

NAME : \_\_\_\_\_



KAMBALA

MATHEMATICS  
YEAR 10 ADVANCED

HALF-YEARLY EXAMINATION  
MAY 2003

*Time Allowed : 2 hours  
Plus 5 minutes Reading Time*

Paper for Mr Maitland's class

Instructions :

- This Examination consists of 3 sections :

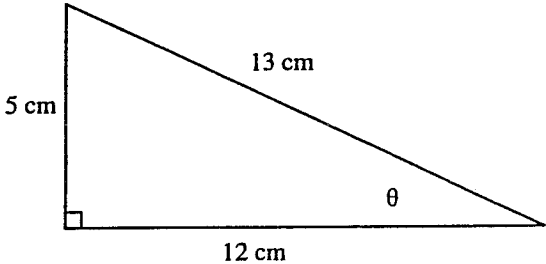
Section A ( 10 marks )	Year 9 Topics
Section B ( 50 marks )	Geometry, Probability, Quadratics
Section C ( 40 marks )	Geometry, Probability, Quadratics, Consumer Arithmetic, Graphing
Total ( 100 marks )	

- Write your answers on the paper provided.
- Show all necessary working.
- Calculators may be used.

## Section A

## Year 9 Topics

10 Marks

1. Find the surface area of a cube whose side is 8 cm 1
2. Solve the equation  $\frac{n}{4} = 8$  1
3. Rationalise the denominator of  $\frac{3}{\sqrt{3}}$ . 1
4. Simplify  $(3 - \sqrt{2})(3 + \sqrt{2})$ . 1
5. Write down the gradient of the line  $y = \frac{2}{3}x + 5$ . 1
6. Solve the simultaneous equations below for  $x$  only: 1  
$$x + y = 2$$
$$x - y = 4$$
7. Find the exact value of  $\sin \theta$  from the triangle. 1  


The diagram shows a right-angled triangle. The vertical side (left) is labeled 5 cm. The horizontal side (bottom) is labeled 12 cm. The hypotenuse (slanted) is labeled 13 cm. A right-angle symbol is at the bottom-left vertex. The angle  $\theta$  is at the bottom-right vertex.
8. If the circumference of a circle is 25cm, find its diameter to 1 decimal place. 1
9. If  $a = \frac{3}{2}$ , evaluate  $\frac{2}{3}a^3$ . 1
10. The length of a rectangular lawn is 4 metres longer than its width. If the width is  $x$  metres, and the area is 192 square metres write down an **equation** which may be used to find its width. (Do not solve) 1

END OF SECTION A

**Section B**      **Geometry, Probability, Quadratics**

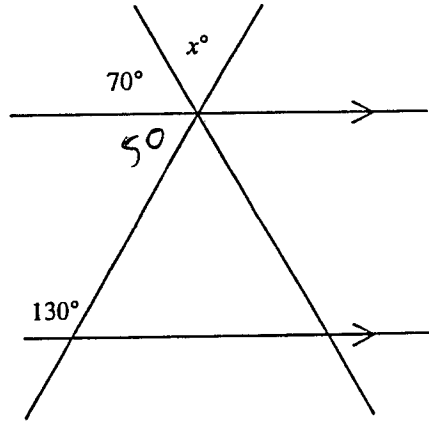
**50 Marks**

**Question 1.**    (Start a NEW Page)

**10 marks**

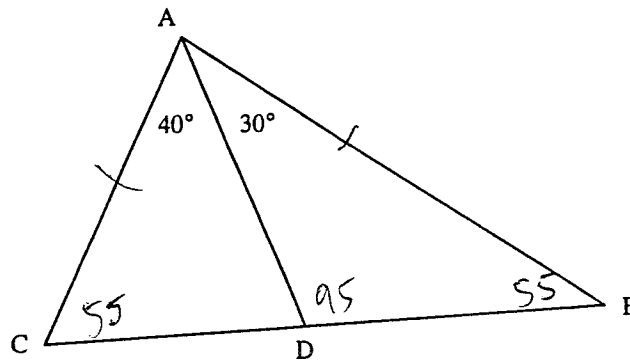
- (a) Find the value of  $x$ .

