KAMBALA

Extension 1 Mathematics

Preliminary Assessment Task 2

MAY 2005

Time Allowed: 1.5 hours working time, plus 5 minutes for reading at the beginning.

INSTRUCTIONS

- This examination contains 5 questions of 12 marks each. Marks for each question are shown.
- Answer all questions on the writing paper provided. Start each question on a new page.
- Calculators should be used.
- Show all necessary working.
- Marks may be deducted for careless or badly arranged work.

Question 1: 12 marks

(a) Identify the solutions to the following equations as rational or irrational. (You do not need to find the solutions) 2

(i) $3x^2 - 9 = 0$

(ii) $\dfrac{7x-8}{2} = 5$

(b) Solve the inequalities: 4

(i) $\dfrac{5-2x}{3} > -5$

(ii) $x^2 - 3x > 4$

(c) Solve the inequality: $|4x-1| \le 9$, indicating your solution on a number line. 3

(d) (i) Factorise $2^{n+1} + 2^n$ 3

(ii) Hence write $\dfrac{2^{1001}+2^{1000}}{3}$ as a power of 2.

Question 2: (Begin on a new page) 12 marks

(a) (i) Sketch the graphs of $x - 2y = 0$ and $xy = 2$ on the same number plane. 5

(ii) From your graph state the number of solutions there would be to the simultaneous solution of $x - 2y = 0$ and $xy = 2$

(iii) Solve simultaneously, $x - 2y = 0$ and $xy = 2$.

(b) Solve for x: $\dfrac{4}{5-x} \ge 1$ 3

(c) Find the value of θ in the following (θ is acute): 2

(i) $\sin\theta = \cos 55°$

(ii) $\cos(270° - \theta) = -\dfrac{1}{2}$

(d) Explain why $\sin^2\theta + \cos^2\theta = 1$ 2

Question 3: (Begin on a new page) 12 marks

(a) Given that $\cot\beta = -\frac{3}{5}$ and that β is obtuse, find the exact value of $\sin\beta$. 2

(b) On the same set of axes, sketch the graphs of $y = \sin x$ and $y = \operatorname{cosec} x$, for $0° \le x \le 360°$ 2

(c) Prove the identities: 5

(i) $\sin\theta\cos\theta\operatorname{cosec}^2\theta = \cot\theta$

(ii) $\dfrac{1+\tan^2 x}{1+\cot^2 x} = \tan^2 x$

(d) Solve the equation: $\cos 2x = -\frac{1}{\sqrt{2}}$ for $0° \le x \le 360°$ 3

Question 4: (Begin on a new page) 12 marks

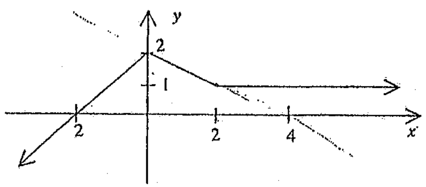
(a) 6

(i) Sketch the graph of $y = 16 - x^2$

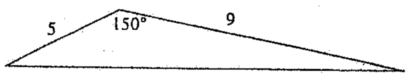
(ii) State the domain and range of $f(x) = 16 - x^2$

(iii) On the same number plane as in 4(a)(i), sketch the graph of $y = 2x^2 + 1$, and shade the region that describes where $y \le 16 - x^2$ and $y \ge 2x^2 + 1$ are both true.

(b) Define the piecewise function that is shown in the graph below by stating the function and the domain for each section of the graph. 3

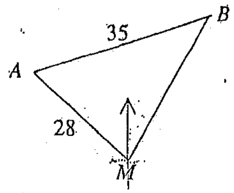


(c) Find the exact value of the area of the triangle shown. Measurements are in cm. 3



Question 5: (Begin on a new page) 12 marks

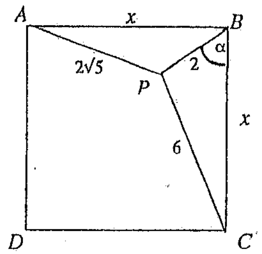
(a) Town A is 28 km from a monument (M) and on a bearing of 312° from it. Town B is on a bearing of 025° from the monument. The towns are 35 kilometres apart. This information is shown in the diagram below: 4



(i) Copy the diagram on to your writing paper and show that $\angle AMB = 73°$.

