KAMBALA

MATHEMATICS

YEAR 11

2 Unit

Preliminary Examination

September 2004

Time Allowed: 2 hours
Reading time : 5 minutes

INSTRUCTIONS
Calculators may be used.
All questions are to be answered on the writing paper provided.
All questions are of equal value.
Start each question on a new page.
Show all necessary working.
Marks will be deducted for careless or badly arranged work.

Question 1. (15 marks)

(a) If $V = \pi r^2 h$, find h, correct to one decimal place, given V=1200 and r=7.9. (2)

(b) Factorise $9x^2 - 100$ (2)

(c) Solve $28 - 5a \geq -27$ (3)

(d) Find x and y, such that $\frac{16\times 9}{27\times 81} = 2^x \times 3^y$ (2)

(e) Simplify $1 + \tan^2\theta$ (1)

(f) Solve $|7-3x| > 9$. (3)

(g) Find the integers a and b such that $\frac{\sqrt{5}}{2+\sqrt{5}} = a + b\sqrt{5}$ (2)

Question 2. (15 marks)

(a) (i)On a number plane draw the points A (-4,-3), B (6,-3) and C (4,1) (1)

(ii) Find the gradient of BC. (1)

(iii) Prove A,B and C are the vertices of a right-angled triangle. (2)

(iv) Find the distance BC in exact form (2)

(v) Find the size of angle CAB to the nearest degree. (2)

(vi) Show that the co-ordinates of M, the mid-point of AB, are (1,-3) (2)

(b) A(4,-1) and B are the endpoints of an interval. C $(\frac{1}{2}, 1\frac{1}{2})$ is the midpoint of this interval. Find the coordinates of point B. (2)

(c) 11, 7, 3, -1, -5 form an arithmetic sequence..
By first finding the first term and the common difference, find the seventh term (3)

Question 3. (15 marks)

(a) Consider the equation $2x^2-(3+k)x+2=0$ (4)
For what values of k does the equation have
(i) equal roots
(ii) distinct real roots.

(b) Find the value of $\sum_{k=1}^{50} 2^k$ correct to 3 significant figures. (3)

(c) The third term of a geometric sequence is $\frac{3}{4}$ and the seventh term is 12. (4)
Find the first term and the constant ratio.

(d) If $\tan\alpha=\frac{3}{5}$ and $0°\le\alpha\le360°$, find the **exact** values of (4)
(i) $\cos\alpha$
(ii) $\operatorname{cosec}\alpha$

Question 4.(15 marks)

(a) Sketch the graph of the following function $y=(x-3)^2+4$, showing any intercepts and the vertex. From the graph, find the range of the function with domain $-1\le x\le2$ (4)

(b) Solve $3a^4-10a^2+8=0$ (4)

(c) If α and β are the roots of the equation $2x^2-8x-3=0$, find the (5)
value of:
(i) $\alpha+\beta$
(ii) $\alpha\beta$
(iii) $\frac{1}{\alpha}+\frac{1}{\beta}$
(iv) $\alpha^2+\beta^2$

(d) The lines $ax+2y=6$ *and* $4y=bx-9$ are parallel. Find the value of $\frac{a}{b}$ (2)

Question 5. (15 marks)

(a) Prove the identity $(\sec\alpha-\cos\alpha)^2=\tan^2\alpha-\sin^2\alpha$ (3)

(b) (i) On the same axes draw the graphs of $y=\sin x$ and $y=\cos x$, $0°\le x\le360°$
Clearly labelling each graph. (2)

(ii) Use the graphs to write down the number of solutions for $\sin x=\cos x$ (2)

(iii) Solve the equation $\sin x=\cos x$ to find these solutions (for $0°\le x\le360°$). (3)

(c) A length of railway track is to be relaid and supplies of materials are to be deposited at 24 points along the track at $\frac{1}{4}$km intervals. Supplies are taken by train from a depot which is 15 km from the closest of these points. The train must return to the depot and reload after depositing supplies at each individual point.

