Student ID: _____

KAMBALA

Mathematics Extension 1

HSC Assessment Task 1

November 2006

Circle Geometry, Further Trigonometry, Polynomials, Inequalities

Time Allowed: 50 minutes working time

Outcomes Assessed

H5 applies appropriate techniques from the study of calculus, geometry and trigonometry
H6 uses the derivative to determine the features of the graph of a function
PE3 solves problems involving inequalities, polynomials and circle geometry
PE6 makes comprehensive use of mathematical language, diagrams and notation for communicating in a wide variety of situations
HE7 evaluates mathematical solutions to problems and communicates them in an appropriate form

INSTRUCTIONS

- This examination contains 3 questions of 12 marks each. Marks for each question are shown.
- Answer all questions on the writing paper provided. **Start each question on a new page.**
- Calculators may be used.
- Show all necessary working.
- Marks may be deducted for careless or badly arranged work.

Question 1 (12 Marks) (*Start a new page.*) Marks

(a) If α,β,γ are the roots of $2x^3-4x^2+5=0$ find:

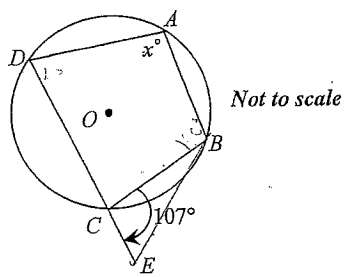
(i) $\alpha\beta\gamma$ 1

(ii) $\alpha\beta+\alpha\gamma+\beta\gamma$ 1

(iii) $\frac{1}{\alpha\beta}+\frac{1}{\alpha\gamma}+\frac{1}{\beta\gamma}$ 2

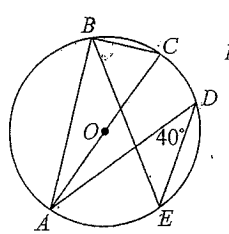
(b) Find the acute angle between the lines $y=2x+5$ and $3x+y=1$. 2

(c) (i) A, B, C and D are points on the circumference and O is the centre of the circle. Find the value of the x, giving reasons for your answer. 1



Not to scale

(ii) In the diagram A, B, C, D and E are points on the circumference, O is the centre of the circle and $\angle ADE = 40°$. Find $\angle EBC$ giving reasons. 2



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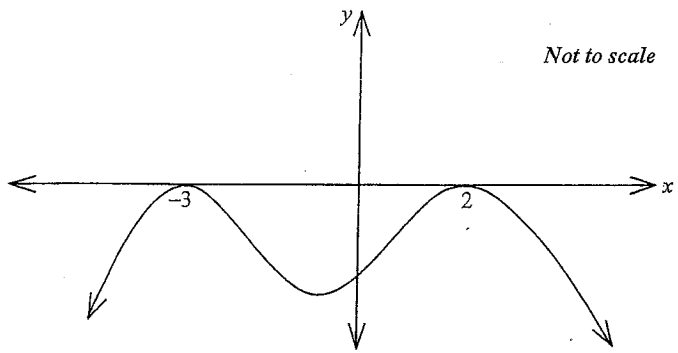
(d) (i) Expand $\sin(A+B)$. 1

(ii) The angles A and B are acute and $\sin A=\frac{4}{5}$ and $\sin B=\frac{8}{17}$. 2

Find the exact value of $\sin(A+B)$.

Question 2 (12 Marks) *(Start a new page.)* **Marks**

(a) Write down a possible equation for this graph. **1**



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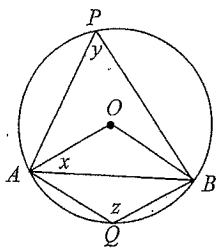
(b) In the diagram O is the centre of the circle.



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(i) Copy or trace the diagram onto your answer sheet and prove that $x+y=90°$ **2**

(ii) Prove that $z=2x+y$ **2**

(c) (i) Show that $P(x)=2x^3-9x^2+12x-2$ has a real root in the interval $0<x<2$. **2**

(ii) Show that $P(x)$ has stationary points when $x=1$ and $x=2$. **2**

(iii) Why would $x=1$ be an inappropriate choice as a first approximation for finding the root between $x=0$ and $x=2$ by Newton's Method? **1**

(iv) Starting with $x=\frac{1}{2}$, use Newton's Method once to find a better approximation for the root. **2**