2



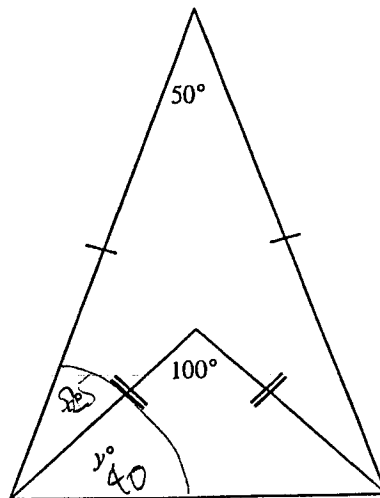
- (b) If  $AB = AC$ , what is the size of  $\angle BDA$ ? Give reasons for your answer.

4



- (c) Find the value of  $y$  and  $x$ , giving reasons.

4



## Section B Continued

**Question 2. (Start a NEW Page)****10 marks**

- (a) Solve the equation  $x^2 = 25$ . 2
- (b) Solve the equation  $(x + 1)(2x - 3) = 0$ . 2
- (c) Solve  $3x^2 - 10x + 5 = 0$  using the quadratic formula, leaving your answers in simplified surd form. 3
- (d) Solve  $x^2 - 6x + 1 = 0$  by completing the square, expressing your answers to 2 decimal places. 3

**Question 3. (Start a NEW Page)****10 marks**

- (a) A box has 4 Geography books, 3 Mathematics books and 5 French books.  
One book is drawn at random from the box.  
Find the probability that the book drawn is :
- (i) a French book 1
- (ii) not a French book 1
- (iii) a French or a Mathematics book 1
- (iv) a Science Book 1
- (b) Two dice numbered 1 to 6 are rolled simultaneously.  
The sum of the 2 uppermost faces is noted.
- (i) Draw a diagram to show the possible outcomes 1
- Find the probability that the sum is:
- (ii) 9 1
- (iii) not 9 1
- (iv) greater than 9 1
- (c) Consider the statement:  
“ If two coins are thrown the possible outcomes are two heads, two tails or one head and one tail. So the probability of getting two heads when two coins are thrown is one third”
- Do you agree or disagree with the statement? Give reasons for your answer. 2

## Section B

## Continued

## Question 4. (Start a NEW Page)

10 marks

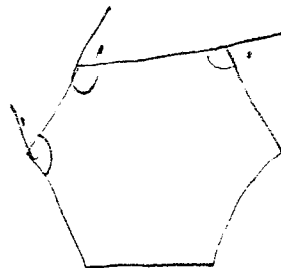
(a) A decagon is a polygon with 10 sides.

- (i) Find the interior angle sum of a decagon. 2
- (ii) If this decagon is regular, find the size of each interior angle. 2
- (iii) If this decagon is regular, find the size of each exterior angle. 2

(b) The number of diagonals,  $D$ , in a polygon is given by the formula  $D = \frac{1}{2}n(n-3)$ . 4

If a polygon has 14 diagonals, how many sides does it have?

$$\frac{1}{2}n^2 - \frac{3}{2}n = 14$$

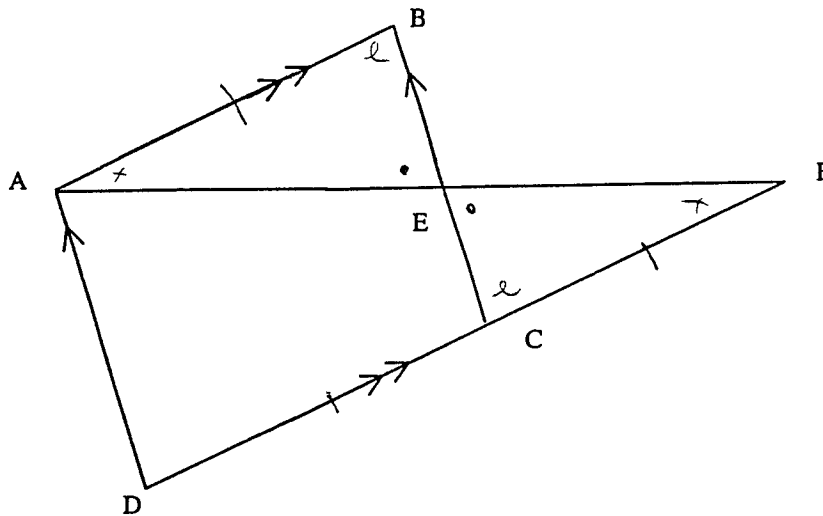


**Question 5. (Start a New Page)****10 marks**

- (a) ABCD is a parallelogram. The side DC is produced to F so that  $DC = CF$ . The line AF meets BC at E.