(i) How far has the train travelled when it returns to the depot after it has deposited the materials at the point closest to the depot. (1)

(ii) How far is the farthest point from the depot? (2)

(iii) Use the sum of an arithmetic progression to find the total distance that the train must travel in order to return to the depot having supplied all the points with materials. (3)

Question 6. (15marks)

(a) Write $0.2\dot{6}$ as the sum of the terms of a geometric series. Hence find the value of $0.2\dot{6}$ as a fraction in its simplest form. (2)

(b) Find the values of m, p and q for which (3)

$$3x^2 - x - 5 \equiv m(x+1)^2 + p(x-3) + q - 4$$

(c) If $\cos\theta = -\frac{2}{5}$ and $0° \le \theta \le 360°$, find the exact value(s) of $\tan\theta$. (3)

(d) Find the value of a for which the function $2x^2 - (3a-1)x + (2a-5) = 0$ has

i. one root equal to zero (2)

ii. the roots as reciprocals of one another (2)

(e) Solve $\frac{4x+1}{2x-3} = \frac{2x+1}{1+x}$ (3)

End of Test

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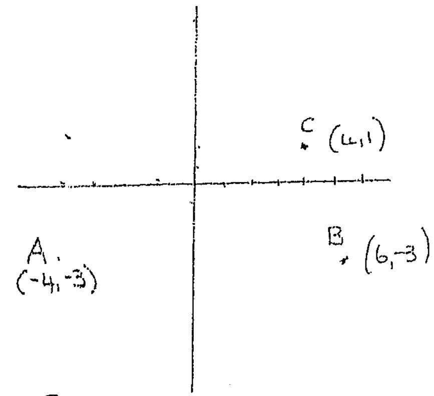
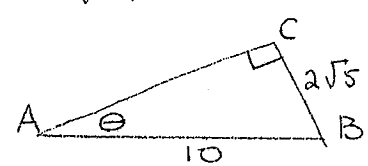
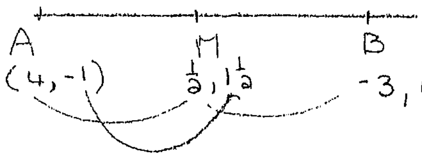
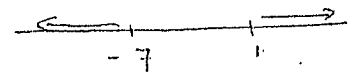
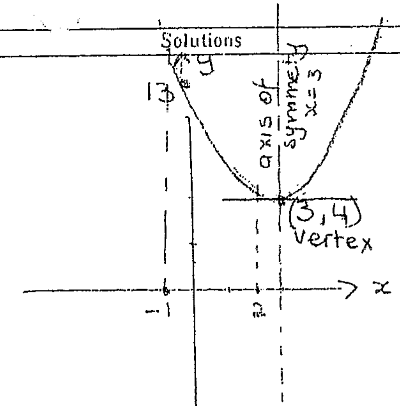
Qn	Solutions	Marks	Comments+Criteria
1a	$V=\pi r^2 h = \pi \times 7.9^2 \times h$	✓ $\frac{1200}{\pi \times 7.9^2}$	
	$\frac{1200}{(\pi \times 7.9^2)} = h = 6.12036\ldots$	✓	
	6.1 1dp		
(b)	$9x^2-100 = (3x)^2 - 10^2$	✓	
	$= (3x-10)(3x+10)$	✓	
(c)	$28-5a \geq -27$		
	$-5a \geq -27-28$	✓	
	$-5a \geq -55$	✓	
	$a \leq 11$	✓ sign	
(d)	$\frac{16 \times 9^1}{27 \times 81_9} = \frac{2^4}{3^3 \times 3^2} = 2^4 . 3^{-5}$		
	$x=4 \quad y=-5$	✓✓	
(e)	$1+\tan^2\theta = \sec^2\theta$	✓	
(f)	$7-3x > 9$ or $7-3x < -9$		
	$-3x > 2$ or $-3x < -16$		
	$x \cancel{>} \frac{2}{-3}$ $x > \frac{16}{3}$		
	$x > 5\frac{1}{3}$		
(g)	$\frac{\sqrt5}{(2+\sqrt5)} \frac{(2-\sqrt5)}{(2-\sqrt5)} = \frac{\sqrt5(2-\sqrt5)}{4-5}$	✓	
	$= \frac{2\sqrt5-5}{-1}$		
	$= 5-2\sqrt5$		
	$a=5 \quad b=-2$	✓ $-\frac{1}{2}$ signs wrong	

