11-2U PRELIM EXAM 2003

Question 1 *(Start a new page)* Marks

(a) Expand and simplify $5a + (2a - 3)^2$ 2

(b) Factorize fully $a^3 + 8$ 2

(c) Express $\frac{2}{4+\sqrt{3}}$ with a rational denominator. 2

(d) Simplify fully: $\sqrt{27} + \sqrt{75}$ 2

(e) Simplify $\frac{2x}{3} - \frac{x+4}{7}$. 2

(f) Solve $9x^2 - 121 = 0$ 2

(g) Find the domain and range of: $y = -\sqrt{25 - x^2}$. 2

Question 2 *(Start a new page)* Marks

(a) If $f(x) = x^3 - 3x^2 + 4x$, find $f(-a)$. 2

(b) A function $f(x)$ is defined as:

$$f(x) = \begin{cases} -4 & \text{for } x \le -3 \\ 3x & \text{for } -3 < x < 0 \\ x^2 & \text{for } x \ge 0 \end{cases}$$

Find the value of f(-1) and f(2). 2

(c) Determine if the function $f(x) = x^2 + 2$ is odd, even or neither. 2

(d) Solve $|2x - 3| = 12$ 2

(e) Sketch each of the following indicating all important features:

(i) $y = x - 2$ 2

(ii) $xy = 8$ 2

(iii) $(x + 5)^2 + y^2 = 16$ 2

Question 3 *(Start a new page)* Marks

(a) Find the exact value of cos135° 2

(b) If $\sin A = \frac{5}{13}$ and $\cos A < 0$ what is the exact value of tanA 2

(c) Simplify (i) $5 - 2\cos^2 x - 2\sin^2 x$ 2

(ii) $\sec x \cot x \sin x$ 2

(d) Solve $\cos 3x = -\frac{1}{2}$ for $0° \le x \le 360°$ 3

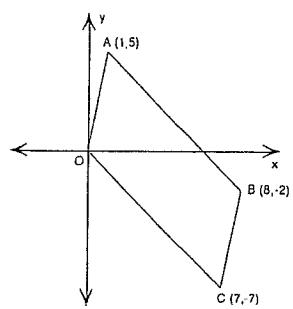
(e) The bearings from a point P of two ships A and B are 35° and 125° and their distances from P are 350m and 675m respectively.

(i) Draw a neat diagram showing this information 1

(ii) Hence find the bearing of A from B (to the nearest minute) 2

Question 4 *(Start a new page)* Marks

(a) In the diagram below O(0,0), A(1,5), B(8,-2) and C(7,-7) are the vertices of quadrilateral OABC.



(i) Find the midpoint of the interval AC 1

(ii) Find the gradient of AB. 1

(iii) Show that the equation of AB is $x + y = 6$. 2

(iv) Show that AB is parallel to OC. 2

(v) Show that the diagonals AC and OB bisect each other 2

(b) Using the perpendicular distance or otherwise, determine whether the line $3x + 5y + 15 = 0$ is a tangent to the circle with centre (0,0) and radius 2 units. 3

(c) Find the equation of the line through the intersection of the lines $x + y - 2 = 0$ and $2x - y - 1 = 0$ and the point (1,3). 3

Question 5 *(Start a new page)* Marks

(a) The fourth term of an arithmetic sequence is 14 and the ninth term is 39. Find:

(i) the common difference 2

(ii) the first term 1

(b) Evaluate: $\sum_{k=2}^{20} 3k-4$ 3

(c) In an arithmetic sequence, the seventh term is 8 while the sum of the third and eighth terms is 7. Find the sum of the first 50 terms. 3

(d) For what values of x does the geometric series $9 + 15x + 25x^2 + \ldots$ have a limiting sum? 2

(e) Express the recurring decimal $0.2\dot{6}$ as a fraction in its lowest form, by first writing the decimal as an infinite geometric series. 3

Question 6 *(Start a new page)* Marks

(a) Consider the quadratic function: $y = x^2 + 6x - 7$

(i) Express the quadratic in the form: $y = (x+h)^2 + k$ by completing the square 1

(ii) Find the axis of symmetry 1

(iii) Find the x and y intercepts 3

(iv) Use the information above to sketch the curve 2

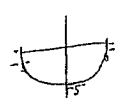
(v) Hence solve for x: $x^2 + 6x - 7 < 0$ 1

