KAMBALA GIRLS' SCHOOL

EXTENSION 1 MATHEMATICS

Preliminary Task #2

June 2003

Time Allowed: 45 minutes

Outcomes to be assessed by this task

- P1, P2, P3, P4, P5
- PE1, PE2, PE3, PE6

Syllabus references assessed by this task

- 1.1, 1.2, 1.3, 1.4, 1E
- 4.1, 4.2, 4.3, 4.4
- 5.1, 5.2, 5.3, 5.4, 5.5, E5.6, E5.7, E5.8, E5.9 (Not including general solutions)
- 9.4
- 13.2

Instructions

- Time allowed is 45 minutes.
- There are 3 questions, each worth 12 marks.
- The mark value of each part is indicated in **bold** next to that part.
- Start each question on a new page.

Question One (Start a new page) **12 Marks**

(a) Solve $\dfrac{x+1}{x-1} \le 2$ **3**

(b) Consider the function $f(x) = \dfrac{x-2}{x-1}$

(i) Find the vertical asymptote(s). **1**

(ii) Find the horizontal asymptote(s). **1**

(iii) Sketch the function, showing all relevant features. **2**

(c)

(i) On the same set of axes, neatly sketch the graphs of $y = \sin x$ and $y = \cos x$, for $0^\circ \le x \le 360^\circ$. **2**

(ii) On the same set of axes, and using addition of ordinates, neatly sketch the graph of $y = \sin x + \cos x$, for $0^\circ \le x \le 360^\circ$. **1**

(iii) Hence or otherwise, solve $\sin x + \cos x = -1$ for $0^\circ \le x \le 360^\circ$. **2**

Question Two (Start a new page) **12 Marks**

(a) Find the exact value of $\tan 15°$ in simplest form. 3

(b)

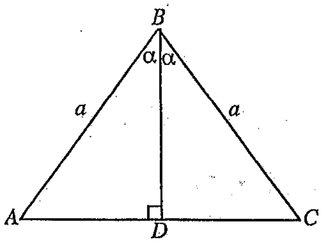
(i) Show that $\sin 3\theta = \sin\theta\cos 2\theta + 2\sin\theta\cos^2\theta$ 1

(ii) Show that $\cos 3\theta = \cos\theta\cos 2\theta - 2\sin^2\theta\cos\theta$ 1

(iii) Hence, or otherwise, prove that $\dfrac{\sin 3\theta}{\sin\theta} - \dfrac{\cos 3\theta}{\cos\theta} = 2$ (for $\sin\theta \neq 0, \cos\theta \neq 0$). 2

(c) In the diagram below, ΔABC is isosceles, $AB = BC = a$ and $BD \perp AC$.

Let $\angle ABD = \angle CBD = \alpha$



(i) Show that $BD = a\cos\alpha$ 1

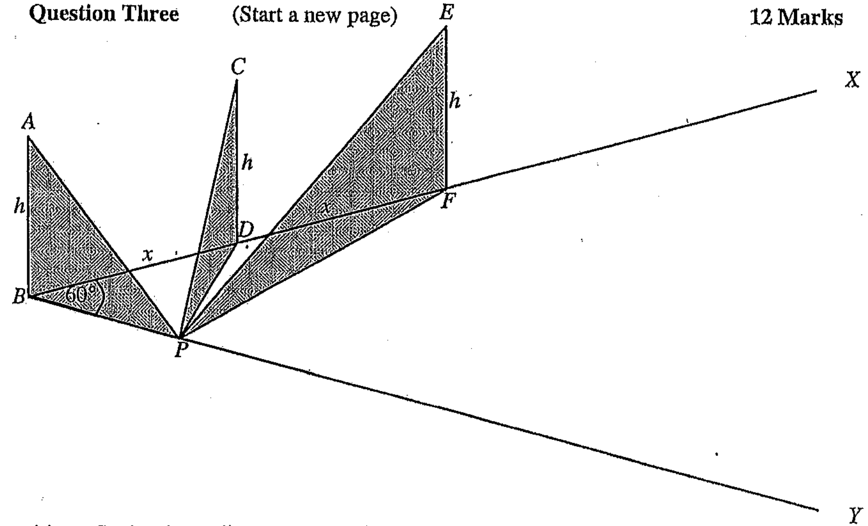
(ii) Hence, show that the area of ΔABD is: 1