- (i) Explain why  $AB = CF$   
 (ii) Prove that  $\triangle FCE \cong \triangle ABE$   
 (iii) Show that E is the midpoint of BC

1  
 3  
 1



- (b) "The sum of a positive integer and three times its square is equal to 52. Find the integer."

5

Solve the above problem by constructing and solving a suitable quadratic equation.  
 (Hint: let the integer be  $x$ )

**END OF SECTION B**

## Section C

40 Marks

## Question 1:

Start a new page

10 marks

- (a) Ali gets a gift from her aunty Hariette of \$10 000 on her 16<sup>th</sup> birthday. Because she wants to buy a car after uni she is going to invest the \$10000 for 5 years. She can't decide whether to invest it with the *Trustme Bank* at  $6\frac{1}{2}\%$  per annum compounded annually, or the *Flybinite Bank* at 6% per annum compounded monthly.
- (i) To what amount will the \$10 000 grow if it is invested with the *Trustme Bank* for five years? 2
- (ii) What would be the better investment of the two banks? Justify your answer with working and reasoning. 2
- (b) Lena decides to buy a new *Pear 'uMac'* laptop for \$3500. Her friend Georgie tells her that it will depreciate at 28% a year for each year she owns it.
- (i) What will the laptop be worth when she decides to sell it at the end of 4 years? 2
- (ii) Her friend Christine says she will buy the laptop when it is worth less than \$200. How long will Christine have to wait? 2
- (c)  $4x^2 - 12x + k^2$  is a perfect square. Find the possible values of  $k$  2

## Question 2:

Start a new page

10 marks

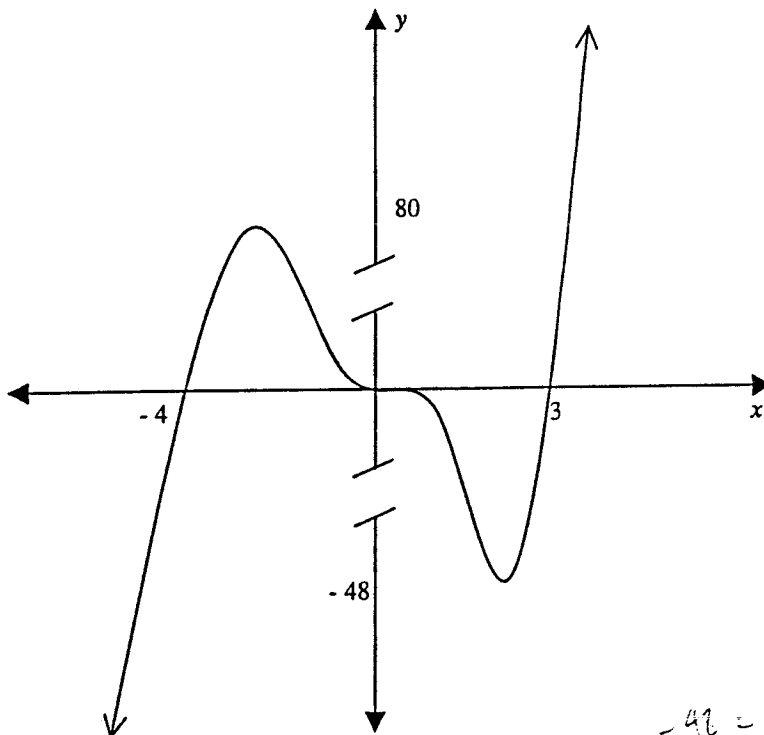
- (a) For the parabola  $y = x^2 - 2x - 3$
- (i) Find the  $y$  intercept 1
- (ii) Find the  $x$  intercepts 2
- (iii) Give the equation of the axis of symmetry 1
- (iv) Find the vertex 1
- (v) Sketch the curve on suitably labelled axes 2