(b) Simplify $\cos(180-A)\sin(90-A) + \sin(180+A)\cos(90-A)$ 3

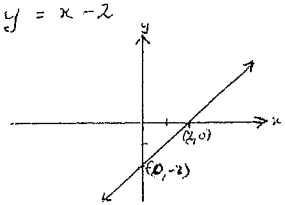
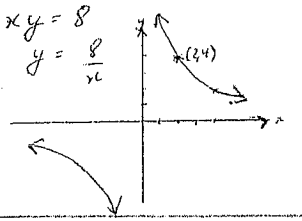
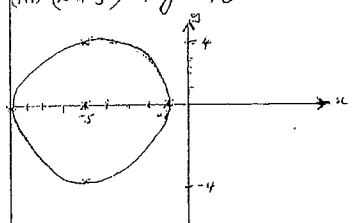
(c) How many terms of the sequence 3, 6, 12,...must be taken for the sum to exceed one million? 3
(Hint: First find an expression for S_n)

END OF EXAMINATION

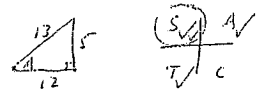
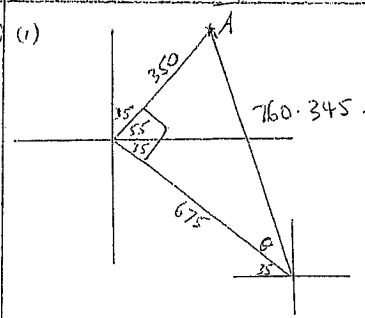
RMBL

Qn	Solutions	Marks	Comments+Criteria
1(a)	$5a + (2a-3)^2$ $= 5a + 4a^2 - 12a + 9$ $= 4a^2 - 7a + 9$	1 1 ½	1 mark off per mistake
(b)	$a^3 + 8$ $= (a+2)(a^2 - 2a + 4)$	1,1 ½	
(c)	$\frac{2}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}} = \frac{2(4-\sqrt{3})}{16-3}$ $= \frac{8-2\sqrt{3}}{13}$	1 1 ½	
(d)	$\sqrt{27} + \sqrt{75}$ $= 3\sqrt{3} + 5\sqrt{3}$ $= 8\sqrt{3}$	1 1 ½	
(e)	$\frac{2x}{3} - \frac{x+4}{7} = \frac{7(2x) - 3(x+4)}{21}$ $= \frac{14x - 3x - 12}{21}$ $= \frac{11x-12}{21}$	1 1 ½	1 mark numerator $11x+12 = ½$ 1 mark denominator
(f)	$9x^2 - 121 = (3x-11)(3x+11) = 0$ (½) $\therefore x = \frac{11}{3}, -\frac{11}{3}$	1,1 ½	½ factorise 1 ½ 11/3 2/2 both answers
(g)	$y = -\sqrt{25-x^2}$ $D: -5 \le x \le 5$ $R: -5 \le y \le 0$ 	1 1 ½	1 each NO part marks.

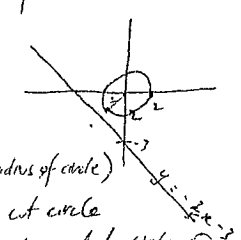
KMBL a-c, PMd-f

Qu	Solutions	Marks	Comments+Criteria
2a	$f(-a) = (-a)^3 - 3(-a)^2 + 4(-a)$ $= -a^3 - 3a^2 - 4a$ OR $-(a^3 + 3a^2 + 4a)$	1 1 ½	
(b)	$f(-1) = 3\times -1 = -3$ $f(2) = 2^2 = 4$	1 1 ½	
(c)	$f(x) = x^2 + 2$ $f(-x) = (-x)^2 + 2$ $= x^2 + 2$ $= f(x)$ ∴ even function	1 1 ½	
(d)	$\lvert 2x - 3\rvert = 12$ ① $2x - 3 = 12$, $2x = 15$, $x = 7\frac{1}{2}$ ② $-(2x-3) = 12$, $-2x + 3 = 12$, $-2x = 9$, $x = -4\frac{1}{2}$	1 1 ½	
(e) (i)	$y = x - 2$ 	2	1 mk shape 1 mk main points
(ii)	$xy = 8$ $y = \frac{8}{x}$ 	2	
(iii)	$(x+5)^2 + y^2 = 16$  Centre $(-5, 0)$ Radius 4	½	

