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KAMBALA

MATHEMATICS

2 UNIT

YEAR 11 PRELIMINARY EXAMINATION

August 2005

Time Allowed: 2 hours
Reading Time: 5 minutes

INSTRUCTIONS

- This examination contains 6 questions of equal value. Marks for each question are shown.
- Answer all questions on the writing paper provided. Start each question on a new page.
- Calculators may be used.
- Show all necessary working.
- Marks will be deducted for careless or badly arranged work.

QUESTION 1 *(Start a new page)* MARKS

(a) Factorise $4x^2 - 9$. 1

(b) Evaluate $\frac{2\sqrt{37}}{3}$ correct to 3 significant figures. 2

(c) Solve $1 - 3x \le 7$ and show your answer on a number line. 2

(d) Simplify $\frac{x}{4} - \frac{3x+1}{6}$ 1

(e) Solve for x and y: 2

$$2x + y - 2 = 0$$
$$6x - 5y + 18 = 0$$

(f) Express 0.0000038 in scientific notation. 1

(g) A function is defined by the rule: 3

$$f(x) = \begin{cases} x+2, & x \ge 0 \\ \frac{1}{x}, & x < 0 \end{cases}$$

Find:
(i) $f(3)$ (ii) $f(-2)$ (iii) $f(p^2)$

QUESTION 2 *(Start a new page)* MARKS

(a) Shade the region in the Cartesian plane for which the following inequalities hold simultaneously: 2

$$x - y - 3 > 0$$
$$x \le 6$$
$$y \ge 0$$

(b) If θ is acute, and $\tan\theta = \frac{4-\sqrt{2}}{4+\sqrt{2}}$, write down the value of $\sin\theta$ 3

(c) Prove the trigonometric identity: 2

$$1 + \tan^2\theta = \frac{1}{1-\sin^2\theta}$$

(d) For the function $y = x + \frac{1}{2x}$ find the gradient of the curve when $x = \frac{1}{2}$. 3

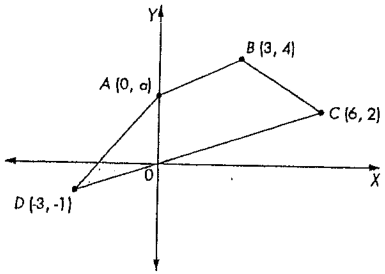
(e) Simplify $\frac{\log x^3}{\log\sqrt{x}}$ 2

QUESTION 3 *(Start a new page)*

MARKS

(a) Differentiate from first principles $y = x^2 + 3x$ **3**

(b)



(i) Find the gradient of DC. **1**

(ii) If AB is parallel to DC, show that the value of $a = 3$. **1**

(iii) Show that the equation of DC is $x - 3y = 0$. **1**

(iv) Find the perpendicular distance between B and DC. **2**

(v) Prove that $DC = 3AB$. **2**

(vi) Find the area of trapezium $ABCD$. **2**

QUESTION 4 *(Start a new page)*

MARKS

(a) Find all the values of θ, with $0° \le \theta \le 360°$, for which $\tan\theta = \frac{1}{\sqrt{3}}$ **1**

(b) Solve for x: $|x-2| = 2x - 1$ **3**

(c) Solve for x: $9^x = \frac{\sqrt{3}}{3}$ **2**

(d) A continuous function $f(x)$ has the following properties :

$f'(-1) = 0$
$f'(x) > 0$ when $x < -1$
$f'(x) < 0$ when $x > -1$
$f(2) = f(-4) = 0$

(i) Explain the geometrical significance of each property **3**

(ii) Sketch the curve of $y = f(x)$ **1**

(iii) On a different set of axes, sketch the graph of the gradient function $y = f'(x)$ **2**

QUESTION 5 *(Start a new page)* **MARKS**

(a) Differentiate

(i) $y = (3x^3 + 4)^5$ **2**

(ii) $y = \frac{x-1}{x^2}$ **2**

(b) Find the equation of the normal to the curve $y = x^4 + 2x^2 - 5$ at the point where $x = 1$. **4**

(c) Find $\lim_{x \to 1} \frac{x-1}{x^2 - x}$ **2**

(d) Solve for x:

$\log_4 x - \log_4 (x-1) = 1$ **2**

QUESTION 6 *(Start a new page)* **MARKS**

(a) Consider the curve given by $y = x^3 - 12x + 4$

(i) Find the co-ordinates of the stationary points and determine their nature. **3**

(ii) Sketch the curve for the domain $-3 \le x \le 3$ showing all relevant points. **3**
(There is no need to show any point(s) of inflexion or x intercepts)

(b) The series 9, 21, 69,.......has $T_n = 5 + 4^n$

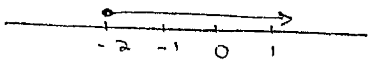
(i) Find the 4th term **1**

(ii) Find an expression for S_n of this series. **3**

(c) A sequence has a limiting sum of 12 and a first term of 40. Find r and explain the relevance of your answer. **2**