- (b) Write down an equation of a parabola with  $x$  intercepts of  $-2$  and  $+4$  1
- (c) What is the vertex of the parabola given by  $y = 3x + 5 - 3x^2$  2

**Question 3:** **Start a new page** **10 marks**

- (a) Sketch the following curves on suitably labelled axes:
- (i)  $y = x(x - 1)(x + 2)$  2
- (ii)  $y = x^3(x - 2)^3(x + 1)(x - 1)^2$  (show relative heights only) 3
- (iii)  $xy = -6$  2
- (iv)  $x^2 + y^2 = 12$  1

- (b) Give the equation of the curve sketched below 2



*Handwritten notes:*  
 $-48 = x^3(x-4)(x-3)$   
 $80 = x^3(x-4)(x-3)$

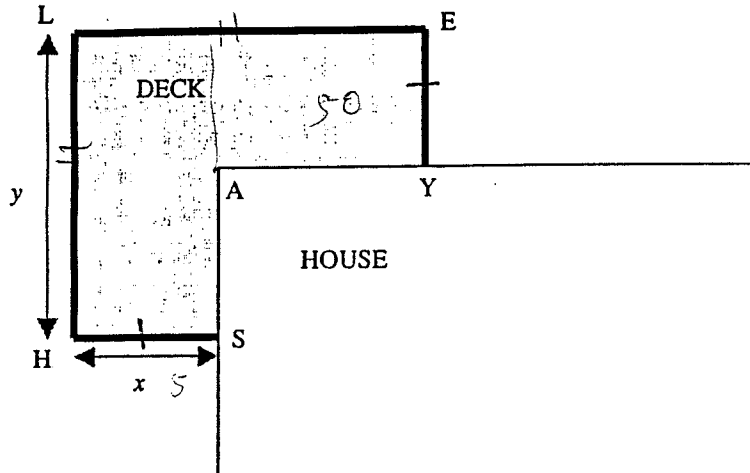


**Question 4:**

**Start a new page**

**10 marks**

(a) Xiaolei decides to build a deck on the corner of her house. The plan for the deck is drawn below. A railing is to be built around the four outer edges of the deck, SH, HL, LE, and EY. The railing has a total length of 30 metres and  $SH = EY$  and  $HL = LE$ .

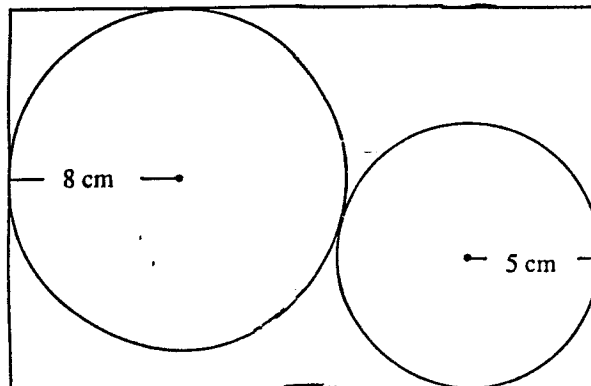


- (i) If SH is  $x$  metres, and HL is  $y$  metres, show that: 1  

$$y = 15 - x$$
- (ii) Show that the area  $A$  of the deck is: 2  

$$A = 2xy - x^2$$
- (iii) Using (i) and (ii) show that  $A = 30x - 3x^2$  1
- (iii) For what value of  $x$  will the area of the deck be largest? 2  
 Explain how you found your answer.