(ii) Find the bearing of town A from town B.

(b) The diagram below shows a square $ABCD$ of side x cm. The point P is inside the square, such that $PC = 6$ cm; $PB = 2$ cm and $AP = 2\sqrt{5}$ cm. Angle $PBC = \alpha$. 8



(i) Using the cosine rule in the triangle PBC, show that $\cos\alpha = \dfrac{x^2 - 32}{4x}$

(ii) By considering triangle PBA and $\cos\angle ABP$, show that $\sin\alpha = \dfrac{x^2 - 16}{4x}$

(iii) Hence, or otherwise, show that the value of x is a solution of $x^4 - 56x^2 + 640 = 0$

(iv) Find the value(s) of x.

End of examination

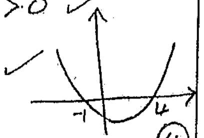
Preliminary HT #2 Extension 1 Mathematics 2005

1 a) i) Irrational ✓
ii) Rational ✓ (2)

b) i) $\frac{5-2x}{3} > -5$
$5-2x > -15$ ✓
$20 > 2x$ ✓
$x < 10$

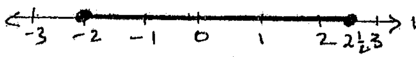
ii) $x^2-3x-4 > 0$
$(x-4)(x+1) > 0$ ✓

$\therefore$ $x > 4,\ x < -1$ ✓



(4)

c) $|4x-1| \le 9$
$-9 \le 4x-1 \le 9$ ✓
$-8 \le 4x \le 10$
$-2 \le x \le 2\frac{1}{2}$ ✓

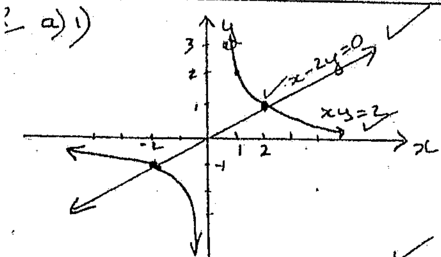


✓ (3)

d) i) $2^{n+1}+2^n$
$= 2^n(2+1)$ ✓
$= 2^n \times 3$

ii) $\frac{2^{1001}+2^{1000}}{3} = \frac{2^{1000}\times 3}{3}$ ✓
$= 2^{1000}$ ✓ (3)

2 a) i)



✓

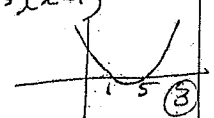
ii) 2 solutions (2 pts of intersection) ✓

iii) $x=2,\ y=1$
$x=-2,\ y=-1$ } ✓ (5)

b) $\frac{4}{5-x} \ge 1$

$\frac{4\times(5-x)^2}{5-x} \ge (5-x)^2$ ✓

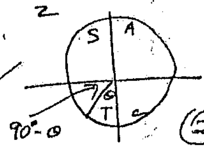
$4(5-x) \ge 25-10x+x^2$
$20-4x \ge 25-10x+x^2$
$0 \ge x^2-6x+5$ ✓
$0 \ge (x-5)(x-1)$
or $(x-5)(x-1) \le 0$
$1 \le x \le 5$ ✓



(3)

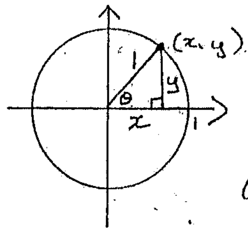
c) i) $\sin\theta = \cos 55^\circ$
$\theta = 35^\circ$ ✓ ($\sin 35^\circ = \cos 55^\circ$ since $35^\circ+55^\circ = 90^\circ$)

ii) $\cos(270^\circ-\theta) = -\frac{1}{2}$
$90^\circ-\theta = 60^\circ$
$\theta = 30^\circ$ ✓