$$\frac{a^2 \sin\alpha\cos\alpha}{2}$$

(iii) By considering the area of ΔABC, prove that: 3

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

Question Three (Start a new page) **12 Marks**



(a) In the above diagram, BX and BY represent two roads intersecting at an angle of 60°. On the road BX are situated three telegraph poles AB, CD and EF, all of equal height, h metres, and the same distance, x metres, apart (i.e. $BD = DF = x$). P is a point on the road BY and the angles of elevation of A and C from P are 45° and 30° respectively.

(i) Show that $BP = h$ and $DP = h\sqrt{3}$. 2

(ii) By the use of the Sine Rule in ΔBDP, show that $\angle BDP = 30°$ and hence show that ΔBDP is right angled at P. 2

(iii) Using ΔBDP, prove that $x = 2h$. 2

(iv) By the use of the Cosine Rule in ΔPDF, show that $PF = h\sqrt{13}$. 2

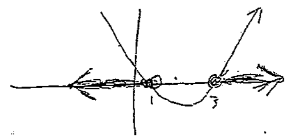
(v) Hence find the angle of elevation of E from P to the nearest degree. 1

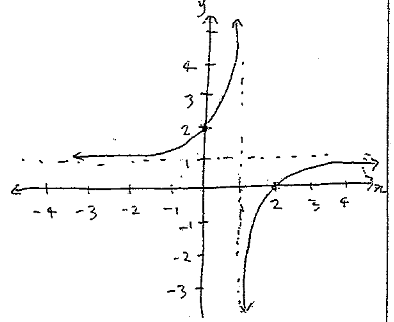
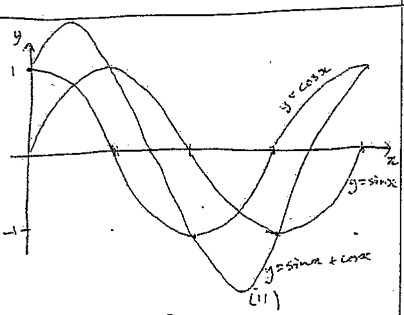
(b) Solve the following for x, given $0° \le x \le 360°$: 3

$$4^{\sin x} - \frac{5}{2}(2^{\sin x}) + 1 = 0$$

END OF TASK

Notes - T.E. - Transcription Error
- B.A.S. - Buy a Stapler

Qn	Solutions	Marks	Comments+Criteria
1) a)	$\frac{x+1}{x-1} \le 2 \qquad x \ne 1$ $\boxed{\times (x-1)^2}$ $(x+1)(x-1) \le 2(x-1)^2$ $x^2 - 1 \le 2x^2 - 4x + 2$ $0 \le x^2 - 4x + 3$ $0 \le (x-3)(x-1)$  $x < 1 \quad (x \ne 1)$ $x \geq 3$	3	$1\frac{1}{2}$ marks - 1 soln + $x \ne 1$ 1 marks - 1 soln 2 marks - factorised, diagram, wrong soln Alternative $2(x-1)^2 - (x+1)(x-1) \geq 0$ $(x-1)(2(x-1) - (x+1)) \geq 0$ $(x-1)(2x-2-x-1) \geq 0$ $(x-1)(x-3) \geq 0$
b)	$f(x) = \frac{x-2}{x-1}$		Note: $x = 1$ NOT $x \ne 1$
i)	vertical occurs when $x - 1 = 0$ $x = 1$	1	
ii)	$f(x) = \frac{x-2}{x-1} \begin{matrix} \div x \\ \div x \end{matrix}$ $= \frac{\frac{x}{x} - \frac{2}{x}}{\frac{x}{x} - \frac{1}{x}}$ as $x \to \infty$ $= \frac{1 - \frac{2}{x}}{1 - \frac{1}{x}}$ $\quad f(x) = \frac{1-0}{1-0} = 1$	1	