(b) Two circles are placed in a rectangle so that they just touch the sides, as in the diagram. One circle has a radius of 8 cm and the other a radius of 5 cm. If one side of the rectangle is 25 cm, what is the other? Show your reasoning. 3



*End of Examination*

Section A

Year 9 Topics

10 Marks

1. Find the surface area of a cube whose side is 8 cm

1

$384 \text{ cm}^2 \checkmark$

2. Solve the equation  $\frac{n}{4} = 8$

1

$n = 32 \checkmark$

3. Rationalise the denominator of  $\frac{3}{\sqrt{3}}$

1

$\frac{3\sqrt{3}}{3} = \sqrt{3} \checkmark$

4. Simplify  $(3-\sqrt{2})(3+\sqrt{2})$

1

$9 + 3\sqrt{2} - 3\sqrt{2} - 2 = 7 \checkmark$

5. Write down the gradient of the line  $y = \frac{2}{3}x + 5$

1

$m = \frac{2}{3} \checkmark$

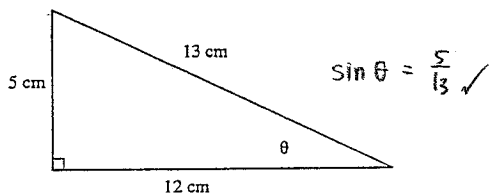
6. Solve the simultaneous equations below for x only:

1

$x + y = 2$   
 $x - y = 4$   
 $2x = 6$   
 $x = 3 \checkmark$

7. Find the exact value of  $\sin \theta$  from the triangle.

1



8. If the circumference of a circle is 25cm, find its diameter to 1 decimal place.

1

$\pi d = 25 \therefore d = \frac{25}{\pi} = 7.96 \checkmark$

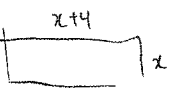
9. If  $a = \frac{3}{2}$ , evaluate  $\frac{2}{3}a^3$

1

$\frac{2}{3} \times (\frac{3}{2})^3 = 2 \frac{1}{4} \checkmark$

10. The length of a rectangular lawn is 4 metres longer than its width. If the width is x metres, and the area is 192 square metres write down an equation which may be used to find its width. (Do not solve)

1



$192 = x(x+4)$   
 $192 = x^2 + 4x \therefore x^2 + 4x - 192 = 0$

END OF SECTION A

10

Section B

Geometry, Probability, Quadratics

50 Marks

Question 1. (Start a NEW Page)

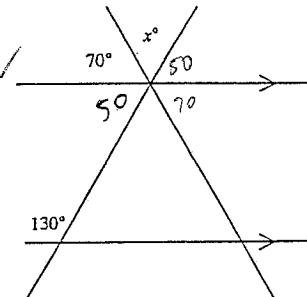
10 marks

- (a) Find the value of x.

2

$360 - 240 = 120 \checkmark$

$x = 60 \checkmark$



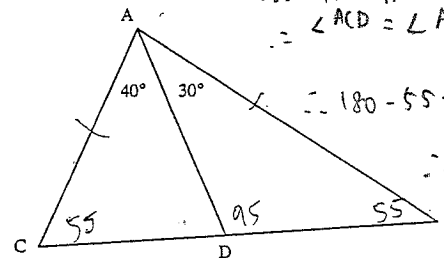
- (b) If  $AB = AC$ , what is the size of  $\angle BDA$ ? Give reasons for your answer.

4

$180 - 70 = 110$   
 $\therefore \angle ACD = \angle ABC = 55^\circ$  (opp. base  $\angle$  of isosceles  $\Delta$ )

$\therefore 180 - 55 - 30 = 95^\circ$  ( $\Delta$  sum of  $\Delta = 180^\circ$ )

$\therefore \angle BDA = 95^\circ \checkmark$

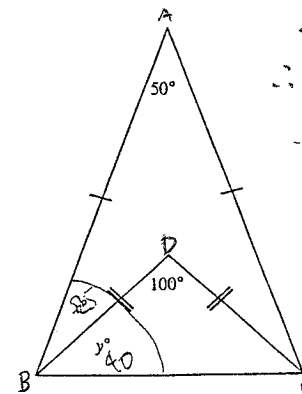


- (c) Find the value of y and x, giving reasons.

$\angle DBC = \angle DCB = 40^\circ$  (opp base  $\angle$  of isos  $\Delta$ )

$\therefore y = 40^\circ$   
 $\therefore \angle ABC = \angle ACB = 65^\circ$  (opp / base  $\angle$  of isos  $\Delta$  are =)

$\therefore x = 65 - 40 = 25 \checkmark$



6

Section B Continued

Question 2. (Start a NEW Page)

10 marks

(a) Solve the equation  $x^2 = 25$ .