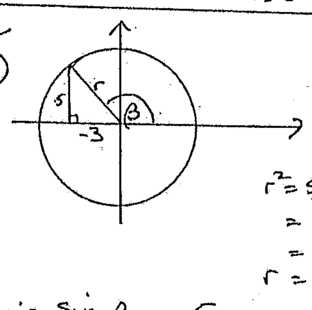
(2)

d)



$\sin\theta = \frac{y}{1} = y$ ✓
$\cos\theta = \frac{x}{1} = x$
$x^2+y^2=1$ ✓ (equation of circle)
ie $\cos^2\theta + \sin^2\theta = 1$ (2)

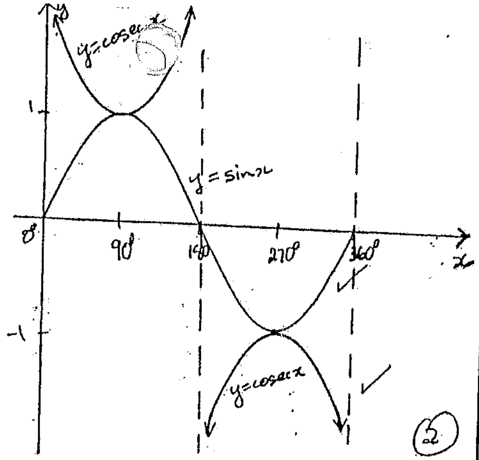
3 a)



$\cot\beta = -\frac{3}{5}$
$\tan\beta = -\frac{5}{3}$
$r^2 = 5^2+3^2$
$= 25+9$
$= 34$
$r = \sqrt{34}$ ✓

$\therefore \sin\beta = \frac{5}{\sqrt{34}}$ ✓ (2)

3 b)

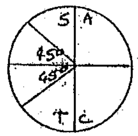


✓ (2)

c) i) LHS $= \sin\theta\cos\theta \times \frac{1}{\sin^2\theta}$ ✓
$= \frac{\cos\theta}{\sin\theta}$ ✓
$= \cot\theta =$ RHS

ii) LHS $= \frac{1+\tan^2 x}{1+\cot^2 x}$
$= \frac{\sec^2 x}{\csc^2 x}$ ✓
$= \frac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}}$
$= \frac{1}{\cos^2 x} \times \frac{\sin^2 x}{1}$ ✓
$= \frac{\sin^2 x}{\cos^2 x}$
$= \tan^2 x =$ RHS ✓ (5)

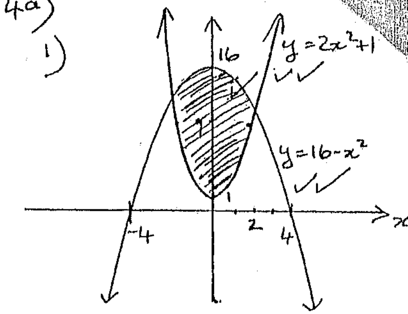
d) $\cos 2x = -\frac{1}{\sqrt{2}}$ $\quad 0^\circ \le x \le 360^\circ$
ie $0^\circ \le 2x \le 720^\circ$



Related acute angle $= 45^\circ$ ✓
→ 2nd & 3rd Qs

$\therefore 2x = 180^\circ-45^\circ,\ 180^\circ+45^\circ,\ 360^\circ+180^\circ-45^\circ,\ 360^\circ+180^\circ+45^\circ$
$2x = 135^\circ,\ 225^\circ,\ 495^\circ,\ 575^\circ$
$x = 67\frac{1}{2}^\circ,\ 112\frac{1}{2}^\circ,\ 247\frac{1}{2}^\circ,\ 287\frac{1}{2}^\circ$ ✓ (3)

4 a) i)



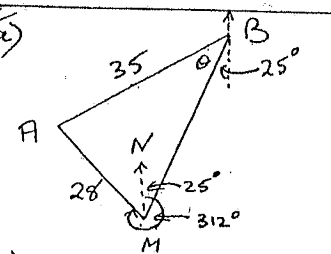
ii) D: all real x
R: all real $y \le 16$ } ✓ (6)

iii) Above

b) $f(x) = \begin{cases} x+2, & x \le 0 \\ 2-\frac{x}{2}, & 0 < x < 2 \\ 2, & x \ge 2 \end{cases}$ ✓ (3)