Qn	Solutions	Marks	Comments+Criteria
1) b) iii)	at $x = 0$, $f(x) = \frac{-2}{-1}$ $= 2$ at $f(x) = 0$, $\frac{x-2}{x-1} = 0$ $x - 2 = 0$ $\therefore x = 2$ 	2	$f(x) = \frac{x-1-1}{x-1}$ $= 1 - \frac{1}{x-1}$
c) i) and ii)		2 1	1 for $\sin x$ 1 for $\cos x$ no $\frac{1}{2}$ marks

Qn	Solutions	Marks	Comments+Criteria
1) c) iii)	$\sin x + \cos x = -1$ Consider the intersection of the two graphs $y = \sin x + \cos x$ and $y = 1$ in the graph, this occurs when $x = 180°$ and $x = 270°$	2	
2) a)	$\tan 15° = (45° - 30°)$ $= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$ $= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \cdot \frac{1}{\sqrt{3}}} \begin{matrix} \boxed{\times\sqrt{3}} \\ \boxed{\times\sqrt{3}} \end{matrix}$ $= \frac{\sqrt{3} - 1}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1}$ $= \frac{3 - 2\sqrt{3} + 1}{3 - 1}$ $= \frac{4 - 2\sqrt{3}}{2}$ $= 2 - \sqrt{3}$	3 1 1 1	$\tan 15°$ $= \tan(60° - 45°)$ $= \frac{\tan 60° - \tan 45°}{1 + \tan 60 \tan 45°}$ $= \frac{\sqrt{3} - 1}{1 + \sqrt{3}} \cdot \frac{1 - \sqrt{3}}{1 - \sqrt{3}}$ $= \frac{\sqrt{3} - 3 - 1 + \sqrt{3}}{1 - 3}$ $= \frac{2\sqrt{3} - 4}{-2}$ $= \frac{2(\sqrt{3} - 2)}{-2}$ $= 2 - \sqrt{3}$

Qn	Solutions	Marks	Comments+Criteria
2) b) i)	$\sin 3\theta = \sin(\theta + 2\theta)$ $= \sin\theta\cos 2\theta + \cos\theta \sin 2\theta$ $= \sin\theta\cos 2\theta + \cos\theta(2\sin\theta\cos\theta)$ $= \sin\theta\cos 2\theta + 2\sin\theta\cos^2\theta$	1	
ii)	$\cos 3\theta = \cos(\theta + 2\theta)$ $= \cos\theta\cos 2\theta - \sin\theta\sin 2\theta$ $= \cos\theta\cos 2\theta - \sin\theta(2\sin\theta\cos\theta)$ $= \cos\theta\cos 2\theta - 2\sin^2\theta\cos\theta$	1	
iii)	LHS $= \frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta}$ $= \frac{\sin\theta\cos 2\theta + 2\sin\theta\cos^2\theta}{\sin\theta}$ $- \frac{(\cos\theta\cos 2\theta - 2\sin^2\theta\cos\theta)}{\cos\theta}$ $= \cos 2\theta + 2\cos^2\theta - \cos 2\theta + 2\sin^2\theta$ $= 2\cos^2\theta + 2\sin^2\theta$ $= 2(\cos^2\theta + \sin^2\theta)$ $= 2(1)$ $= 2$	2 1 1	
c) i)	In $\triangle BAD$ $\cos\alpha = \frac{BD}{a}$ $\therefore BD = a\cos\alpha$	1	