KM

Qu	Solutions	Marks	Comments+Criteria
3a	$\cos 135 = -\cos 45$ $= -\frac{1}{\sqrt{2}}$	1 1 ½	−1 mk (no sign)
(b)	$\sin A = \frac{5}{13}$, $\cos A < 0$  $\tan A = -\frac{5}{12}$	1 1 ½	−1 mk (no sign) or (1 mk) for process
(c) (i)	$5 - 2\cos^2 x - 2\sin^2 x$ OR $5 - 2(1 - \sin^2 x) - 2\sin^2 x$ $= 5 - 2(\cos^2 x + \sin^2 x)$ $= 5 - 2 + 2\sin^2 x - 2\sin^2 x$ $= 5 - 2\times 1$ $= 5 - 2 + 0$ $= 3$ $= 3$.	1 1 ½	1 factorise 1 identity $s^2 + c^2 = 1$ $5 - 2(\cos^2 x - \sin^2 x)$ lose 1 mk.
(ii)	$\sec x \cot x \sin x$ $= \frac{1}{\cos x} \times \frac{\cos x}{\sin x} \times \frac{\sin x}{1}$ $= 1$	1 1 ½	$\frac{1}{\cos x} \cdot \frac{1}{\tan x} \cdot \sin x$ (1 mk)
(d)	$\cos 3x = -\frac{1}{2}$ $0 \le x \le 360°$ $3x = (60), 180-60, 180+60$ (2nd, 3rd quads.) $3x = 120, 240, 120+360, 240+360, 120+720, 240+720$ $3x = 120, 240, 480, 600, 840, 960$ $\therefore x = 40, 80, 160, 200, 280°, 320°$	1 1 1 ½	−1 mk incorrect basic angle
(e) (i)	 $760 \cdot 345\ldots$	1 1 ½	
(ii)	$\tan\theta = \frac{350}{675}$ $\theta = 27°24'$ Bearing $= 270 + 35 + 27°24$ $= 332°24'$	1 1 ½	

PLM

Qn	Solutions	Marks	Comments+Criteria
4 (a)(i)	$M = \left(\frac{1+7}{2}, \frac{5-7}{2}\right)$ $= (4, -1)$	1	
	(ii) $m_{AB} = \frac{-2-5}{8-1}$ $= \frac{-7}{7}$ $= -1$	1	
	(iii) Use $(1,5)$, $m = -1$ $y - 5 = -1(x-1)$ $y - 5 = -x + 1$ $y = -x + 6$ or $x + y = 6$	2	1 mark off per mistake
	(iv) $m_{OC} = \frac{-7-0}{7-0}$ $= \frac{-7}{7}$ $= -1$ $= m_{AB}$ $\therefore$ OC // AB	2	
	(v) Point of intersection: Midpt OB $= (4, -1)$ midpt AC $= (4, -1)$ $\therefore$ diagonals bisect each other	2	
(b)	$3x + 5y + 15 = 0$ Point $(0,0)$ $a = 3$ $b = 5$ $c = 15$ $PD = \left\lvert \frac{3.0 + 5.0 + 15}{\sqrt{3^2 + 5^2}} \right\rvert$ $= \frac{15}{\sqrt{9+25}}$ $= \frac{15}{\sqrt{34}}$ (1½) $\doteq 2.6 > 2$ (½) (radius of circle) $\therefore$ line does not cut circle $\therefore$ line is not a tangent to circle (1) 	3	1 mark reasonable attempt at P.Dist formula 1½ marks correct P.Dist. 2 marks P.Dist vs radius 3 marks radius → not tangent
(c)	$x + y - 2 + k(2x - y - 1) = 0$ sub $(1,1)$ $1 + 1 - 2 + k(2\times1 - 1 - 1) = 0$ $2 + 2k = 0$ $\therefore 2k = 2$ $k = 1$ (2) $x + y - 2 + 1(2x - y - 1) = 0$ $x + y - 2 + 2x - y - 1 = 0$ $3x - 3 = 0$ $3x = 3$ $x = 1$ (3)	3	(or find point of intersection method)