Qn	Solutions	Marks	Comments+Criteria
2. (i)		✓	
a(ii)	B C $(6,-3)\ (4,1)$ $x_1\ y_1 \quad x_2\ y_2$		
	$m = \frac{y_2-y_1}{x_2-x_1} = \frac{1--3}{4-6} = \frac{4}{-2} = -2$	✓	
(iii)	m A C $(-4,-3)\ (4,1)$		
	$m = \frac{1+3}{4+4} = \frac{4}{8} = \frac{1}{2}$.		
	$\therefore AC \perp BC \quad m_1 \times m_2 = -1$ $-2 \times \frac{1}{2} = -1$	✓✓	
	$(\angle ACB = 90^\circ)$		
iv)	BC $(6,-3)\ (4,1)$ $x_1\ y_1 \quad x_2\ y_2$		
	$d = \sqrt{(4-6)^2+(1+3)^2}$	✓	
	$= \sqrt{4+16} = \sqrt{20} = 2\sqrt5$ units	✓	
(v)			
	$\sin\theta = \frac{2\sqrt5}{10}$	✓	
	$\theta = 26°34'$ $= 27°$	✓	don't worry about nearest degree.

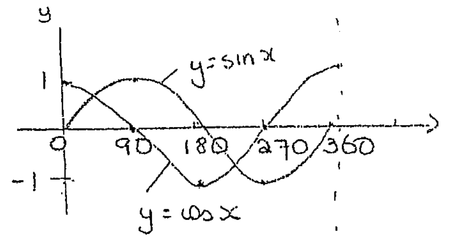
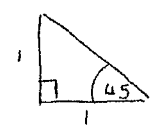
Qn	Solutions	Marks	Comments+Criteria
(vi)	A B $(-4,-3)$ $(6,-3)$ $\frac{-4+6}{2}, \frac{-3+-3}{2}$ $1, -3$	✓ ✓	
(b)	 or use midpoint formula	✓✓	
(c)	$11, 7, 3, -1 \ldots$ $a = 11$ $d = 7-11 = -4$ $T_7 = a + 6d = 11 + 6 \times -4 = -13$	a ✓ d ✓ ✓	

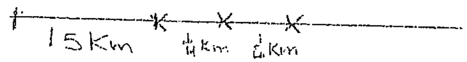
Qn	Solutions	Marks	Comments+Criteria
3 (a)	$2x^2 - (3+k)x + 2 = 0$ $\Delta = 0$ for equal roots $\Delta = b^2 - 4ac$ $= (3+k)^2 - 4 \times 2 \times 2$ $= 9 + 6k + k^2 - 16$ $= k^2 + 6k - 7$ $\Delta = 0$ $k^2 + 6k - 7 = 0$ $(k+7)(k-1) = 0$ $k = -7, k = 1$	1 1 1	$\frac{1}{2}$ mk for $k = 1$
	$\Delta > 0$ $k^2 + 6k - 7 > 0$ $(k+7)(k-1) > 0$  $k > 1$ $k < -7$	1	$\frac{1}{2}$ mk for wrong inequality
(b)	$\sum_{k=1}^{50} 2^k$ T_1 $2^1 = 2$ $\therefore$ GP $T_2 = 2^2 = 4$ $a = 2$ $T_3 = 2^3 = 8$ $r = 2$ $n = 50$ $S_n = \frac{a(r^n - 1)}{r - 1}$ $r > 1$ $= \frac{2(2^{50} - 1)}{2 - 1}$ 2.25×10^{15} 3sf.	1 1 1	or $2^1 + 2^2 + 2^3$ 1mk $\frac{1}{2}$ mk for 2^{49} or $r = \frac{1}{2}$ need correct s.f for full mks $\frac{1}{2}$ mk for wrong s.f