Qn	Solutions	Marks	Comments+Criteria
)	Alternative 1 Area $= \frac{1}{2} ab \sin C$ $= \frac{1}{2} \times a \times BD \times \sin\alpha$ $= \frac{1}{2} \times a \times a\cos\alpha \times \sin\alpha$ $= \frac{a^2 \sin\alpha \cos\alpha}{2}$ or Alternative 2 In ΔABD $\sin\alpha = \frac{AD}{a}$ $\therefore AD = a\sin\alpha$ Area $= \frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} \times AD \times BD$ $= \frac{1}{2} \times a\sin\alpha \times a\cos\alpha$ $= \frac{a^2 \sin\alpha \cos\alpha}{2}$	$\boxed{1}$	
	Using Area Formula Area$(\Delta ABC) = \frac{1}{2} ab \sin C$ $= \frac{1}{2} \times a \times a \times \sin 2\alpha$ $= \frac{a^2 \sin 2\alpha}{2}$ ✓ Also, Area$(\Delta ABC) = 2 \times$ Area(ΔABD) ✓ $\therefore \frac{a^2 \sin 2\alpha}{2} = 2 \times \frac{a^2 \sin\alpha\cos\alpha}{2}$ ✓ $\therefore \sin 2\alpha = 2\sin\alpha\cos\alpha$	$\boxed{3}$	Interesting answer, but doesn't consider area (so not correct) $\frac{AC}{\sin 2\alpha} = \frac{a}{\sin(90-\alpha)}$ $\frac{2AD}{\sin 2\alpha} = \frac{a}{\cos\alpha}$ $\frac{2a\sin\alpha}{\sin 2\alpha} = \frac{a}{\cos\alpha}$ $\therefore 2\sin\alpha\cos\alpha = \sin 2\alpha$

Qn	Solutions	Marks	Comments+Criteria
3a) i)	In ΔBAP $\tan 45° = \frac{h}{BP}$ $\therefore BP = \frac{h}{\tan 45°}$ $= \frac{h}{1}$ $= h$ In ΔCDP $\tan 30° = \frac{h}{DP}$ $\therefore DP = \frac{h}{\tan 30°}$ $= \frac{h}{1/\sqrt{3}}$ $= h\sqrt{3}$	$\boxed{2}$ 1 1	Also, ΔBAP is isosceles, $\therefore BP = BA = h$
ii)	$\frac{\sin\angle BDP}{BP} = \frac{\sin\angle DBP}{DP}$ $\frac{\sin\angle BDP}{h} = \frac{\sin 60°}{h\sqrt{3}}$ $\sin\angle BDP = \frac{h \times \frac{\sqrt{3}}{2}}{h\sqrt{3}}$ $= \frac{1}{2}$ $\therefore \angle BDP = \sin^{-1}\left(\frac{1}{2}\right)$ $= 30°$ $\therefore \Delta BDP$ is right angled at P (angle sum of a triangle)	$\boxed{2}$ 1 1	

Solutions	Marks	Comments+Criteria
Using Pythagoras $x^2 = h^2 + (h\sqrt{3})^2$ $= h^2 + 3h^2$ $= 4h^2$ $x = \sqrt{4h^2}$ $= 2h$	$\boxed{2}$ 1 1	Alternative $\sin 30° = \frac{h}{x}$ $x = \frac{h}{\sin 30°}$ $= \frac{h}{\frac{1}{2}}$ $= 2h$
$a^2 = b^2 + c^2 - 2bc\cos A$ $\angle PDF = 150°$ (supp. adj. $\angle$'s) $PF^2 = (2h)^2 + (h\sqrt{3})^2 - 2(2h)(h\sqrt{3})\cos 150°$ $= 4h^2 + 3h^2 - 4\sqrt{3}h^2(-\cos 30°)$ $= 4h^2 + 3h^2 + 4\sqrt{3}h^2 \frac{\sqrt{3}}{2}$ $= 7h^2 + 6h^2$ $= 13h^2$ $\therefore PF = \sqrt{13h^2}$ $= h\sqrt{13}$	$\boxed{2}$ 1 1	
$\tan\angle EPF = \frac{h}{h\sqrt{13}} = \frac{1}{\sqrt{13}}$ $\therefore \angle EPF = \tan^{-1}\left(\frac{1}{\sqrt{13}}\right)$ $= 16°$ (to the nearest degree)	$\boxed{1}$	