MD

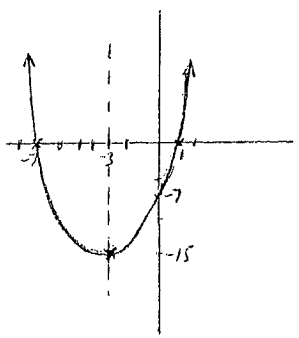
BAS - Buy a Stapler

Qn	Solutions	Marks	Comments+Criteria
5 (a)(i)	$T_4 = a + 3d = 14$ (1) $T_9 = a + 8d = 39$ (2) (2) − (1) $5d = 25$ $d = 5$	1 1 (2)	
(a)(ii)	sub (1) $a + 3\times5 = 14$ $a + 15 = 14$ $a = -1$	1 (1)	
(b)	$\sum_{k=2}^{20} 3k - 4$ $a = 3\times2 - 4 = 2$ $l = 3\times20 - 4 = 56$ $n = 19$ $(d = 3 \Rightarrow$ Series: $2 + 5 + 8 + \ldots + 56)$ $\therefore S_{20} = \frac{19}{2}(2 + 56)$ $= 551$	1 1 1 (3)	−1 for $3(k-4)$
(c)	$T_7 = 8$ $\therefore a + 6d = 8$ (1) $T_3 + T_8 = 7$ $\therefore a + 2d + a + 7d = 7$ $2a + 9d = 7$ (2) (1)×2 $2a + 12d = 16$ $3d = 9$ $d = 3$ sub (1) $a + 6\times3 = 8$ $a + 18 = 8$ $a = -10$ $S_{50} = \frac{50}{2}[2\times-10 + (50-1)3]$ $= 25(-20 + 49\times3)$ $= 25 \times 127$ $= 3175$	1 1 1 (3)	
(d)	$9 + 15x + 25x^2 + \ldots$ $\therefore r = \frac{15x}{9} = \frac{5x}{3}$ $\left\lvert \frac{5x}{3} \right\rvert < 1$ or $-\frac{3}{5} < x < \frac{3}{5}$	1 1 (2)	
(e)	$0.26\ldots = 0.2 + \frac{0.06 + 0.006 + \ldots}{}$ $a = 0.06$, $r = 0.1$ $= 0.2 + \left(\frac{0.06}{1 - 0.1}\right)$ $= 0.2 + \frac{0.06}{0.9}$ $= \frac{2}{10} + \frac{6}{90}$ $= \frac{24}{90}$ $= \frac{4}{15}$	1 1 1 (3)	

TE - Transcription Error

CFPA - Correct for previous answer

2M

Qn	Solutions	Marks	Comments+Criteria
6(a)	(i) $y = x^2 + 6x - 7$ $y = x^2 + 6x + 9 - 7 - 9$ $y = (x+3)^2 - 16$	1	
	(ii) $x = -3$	1	
	(iii) x-int, $y = 0$ $0 = x^2 + 6x - 7$ $0 = (x-1)(x+7)$ $\therefore x = 1$ or -7	2	
	y-int, $x = 0$ $y = 7$	1	
	(iv) 	2	
	(v) $x^2 + 6x - 7 < 0$ $\therefore -7 < x < 1$	1 /8	
(b)	$\cos(180 - A)\sin(90 - A) + \sin(180 + A)\cos(90 - A)$ $= +\cos A \times \cos A + -\sin A \times \sin A$ $= -\cos^2 A - \sin^2 A$ $= -(\cos^2 A + \sin^2 A)$ $= -1$	2 1 /3	
(c)	$3, 6, 12, \ldots \quad a = 3, r = 2$ $S_n = \frac{a(r^n - 1)}{r - 1} = \frac{3(2^n - 1)}{2 - 1} = 3(2^n - 1)$	1	
	Now $3(2^n - 1) > 1,000,000$ $2^n - 1 > \frac{1,000,000}{3}$ $2^n > \frac{1,000,000}{3} + 1$ $2^n > 333,334.33$ $\log 2^n > \log_{10} 333,334.33$ $n > \frac{\log_{10} 333,334.33}{\log_{10} 2}$	1	
	$n > 18.35$ $\therefore n = 19$ ie 19 terms	1 /3	