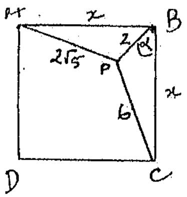
c) $A = \frac{1}{2}\times 5\times 9\times \sin 150^\circ$ ✓
$= \frac{45}{2}\times \sin 30^\circ$
$= \frac{45}{2}\times\frac{1}{2}$ ✓
$= \frac{45}{4}\ \text{cm}^2$ ✓ (3)

5 a)



i) $\angle AMN = 360^\circ - 312^\circ = 48^\circ$
$\angle AMB = 48^\circ + 25^\circ$ ✓
$= 73^\circ$

ii) $\frac{28}{\sin\theta} = \frac{35}{\sin 73^\circ}$ ✓
$\sin\theta = \frac{\sin 73^\circ}{35}\times 28$
$= 0.76504\ldots$
$\theta = 49^\circ 54'\ldots$ ✓
$\doteq 50^\circ$ ✓
$\therefore$ Bearing of A from B $= 180^\circ+25^\circ+50^\circ$
$= 255^\circ$ ✓ (4)



i) $\cos\alpha = \dfrac{x^2+2^2-6^2}{2\times2\times x}$ ✓

$= \dfrac{x^2+4-36}{4x}$

$= \dfrac{x^2-32}{4x}$ ✓

ii) $\cos\angle ABP = \dfrac{x^2+2^2-(2\sqrt{5})^2}{2\times2\times x}$

$= \dfrac{x^2+4-20}{4x}$

$= \dfrac{x^2-16}{4x}$ ✓

Now $\cos\angle ABP = \cos(90^\circ-\alpha)$

$= \sin\alpha$ ✓

$\therefore \sin\alpha = \dfrac{x^2-16}{4x}$

iii) $\sin^2\alpha + \cos^2\alpha = 1$

$\therefore \left(\dfrac{x^2-16}{4x}\right)^2 + \left(\dfrac{x^2-32}{4x}\right)^2 = 1$ ✓

$(x^2-16)^2 + (x^2-32)^2 = 16x^2$

$x^4-32x^2+256+x^4-64x^2+1024 = 16x^2$

$2x^4-112x^2+1280=0$

$x^4-56x^2+640=0$ ✓

iv) $(x^2-40)(x^2-16)=0$

$x^2=40, \quad x^2=16$ ✓

$x=\sqrt{40}, 4 \quad$ ($x>0$ since a length)

but $x \neq 4$ since $\sin\alpha$ would $=0$ & $\alpha = 0^\circ$ or 180° (imposs)

$\therefore x = \sqrt{40}$ cm ✓ (8)

Preliminary M1 #2 Extension 1 Mathematics 2005

1 a) i) Irrational ✓

ii) Rational ✓ (2)

b) i) $\dfrac{5-2x}{3} > -5$

$5-2x > -15$ ✓

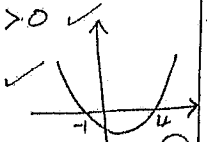
$20 > 2x$ ✓

$x < 10$

ii) $x^2-3x-4 > 0$

$(x-4)(x+1) > 0$ ✓

$\therefore x>4, \ x<-1$ ✓



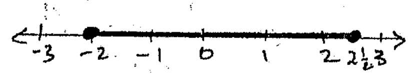
(4)

c) $|4x-1| \le 9$

$-9 \le 4x-1 \le 9$ ✓

$-8 \le 4x \le 10$

$-2 \le x \le 2\frac{1}{2}$ ✓



✓ (3)

d) i) $2^{n+1}+2^n$

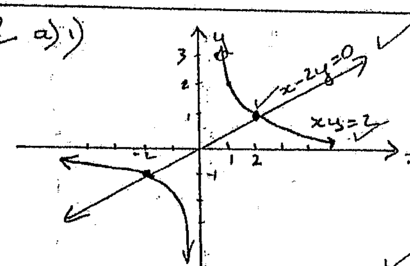
$= 2^n(2+1)$ ✓

$= 2^n \times 3$

ii) $\dfrac{2^{1001}+2^{1000}}{3} = \dfrac{2^{1000}\times3}{3}$ ✓

$= 2^{1000}$ ✓ (3)