$x = \pm 5$  ✓✓

2

(b) Solve the equation  $(x+1)(2x-3) = 0$ .

$2x^2 + 2x - 3x - 3 = 0$

$2x^2 - x - 3 = 0$   
 $2x(x+1) - 3(x+1) = 0$   
 $(2x-3)(x+1) = 0$   
 $x = -1, \frac{3}{2}$  ✓

2

(c) Solve  $3x^2 - 10x + 5 = 0$  using the quadratic formula, leaving your answers in simplified surd form.

$\Delta = \sqrt{40}$   
 $x = \frac{10 \pm \sqrt{40}}{6} = \frac{10 \pm 2\sqrt{10}}{6} = \frac{5 \pm \sqrt{10}}{3}$

9

(d) Solve  $x^2 - 6x + 1 = 0$  by completing the square, expressing your answers to 2 decimal places.

$x^2 - 6x + 9 = 8$   
 $(x-3)^2 = 8$   
 $x-3 = \pm\sqrt{8}$   
 $x = 3 \pm \sqrt{8}$   
 $x = 5.83, 0.17$

3

Question 3. (Start a NEW Page)

10 marks

(a) A box has 4 Geography books, 3 Mathematics books and 5 French books. One book is drawn at random from the box. Find the probability that the book drawn is:

- (i) a French book  $P(F) = \frac{5}{12}$  ✓
- (ii) not a French book  $P(\bar{F}) = \frac{7}{12}$  ✓
- (iii) a French or a Mathematics book  $P(F, M) = \frac{8}{12} = \frac{2}{3}$  ✓
- (iv) a Science Book  $P(S) = 0$  ✓

(b) Two dice numbered 1 to 6 are rolled simultaneously. The sum of the 2 uppermost faces is noted.

(i) Draw a diagram to show the possible outcomes

Find the probability that the sum is:

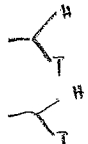
- (ii) 9  $P(9) = \frac{4}{36} = \frac{1}{9}$  ✓
- (iii) not 9  $P(\bar{9}) = \frac{9}{9} = 1$  ✓
- (iv) greater than 9  $P(>9) = \frac{1}{6}$  ✓

(c) Consider the statement:

"If two coins are thrown the possible outcomes are two heads, two tails or one head and one tail. So the probability of getting two heads when two coins are thrown is one third"

Do you agree or disagree with the statement? Give reasons for your answer. Disagree as  $P(HH) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$  not  $\frac{1}{3}$  ✓

1	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12



10

Section B Continued

Question 4. (Start a NEW Page)

10 marks

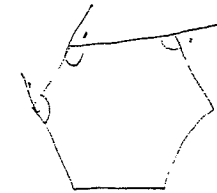
(a) A decagon is a polygon with 10 sides.

- (i) Find the interior angle sum of a decagon.  $1440^\circ$  ✓✓
- (ii) If this decagon is regular, find the size of each interior angle.  $144^\circ$  ✓✓
- (iii) If this decagon is regular, find the size of each exterior angle.  $36^\circ$  ✓✓

(b) The number of diagonals, D, in a polygon is given by the formula  $D = \frac{1}{2}n(n-3)$ .

If a polygon has 14 diagonals, how many sides does it have?

$14 = \frac{1}{2}n(n-3)$   
 $28 = n^2 - 3n$   
 $n^2 - 3n - 28 = 0$   
 $(n-7)(n+4) = 0$   
 $n = -4, 7$   
 $\therefore 7 \text{ sides}$  ✓



10

Question 5. (Start a New Page)

10 marks

(a) ABCD is a parallelogram. The side DC is produced to F so that DC = CF. The line AF meets BC at E.