Qn	Solutions	Marks	Comments+Criteria
3 (c)	$T3 = ar^2 = \frac{3}{4}$	1	
	$T7 = ar^6 = 12$	1	
	$\therefore \frac{ar^6}{ar^2} = \frac{12}{3/4}$	1 r	
	$r^4 = 16 \quad \therefore r = \pm 2$	$\frac{1}{2}$ off if not +, −	
	$ar^6 = 12$ $a \times (\pm 2)^6 = 12 \quad \therefore a = \frac{12}{64} = \frac{3}{16}$	1 a.	0 mks if above not correct method.
(d)	$\tan \alpha = \frac{3}{5} \quad 0° \le \alpha < 360°$ $\therefore$ tan positive $\therefore$ Q1,3 S A / T C	1	2/4 if not both Qs
	3, 5, $\sqrt{34}$, α	1	
	(i) $\cos\alpha = \frac{5}{\sqrt{34}}$ Q1 $-\frac{5}{\sqrt{34}}$ Q3	1	$\frac{1}{2}$ mk for one of these.
	(ii) $\operatorname{cosec}\alpha$ $\sin\alpha = \frac{3}{\sqrt{34}}$ $\therefore \operatorname{cosec}\alpha = \frac{\sqrt{34}}{3}$ − Q1 $-\frac{\sqrt{34}}{3}$ Q3	1	$\frac{1}{2}$ mk for one of these

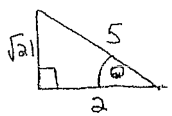
Qn	Solutions	Marks	Comments+Criteria
4 (a)		1 1 1	1 vertex 1 y-intercept 1 happy face concave up
	$\Delta = 0$ $\therefore x = -\frac{b}{2a}$ which is axis of symmetry $x = 3$		
	$x = -1$ $y = (-1-3)^2 + 4 = 16 + 4 = 20$ $x = 2$ $y = (2-3)^2 + 4 = 1 + 4 = 5$ $5 \le y \le 20$	1	
b	$3a^4 - 10a^2 + 8 = 0$ Let $a^2 = \mu$	1	1mk for this
	$3\mu^2 - 10\mu + 8 = 0$ P = 24, S = −10, F −6, −4 $3\mu^2 - 6\mu - 4\mu + 8 = 0$ $3\mu(\mu - 2) - 4(\mu - 2) = 0$ $(3\mu - 4)(\mu - 2) = 0$ $\mu = \frac{4}{3} \quad \mu = 2$	1	1mk for recognising take $\sqrt{\ }$
	$a^2 = \frac{4}{3}$ $\therefore a = \pm\frac{2}{\sqrt{3}} \quad a = \pm\sqrt{2}$	1 1	plus + minus take off 1/2

Qn	Solutions	Marks	Comments+Criteria
4(c)	$2x^2-8x-3=0$ $a=2,\quad b=-8,\quad c=-3$		
(i)	$\alpha+\beta=-\frac{b}{a}=-\frac{-8}{2}=4$	$\frac{1}{2}$	
(ii)	$\alpha\beta=\frac{c}{a}=-\frac{3}{2}$	$\frac{1}{2}$	
(iii)	$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\beta+\alpha}{\alpha\beta}=\frac{4}{-\frac{3}{2}}$	1	
	$=-2\,{}^{2}/_{3}$ $=-8/3$	1	
IV	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$	1	
	$=4^2-2\times-\frac{3}{2}$ $=19$	1	
(d)	$ax+2y=6 \qquad m=-\frac{a}{2}$	$\frac{1}{2}$	
	$4y=bx-9$ $y=\frac{b}{4}x-\frac{9}{4} \qquad m=\frac{b}{4}$	$\frac{1}{2}$	
	$\therefore -\frac{a}{2}=\frac{b}{4}$ $-4a=2b$ $\frac{a}{b}=\frac{2}{-4}=-\frac{1}{2}$	1	