2 a) i)



✓

ii) 2 solutions (2pts of intersection) ✓

iii) $x=2, y=1$
$x=-2, y=-1$ ✓ (5)

b) $\dfrac{4}{5-x} \ge 1$

$\dfrac{4\times(5-x)^2}{5-x} \ge (5-x)^2$ ✓

$4(5-x) \ge 25-10x+x^2$

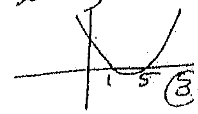
$20-4x \ge 25-10x+x^2$

$0 \ge x^2-6x+5$ ✓

$0 \ge (x-5)(x-1)$

or $(x-5)(x-1) \le 0$

$1 \le x \le 5$ ✓



(3)

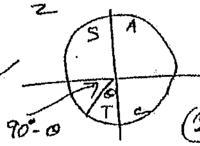
c) i) $\sin\theta = \cos 55^\circ$

$\theta = 35^\circ$ ✓ ($\sin 35^\circ = \cos 55^\circ$ since $35^\circ+55^\circ=90^\circ$)

ii) $\cos(270^\circ-\theta) = -\frac{1}{2}$

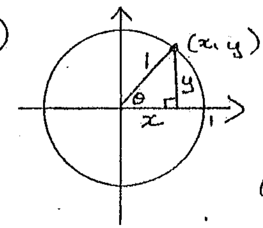
$90^\circ-\theta = 60^\circ$

$\theta = 30^\circ$ ✓



(2)

d)



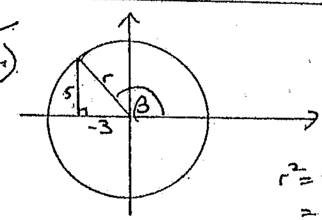
$\sin\theta = \frac{y}{1} = y$ ✓

$\cos\theta = \frac{x}{1} = x$

$x^2+y^2=1$ ✓ (equation of circle)

ie $\cos^2\theta + \sin^2\theta = 1$ (2)

3 a)



$\cot\beta = -\frac{3}{5}$

$\tan\beta = -\frac{5}{3}$

$r^2 = 5^2+3^2$

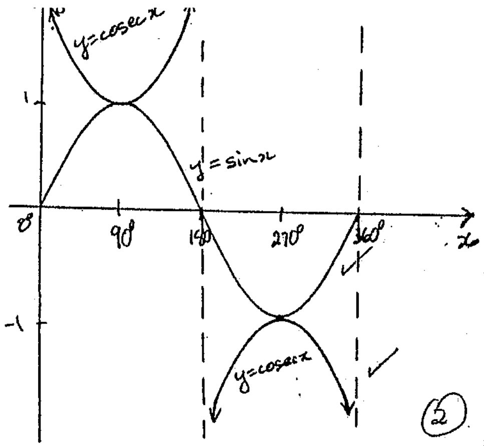
$= 25+9$

$= 34$

$r = \sqrt{34}$ ✓

$\therefore \sin\beta = \dfrac{5}{\sqrt{34}}$ ✓ (2)

3 b)



(2)

c) i) LHS $= \sin\theta\cos\theta \times \frac{1}{\sin^2\theta}$

$= \frac{\cos\theta}{\sin\theta}$

$= \cot\theta = $ RHS

ii) LHS $= \frac{1+\tan^2 x}{1+\cot^2 x}$

$= \frac{\sec^2 x}{\operatorname{cosec}^2 x}$

$= \frac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}}$

$= \frac{1}{\cos^2 x} \times \frac{\sin^2 x}{1}$

$= \frac{\sin^2 x}{\cos^2 x}$

$= \tan^2 x = $ RHS

(5)