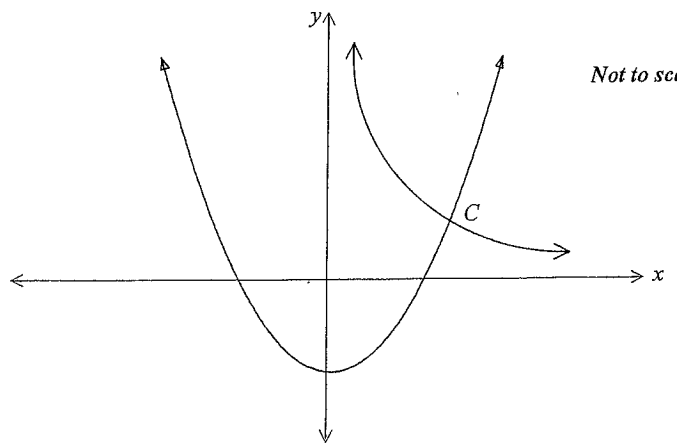
Question 3 (12 Marks) *(Start a new page.)* **Marks**

(a) (i) Show that $\dfrac{1-\cos 2x}{1+\cos 2x}=\tan^2 x$ **2**

(ii) Hence find the value of $\tan^2 22\frac{1}{2}°$ in simplest exact form with a rational denominator. **2**

(b) When $P(x)=x^3+2x^2-ax+1$ is divided by $(x-1)(x+2)$ the remainder is $3x+b$. Find a and b. **3**

(c) The diagram below shows the graph of $y=x^2-3$ and, for $x>0$, the graph of $y=\dfrac{2}{x}$. In the first quadrant the curves intersect at C.



Not to scale

(i) Show that the x co-ordinate of C is given by the equation $x^3-3x-2=0$. **1**

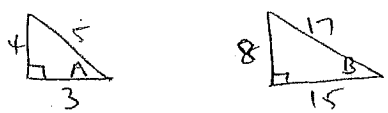
(ii) Factorise $x^3-3x-2=0$ and hence write down the x co-ordinate of C. **2**

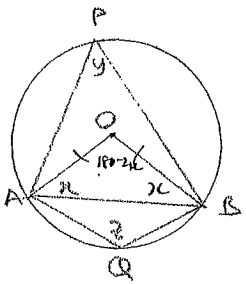
(iii) Solve $x^2-3\le\dfrac{2}{x}$ for all x. **2**

End of Assessment Task

Ext 1 HSC Task #1 2006 SOLUTIONS

Qn	Solutions	Marks	Comments+Criteria
1	(a) $2x^3 - 4x^2 + 5 = 0$ $a=2,\ b=-4,\ c=0,\ d=5$ $\alpha+\beta+\gamma = -\frac{b}{a} = \frac{4}{2} = 2$ $\alpha\beta+\beta\gamma+\alpha\gamma = \frac{c}{a}$ $\alpha\beta\gamma = -\frac{d}{a}$		
	(i) $\alpha\beta\gamma = -\frac{5}{2}$	1	
	(ii) $\alpha\beta + \gamma\alpha + \beta\gamma = 0$	1	
	(iii) $\frac{1}{\alpha\beta} + \frac{1}{\alpha\gamma} + \frac{1}{\beta\gamma}$ $= \frac{\gamma+\beta+\alpha}{\alpha\beta\gamma}$ $= \frac{2}{-\frac{5}{2}}$	1	
	$= -\frac{4}{5}$	1	
	(b) $y = 2x+5 \quad m_1 = 2$ $y = -3x+1 \quad m_2 = -3$	$\frac{1}{2}$	
	$\tan\theta = \left\lvert \frac{m_1 - m_2}{1+m_1m_2} \right\rvert$ $= \left\lvert \frac{2+3}{1-6} \right\rvert$ $= \left\lvert \frac{5}{-5} \right\rvert$	1	
	$\tan\theta = \lvert -1 \rvert$ $= 1$ $\theta = 45^\circ$	$\frac{1}{2}$	