- (i) Explain why AB = CF
- (ii) Prove that  $\Delta FCE \cong \Delta ABE$
- (iii) Show that E is the midpoint of BC

1  
3  
1

(ii) In  $\Delta ABE$ ,  $\Delta FCE$   
 $\angle ABE = \angle ECF$  (alt  $\angle$  of  $\parallel$  lines  $AB, DF$ )  
 $\angle AEB = \angle CEF$  (vertically opp  $\angle$ )  
 $AB = CF$  (proved above)  
 $\therefore \Delta ABE \cong \Delta FCE$  (ASA)

(iii) Since  $\Delta ABE \cong \Delta FCE$   
 $BE = EC$   
 $\therefore E$  is midpoint of BC

(i)  $AB = DC$  (opp sides of  $\parallel$ ogram)  
 $DC = CF$  (given)  
 $\therefore AB = CF$

(b) "The sum of a positive integer and three times its square is equal to 52. Find the integer."

5

Solve the above problem by constructing and solving a suitable quadratic equation.  
 (Hint: let the integer be x)

$x + 3x^2 = 52$   
 $3x^2 + x - 52 = 0$

5  
P-154  
-13, 2

END OF SECTION B

$3x^2 + 12x - 13x - 52 = 0$   
 $3x^2 + 13x - 12x - 52 = 0$   
 $x(3x + 13) - 4(3x + 13) = 0$   
 $(x - 4)(3x + 13) = 0$   
 $\therefore x = 4$  (the integer)

10

Section C

40 Marks

Question 1:

Start a new page

10 marks

(a) Ali gets a gift from her aunty Hariette of \$10 000 on her 16<sup>th</sup> birthday. Because she wants to buy a car after uni she is going to invest the \$10000 for 5 years. She can't decide whether to invest it with the Trustme Bank at  $6\frac{1}{2}\%$  per annum compounded annually, or the Flybinite Bank at 6% per annum compounded monthly. 13488.50 ✓

(i) To what amount will the \$10 000 grow if it is invested with the Trustme Bank for five years? 2

\$13700.87 ✓

(ii) What would be the better investment of the two banks? Justify your answer with working and reasoning. 2

Better at Trustme Bank as earn more of \$212.37 ✓

(b) Lena decides to buy a new Pear 'uMac' laptop for \$3500. Her friend Georgie tells her that it will depreciate at 28% a year for each year she owns it.

(i) What will the laptop be worth when she decides to sell it at the end of 4 years? 2

\$940.58 ✓

(ii) Her friend Christine says she will buy the laptop when it is worth less than \$200. How long will Christine have to wait? 2

7 years ✓

8.71 years.

(c)  $4x^2 - 12x + k^2$  is a perfect square. Find the possible values of k 2

$(2x - k)^2 = 4x^2 - 12x + k^2$

$k = 3$  ✓  $k = 3$

Question 2:

Start a new page

10 marks

(a) For the parabola  $y = x^2 - 2x - 3$

(i) Find the y intercept  $(0, -3)$  ✓

1

(ii) Find the x intercepts  $(3, 0)$  and  $(-1, 0)$  ✓

2

(iii) Give the equation of the axis of symmetry  $x = \frac{2}{2} = 1$  ✓

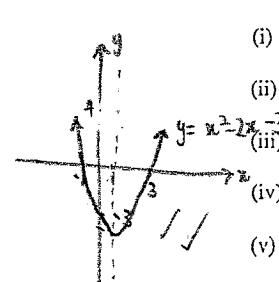
1

(iv) Find the vertex  $y = 1 - 2 - 3 = -4$   $(1, -4)$  ✓

1

(v) Sketch the curve on suitably labelled axes 2

2



10

$y = ax^2 + bx + c$

$y = x^2 - 2x - 8$

(b) Write down an equation of a parabola with x intercepts of -2 and +4

$(x+2)(x-4) = y$

(c) What is the vertex of the parabola given by  $y = 3x + 5 - 3x^2 = -3x^2 + 3x + 5$

vertex =  $(\frac{1}{2}, 10\frac{3}{4})$

$x = \frac{-3}{-6}$   
 $y = 3(\frac{1}{2}) + 5 - 3(\frac{1}{2})^2$   
 $y = 10\frac{3}{4}$

Question 3: Start a new page

(a) Sketch the following curves on suitably labelled axes:

(i)  $x^2 + x - 2$

$y = x(x-1)(x+2)$

$= x^3 + x^2 - 2x$

(ii)  $y = x^3(x-2)^2(x+1)(x-1)^2$  (show relative heights only)

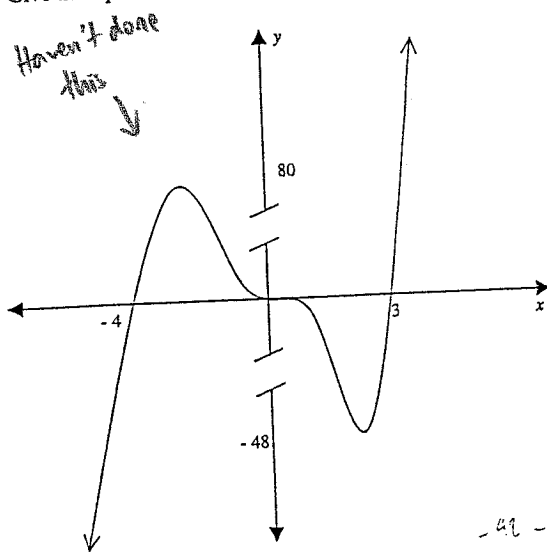
(iii)  $xy = -6$

(iv)  $x^2 + y^2 = 12$

Have a 4  
done for polynomials

7

(b) Give the equation of the curve sketched below



$y = x^3(x+4)(x-3)$

Haven't done this

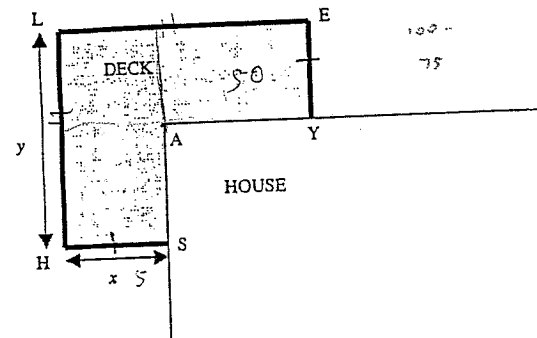
$-4x - x^2 = x^2 + 4x$   
 $20 = x^2 + 4x + 4 - 4$   
 $20 = (x+2)^2 - 4$

Question 4:

Start a new page

10 marks

(a) Xiaolei decides to build a deck on the corner of her house. The plan for the deck is drawn below. A railing is to be built around the four outer edges of the deck, SH, HL, LE, and EY. The railing has a total length of 30 metres and SH = EY and HL = LE.



(i) If SH is x metres, and HL is y metres, show that:

$y = 15 - x$        $2x + 2y = 30 \rightarrow x + y = 15$   
 $\therefore y = 15 - x$

(ii) Show that the area A of the deck is:

$A = 2xy - x^2$   
 $A = xy + x(y-x) = xy + xy - x^2 = 2xy - x^2$

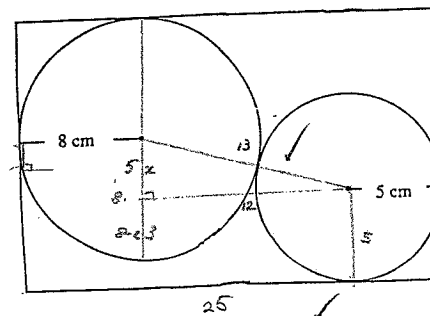
(iii) Using (i) and (ii) show that  $A = 30x - 3x^2$

$A = 2x(15-x) - x^2 = 30x - 2x^2 - x^2 = 30x - 3x^2$

(iii) For what value of x will the area of the deck be largest? Explain how you found your answer.

$x = \frac{-30}{-6}$   
 $y = 10$   
 $\therefore (5, 10) \text{ is vertex}$   
 $\therefore 5 \text{ (turning point)} = 5$

(b) Two circles are placed in a rectangle so that they just touch the sides, as in the diagram. One circle has a radius of 8 cm and the other a radius of 5 cm. If one side of the rectangle is 25 cm, what is the other? Show your reasoning.



18 cm

10