prove $\angle ACB=90^0$, giving reasons for your answer.	2
	c) Prove the perimeter ΔAOD is half the perimeter ΔABC, giving reasons for your answer.	3

4.	In the rectangle $ACEF$, B,D and G are points on the sides as shown. $BG = 6$ cm, $BD = 2$ cm and $BE = 7$ cm and BF $= x$ cm. Find the value of x.  	4

End of Exam paper

Section A

1. a) $3\sqrt{96} - \sqrt{150}$

$= 3 \times 4\sqrt{6} - 5\sqrt{6}$ 1

$= 7\sqrt{6}$ 1

b) $2x+1-3(2-x)$

$= 2x+1-6+3x$

$= 5x-5$ 1

2. $\frac{2x}{3} = \frac{1}{8}$

$x = \frac{1}{8} \times \frac{3}{2}$ 1

$\therefore x = \frac{3}{16}$

3. $64^{-\frac{4}{3}} = \frac{1}{64^{4/3}}$

$= \frac{1}{256}$ 1

4. $F = H + \frac{1}{G}$

$F - H = \frac{1}{G}$ 1

$\frac{1}{F-H} = G$ 1

5. a) $8 + 27x^3$

$= (2+3x)(4-6x+9x^2)$ 1 1

b) $-2t^2 + 11t - 12$

$-2t^2 + 8t + 3t - 12$ $\quad \times 24$, $+11$, $8, 3$

$-2t(t-4) + 3(t-4)$ 1

$(t-4)(3-2t)$ 1

c) $x^5 - x$

$= x(x^4 - 1)$

$= x(x^2+1)(x^2-1)$ 1

$= x(x^2+1)(x+1)(x-1)$ 1

6. $\frac{x^3 - 125}{5-x}$

$= \frac{(x-5)(x^2+5x+25)}{(5-x)}$ 1

$= -(x^2+5x+25)$ OR $-x^2-5x-25$ 1

Section B

1. x = green

$x+1$ = blue

$x+5$ = white

$x+6$ = Red

a) $x + x+1 + x+5 + x+6 = 20$ 1

$4x + 12 = 20$

$4x = 8$ 1

$x = 2$ 1

$\therefore$ Red $= 2+6 = 8$ marbles

b) $\frac{8}{20} = \frac{2}{5}$ 1

c) $\frac{8}{20} \times \frac{7}{19} = \frac{28}{195}$ 1

2. $y = \frac{x+2}{x-3}$

$yx - 3y = x + 2$ 1

$yx - x = 3y + 2$ 1

$x(y-1) = 3y + 2$

$x = \frac{3y+2}{y-1}$ 1

3. Ordered stem-leaf:

3	6 8
4	2 5 5 6
5	3 4 6 7 7 8
6	5 7 9
7	5 6

a) 56

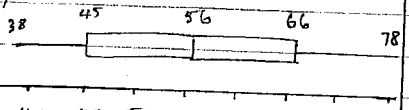
b) LQ = 45 UQ = 66

IQR = 66 − 45 = 21 1

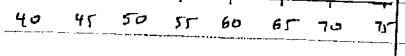
c) 55.2 1

d) 11.9 1

e)



2



4. $\frac{4}{3x^2+16x+5} - \frac{2}{x^2+3x-10} = 0$

$\frac{4}{(x+5)(3x+1)} - \frac{2}{(x+5)(x-2)} = 0$ 1

$4(x-2) - 2(3x+1) = 0$; $x \neq -5$

$4x - 8 - 6x - 2 = 0$

$-2x - 10 = 0$

$-10 = 2x$

$-5 = x$ 1

but $x \neq -5$, so

x has no soln

Section C

1. a) $\left(x + \frac{1}{x}\right)^2$

$= x^2 + \frac{1}{x^2} + 2$ 1

b) $x^2 + \frac{1}{x^2}$

$= \left(x + \frac{1}{x}\right)^2 - 2$

$= 3^2 - 2$ 1

$= 7$ 1

2. $\frac{3^{2n} - 1}{3^{n+1} - 3}$

$= \frac{(3^n - 1)(3^n + 1)}{3(3^n - 1)}$ 1

$= \frac{3^n + 1}{3}$ 1

3.

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K	○ ○ ○ × . . . . .
Q	. . . .
Q	. . . .
Q	. . . .
Q	. . . .
J	. . . .
J	. . . .
J	. . . .
J	. . . .