Qn	Solutions	Marks	Comments+Criteria
1 cont'd	(c)(i) $x = 107^\circ$ ext $\angle$ of cyclic quad equals int. opp. $\angle$	1	
	(ii) $\angle ABE = \angle ADE = 40^\circ$ ($\angle$s in same segment) $\angle ABC = 90^\circ$ ($\angle$ in semicircle) $\angle EBC = \angle ABC - \angle ABE$ $= 90 - 40$ $= 50^\circ$	1 1	
	(d)(i) $\sin(A+B)$ $= \sin A\cos B + \cos A\sin B$	1	
	(ii) $\sin A = \frac{4}{5}$, $\sin B = \frac{8}{17}$.  $\sin(A+B) = \frac{4}{5}\times\frac{15}{17} + \frac{3}{5}\times\frac{8}{17}$ $= \frac{60}{85} + \frac{24}{85}$ $= \frac{84}{85}$	1 1	

Qn	Solutions	Marks	Comments+Criteria
2	(a) $y = -(x+3)^2(x-2)^2$		
	(b) (i)  $180 - 2x = 2y$ $\therefore 90 - x = y$ ie $x + y = 90°$		
	(ii) $\angle ADB = 2\angle APB$ $\therefore 180 - 2x = 2y$. $z = 180 - y$ $= (2x + 2y) - y$ $= 2x + y$		
	(c) (i) $P(x) = 2x^3 - 9x^2 + 12x - 2$ $P(0) = -2 < 0$ $P(2) = 16 - 36 + 24 - 2$ $= 2 > 0$ $\therefore$ roots lie in interval $0 < x < 2$		

Qn	Solutions	Marks	Comments+Criteria
2 ctd	(c) ctd (ii) $P(x) = 2x^3 - 9x^2 + 12x - 2$ $P'(x) = 6x^2 - 18x + 12$ $6(x^2 - 3x + 2) = 0$ $(x-1)(x-2) = 0$ $x = 1, x = 2.$		
	(iii) $x = 1$ is a stat. pt $\therefore$ its tangent never crosses x-axis. $\therefore$ cannot use Newton's method with $x = 1$ as 1st approx.		
	(iv) $P(\frac{1}{2}) = 2$ $P'(\frac{1}{2}) = 4.5$ $x_2 = x_1 - \frac{P(x_1)}{P'(x_1)}$ $= \frac{1}{2} - \frac{2}{\frac{9}{2}}$ $= \frac{1}{2} - \frac{4}{9}$ $= \frac{9}{18} - \frac{8}{18}$ $= \frac{1}{18}$		

Qn	Solutions	Marks	Comments+Criteria
3	(a)(i) Show $\frac{1-\cos 2x}{1+\cos 2x} = \tan^2 x$		
	LHS $= \frac{1-(\cos^2 x - \sin^2 x)}{1+\cos^2 x - \sin^2 x}$		
	$= \frac{1-\cos^2 x + \sin^2 x}{1-\sin^2 x + \cos^2 x}$		
	$= \frac{\sin^2 x + \sin^2 x}{\cos^2 x + \cos^2 x}$		
	$= \frac{2\sin^2 x}{2\cos^2 x}$		
	$= \tan^2 x$		
	= RHS		
	(ii) $\tan^2 22\frac{1}{2}^\circ = \frac{1-\cos 45^\circ}{1+\cos 45}$		
	$= \frac{1-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}$		
	$= \frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$		
	$= \frac{2-2\sqrt{2}+1}{2-1}$		
	$= 3-2\sqrt{2}$		

Qn	Solutions	Marks	Comments+Criteria
3 cont'd	(b) $P(x) = x^3+2x^2-ax+1$		
	$P(1) = 3x+b$. $P(-2) = 3x+b$		
	$3+b = 4-a$ $b-6 = 2a+1$		
	$a+b=1$ ① $2a-b=-7$ ②		
	① + ②: $3a = -6$		
	$a = -2$		
	$-2+b = 1$		
	$b = +3$		
	(c)(i) $y = x^2-3$ ① $y = \frac{2}{x}$ ②		
	① = ②: $x^2-3 = \frac{2}{x}$		
	$\therefore x^3-3x = 2$		
	ie $x^3-3x-2=0$		
	(ii) Let $P(x) = x^3-3x-2$		
	$P(2) = 8-6-2$		
	$= 0$		
	$P(-1) = -1+3-2$		
	$= 0$		
	$(x+1)(x-2) = x^2-x-2$		
	$x^2-x-2 \overline{)\, x^3-3x-2}$; $x+1$; x^3-x^2-2x; x^2-x-2		
	$P(x) = (x+1)(x-2)(x+1)$		
	$\therefore$ Char x-coord of $x=2$.		
	(iii) $x^2-3 \le \frac{2}{x}$ for $0<x\le 2$, $x=-1$		