Qn	Solutions	Marks	Comments+Criteria
Q5 (a)	LHS $=(\sec\alpha-\cos\alpha)^2$ $=\sec^2\alpha-2\sec\alpha.\cos\alpha+\cos^2\alpha$ $=\sec^2\alpha-\frac{2}{\cos\alpha}.\cos\alpha+\cos^2\alpha$ $=(\tan^2\alpha+1)-2+(1-\sin^2\alpha)$ $=\tan^2\alpha+\cancel{1}-\cancel{2}+\cancel{1}-\sin^2\alpha$ $=$ RHS	pt marks for each part 3	
b(i)		2	
(ii)	2 **Only worth 1 mark**	1 dep on their graph.	
(iii)	$\frac{\sin x}{\cos x}=\frac{\cos x}{\cos x}$ $\tan x=1$ $x=45^\circ$ $=180+45$ $\therefore x=45^\circ, 225^\circ$  S | A ✓ T ✓ | C		

Qn	Solutions	Marks	Comments+Criteria
5 (c)	 		
(i)	$15+15=30$ Km	1	
(ii)	$T_{24} = 15 + 23 \times \frac{1}{4}$ $= 20.75$ Km	1 1	
(iii)	$a = 30$ $d = \frac{1}{2} \quad (\frac{1}{4}+\frac{1}{4})$ $S_n = \frac{n}{2}\left(2a+(n-1)d\right)$ $= \frac{24}{2}\left(2\times 30 + 23 \times \frac{1}{2}\right)$ $= 12\left(60 + 11\frac{1}{2}\right)$ $= 858$ Km is the total distance travelled	✓ ✓ ✓	

Qn	Solutions	Marks	Comments+Criteria
6 (a)	$0.2\dot{6} = 0.26666$ $\frac{2}{10} + \frac{6}{100} + \frac{6}{1000} \ldots$ $\frac{2}{10} + S_\infty \qquad a = \frac{6}{100} \quad r = \frac{1}{10}$ $S_\infty = \frac{a}{1-r} \qquad \lvert r \rvert < 1$ $= \dfrac{\frac{6}{100}}{1-\frac{1}{10}}$ $= \frac{6}{100} \div \frac{9}{10} = \frac{1}{15}$ Total $= \frac{2}{10} + \frac{1}{15} = \frac{4}{15}$	½ ½ or ✓ ✓	
(b)	$m(x+1)^2 + p(x-3) + q - 4$ $m(x^2+2x+1) + px - 3p + q - 4$ $mx^2 + \underline{2mx} + m + \underline{px} - 3p + q - 4$ $mx^2 + (p+2m)x + m + q - 3p - 4$ $3x^2 - 1x - 5$ $\underline{m = 3}$ $p + 2m = -1$ $p + 6 = -1$ $p = -6 - 1 = -7$ $m + q - 3p - 4 = -5$ $3 + q + 21 - 4 = -5$ $q = -5 + 4 - 20 - 3$ $= -25$		

Qn	Solutions	Marks	Comments+Criteria
6 c)	$\cos\theta = -\frac{2}{5}$ $\times$S | A T$^{\times}$ | C Q 2+3.  $\tan\theta = -\frac{\sqrt{21}}{2}$ Q 2 $+\frac{\sqrt{21}}{2}$ Q 3.	✓ ✓ ✓	
(d) (i)	$2x^2 - (3a-1)x + (2a-5) = 0$ Let roots α and 0 $\alpha \times 0 = 0 = \frac{c}{a}$ $0 = \frac{2a-5}{2}$ $\therefore 2a = 5 \quad a = 2\frac{1}{2}$.	✓ ✓	
(ii)	Let roots α and $\frac{1}{\alpha}$. $\alpha \times \frac{1}{\alpha} = 1 \Rightarrow \frac{2a-5}{2} = 1$ $2a - 5 = 2$ $2a = 7$ $a = 3\frac{1}{2}$	✓ ✓	
(e)	$(4x+1)(1+x) = (2x+1)(2x-3)$ $4x + \cancel{4x^2} + 1 + x = \cancel{4x^2} - 6x + 2x - 3$ $5x + 1 = -4x - 3$ $9x = -4$ $x = -\frac{4}{9}$	✓ ✓ ✓	