$P(KK) = \frac{12}{32+32+12}$ 1

$= \frac{12}{76} = \frac{3}{19}$ 1

4

Maths: $\bar{x} = 72$

$\bar{x} + 1\sigma = 76$

$\bar{x} + 2\sigma = 80$

$\therefore$ Mark of 81 is more than 2σ from mean ½

Science: $\bar{x} = 78$

$\bar{x} + 1\sigma = 81$

$\therefore$ Mark of 81 is 1σ from mean ½

Thus, John did better in Maths. 1

5. $a = \dfrac{1}{\sqrt{5} - 2}$

$\dfrac{1}{a} = \sqrt{5} - 2$

LHS: $4 + \dfrac{1}{a}$

$= 4 + \sqrt{5} - 2$

$= 2 + \sqrt{5}$ 1

RHS: a

$= \dfrac{1}{\sqrt{5} - 2}$

$= \dfrac{\sqrt{5} + 2}{5 - 4}$ 1

$= \sqrt{5} + 2$

$\therefore$ LHS = RHS

6. Let x = no. of heads still to throw

$\dfrac{110 + x}{200 + x} = \dfrac{7}{10}$ 1

$10(110 + x) = 7(200 + x)$

$1100 + 10x = 1400 + 7x$

$3x = 300$

$x = 100$ 1

$\therefore$ 100 more heads need to be thrown consecutively.

7. a)

$\angle R = 90 - y$ (angle sum of $\triangle PQR$ is $180°$) 1

$\angle UTR = \dfrac{180 - (90 - y)}{2}$ 1

(angles opposite equal sides of triangle are equal)

(angle sum of $\triangle TRU$ is $180°$)

$= \dfrac{45 - y}{2}$

b) $\angle PTS = \dfrac{180 - y}{2}$ (as above)

$= 90 - \dfrac{y}{2}$ 1

$\angle PTS + x + \angle UTR = 180°$

(angles on straight angle add to $180°$)

$90 - \dfrac{y}{2} + x + 45 + \dfrac{y}{2} = 180°$

$x + 135 = 180$

$x = 45°$ 1

Section D

1. $QU^2 = x^2 + x^2$

$QU^2 = 2x^2$ 1

$PQ^2 + QU^2 = PU^2$

$x^2 + 2x^2 = 54$

$3x^2 = 54$

$x^2 = 18$

$x = \sqrt{18}$ or $3\sqrt{2}$ 1

Volume $= (3\sqrt{2})^3 = 27 \times 2\sqrt{2}$

$= 54\sqrt{2}\ cm^3$ 1

2.a) $\sqrt{7x} - \sqrt{3x}$

$= \sqrt{x}(\sqrt{7} - \sqrt{3})$ 1

b) $\sqrt{x}(\sqrt{7} - \sqrt{3}) = 4$

$\sqrt{x} = \dfrac{4}{\sqrt{7} - \sqrt{3}}$ *

$x = \dfrac{16}{7 + 3 - 2\sqrt{21}}$

$= \dfrac{16}{10 - 2\sqrt{21}} \times \dfrac{10 + 2\sqrt{21}}{10 + 2\sqrt{21}}$

$= \dfrac{16(10 + 2\sqrt{21})}{100 - 4 \cdot 21}$

$= \dfrac{32(5 + \sqrt{21})}{16}$

$= 2(5 + \sqrt{21})$

Alternatively *

$\sqrt{x} = \dfrac{4}{\sqrt{7} - \sqrt{3}}$

$= \dfrac{4(\sqrt{7} + \sqrt{3})}{7 - 3}$ 1

$= \sqrt{7} + \sqrt{3}$

$x = (\sqrt{7} + \sqrt{3})^2$ 1

$= 7 + 3 + 2\sqrt{21}$

$= 10 + 2\sqrt{21}$

3 a) $\angle ADO = 90°$

(a line thru the centre of a circle, bisecting a chord is perpendicular to it) 1

b) $AO = BO$ (equal radii) 1

$OD \parallel BC$ (a line thru the midpoints of 2 sides of a triangle is parallel to it) 1

$\angle ACB = 90°$ (corresponding angles ADO & ACB are equal as $OB \parallel BC$) 1

c) $OD = \frac{1}{2} BC$ (a line thru the midpoint of 2 sides of a triangle is ½ the third side) 1

$AO = \frac{1}{2} AB$ (O is midpoint AB)

$AD = \frac{1}{2} CD$ (O is midpoint AC) 1

$\therefore$ Perimeter $\triangle AOD = \frac{1}{2}$ perimeter $\triangle ABC$

4. $AB^2 + AG^2 = 6^2 \Longrightarrow AB^2 = 6^2 - AG^2$ $(\frac{1}{2})$

$AF^2 + BC^2 = 7^2 \Longrightarrow AF^2 = 7^2 - BC^2$ $(\frac{1}{2})$

$AG^2 + BC^2 = z^2$ 1

$\therefore AB^2 + AF^2 = x^2$

$6^2 - AG^2 + 7^2 - BC^2 = x^2$ 1

$6^2 + 7^2 - (AG^2 + BC^2) = x^2$

$6^2 + 7^2 - z^2 = x^2$

$81 = x^2$

$\therefore x = 9$ 1