d) $\cos 2x = -\frac{1}{\sqrt{2}}$ $\quad 0° \le x \le 360°$

ie $0° \le 2x \le 720°$

Related acute angle = 45°

→ 2nd & 3rd Qs

∴ $2x = 180°-45°, 180°+45°,$

$360°+180°-45°, 360°+180°+45°$

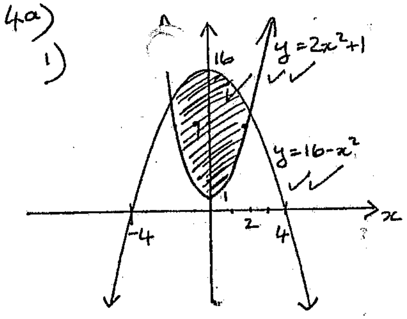
$2x = 135°, 225°, 495°, 585°$

$x = 67\frac{1}{2}°, 112\frac{1}{2}°, 247\frac{1}{2}°,$

$292\frac{1}{2}°$

(3)

4a) i)



ii) D: all real x
R: all real $y \le 16$

(6)

iii) Above

b) $f(x) = \begin{cases} x+2, & x \le 0 \\ 2-\frac{x}{2}, & 0 < x < 2 \\ 1, & x \ge 2 \end{cases}$

(3)

c) $A = \frac{1}{2} \times 5 \times 9 \times \sin 150°$

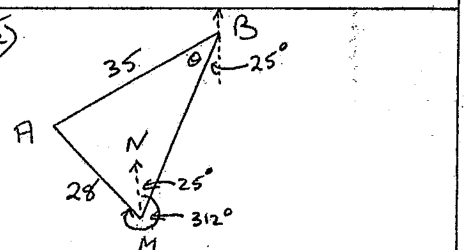
$= \frac{45}{2} \times \sin 30°$

$= \frac{45}{2} \times \frac{1}{2}$

$= \frac{45}{4}$ cm²

(3)

5 a)



i) $\angle AMN = 360° - 312° = 48°$

$\angle AMB = 48° + 25°$

$= 73°$

ii) $\frac{28}{\sin\theta} = \frac{35}{\sin 73°}$

$\sin\theta = \frac{\sin 73°}{35} \times 28$

$= 0.76504\ldots$

$\theta = 49°54'\ldots$

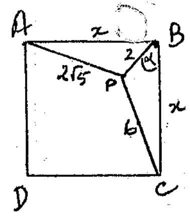
$\doteqdot 50°$

∴ Bearing of A from B $= 180° + 25° + 50°$

$= 255°$

(4)

5 b)



i) $\cos\alpha = \frac{x^2+2^2-6^2}{2 \times 2 \times x}$

$= \frac{x^2+4-36}{4x}$

$= \frac{x^2-32}{4x}$

ii) $\cos\angle ABP = \frac{x^2+2^2-(2\sqrt{5})^2}{2 \times 2 \times x}$

$= \frac{x^2+4-20}{4x}$

$= \frac{x^2-16}{4x}$

Now $\cos\angle ABP = \cos(90° - \alpha)$

$= \sin\alpha$

∴ $\sin\alpha = \frac{x^2-16}{4x}$

iii) $\sin^2\alpha + \cos^2\alpha = 1$

∴ $\left(\frac{x^2-16}{4x}\right)^2 + \left(\frac{x^2-32}{4x}\right)^2 = 1$

$(x^2-16)^2 + (x^2-32)^2 = 16x^2$

$x^4 - 32x^2 + 256 + x^4 - 64x^2 + 1024 = 16x^2$

$2x^4 - 112x^2 + 1280 = 0$

$x^4 - 56x^2 + 640 = 0$

iv) $(x^2-40)(x^2-16) = 0$

$x^2 = 40, \quad x^2 = 16$

$x = \sqrt{40}, 4$ $\quad$ ($x > 0$ since a length)

but $x \ne 4$ since $\sin\alpha$ would $= 0$ & $\alpha = 0°$ or $180°$ (imposs)

∴ $x = \sqrt{40}$ cm

(8)

NOTES