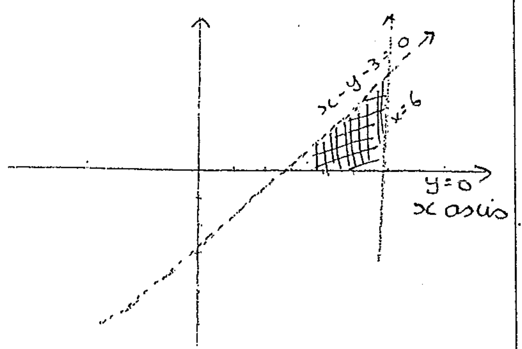
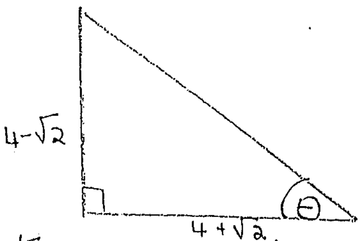
End of paper

Question 1 Solns

Qn	Solutions	Marks	Comments
(a)	$4x^2 - 9 = (2x-3)(2x+3)$	1	
(b)	$\frac{2\sqrt{37}}{3} = 4.05\vert 517502$ 4.06	1 1	1 answer 1 for rounding
(c)	$1 - 3x \leq 7$ $-3x \leq 7-1$ $-3x \leq 6$ $x \geq \frac{6}{-3} \quad x \geq -2$ 		1 answer 1 for number line
(d)	$\frac{x}{4} - \frac{3x+1}{6}$ $\frac{3x}{12} - \frac{2(3x+1)}{12} = \frac{3x-6x-2}{12}$ $= \frac{-3x-2}{12}$ or $-\frac{(3x+2)}{12}$	1	correct answer only.
(e)	$2x+y-2=0$ —① $\times 5$ $6x-5y+18=0$ —② $10x+5y-10=0$ —① $6x-5y+18=0$ —② $16x+8=0$ $16x=-8$ $x = -\frac{8}{16} = -\frac{1}{2}$ Sub into ① $2x-\frac{1}{2}+y-2=0$ $-1+y-2=0$ $\underline{y=3} \quad (-\frac{1}{2}, 3)$	 1 1	 1 x and 1 y

Qn	Solutions	Marks	Comments
(f)	$0.0000038 \quad 3.8 \times 10^{-6}$	1	
(g)	$f(x) = \begin{cases} x+2, & x \geq 0 \\ \frac{1}{x}, & x < 0 \end{cases}$ $f(3) = 3+2 = 5$ $f(-2) = \frac{1}{-2}$ $f(p^2) = p^2+2 \quad (p^2 \geq 0)$	 1 1 1	

Question 2

Qn	Solutions	Marks	Comments
(a)	 Test (4,0) LHS $= x-y-3$ $= 4-0-3 = 1 > 0$ ✓	2	1 correct graphs 1 for correct shading
(b)	 $\tan\theta = \frac{O}{A} = \frac{4-\sqrt{2}}{4+\sqrt{2}}$ $H^2 = (4-\sqrt{2})^2 + (4+\sqrt{2})^2$ $= 16 - \cancel{8\sqrt{2}} + 2 + 16 + \cancel{8\sqrt{2}} + 2$ $= 36$ $H = \sqrt{36} = 6$ $\sin\theta = \frac{O}{H} = \frac{4-\sqrt{2}}{6}$	1 1 1	
(c)	LHS $= 1 + \tan^2\theta = \sec^2\theta$ RHS $= \frac{1}{1-\sin^2\theta} = \frac{1}{\cos^2\theta} = \sec^2\theta$ $\therefore$ LHS = RHS .	1 1	

Qn	Solutions	Marks	Comments
(d)	$y = x + \frac{1}{2x} \quad \therefore \quad y = x + \frac{x^{-1}}{2}$ $y' = 1 - \frac{1}{2}x^{-2}$ $= 1 - \frac{1}{2x^2}$ $= 1 - \frac{1}{2(\frac{1}{2})^2} = 1 - \frac{1}{0.5}$ $=$ $= 1 - 2 = -1$	1 1 1	x many brought the $1/2$ to the top but not in a bracket and then did not use the chain rule even if in a bracket
(e)	$\frac{\log x^3}{\log\sqrt{x}} = \frac{\log x^3}{\log x^{\frac{1}{2}}} = \frac{3\log x}{\frac{1}{2}\log x}$ $= 6$	1 1	

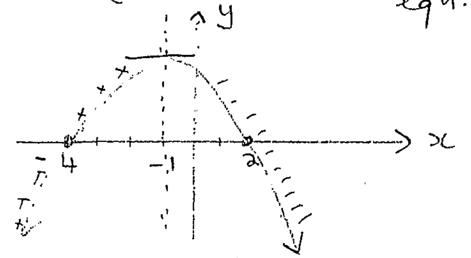
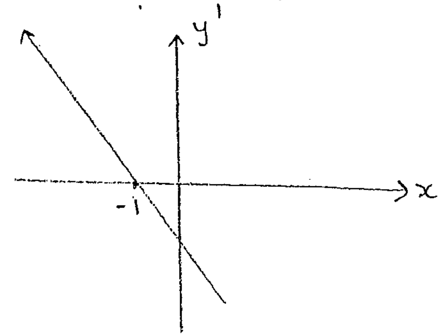
Question 3.

Qn	Solutions	Marks	Comments
(a)	$f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$		Many students merely differentiated $f(x)$ without using first principles & gained 0 marks
	$f(x) = x^2 + 3x$ $f(x+h) = (x+h)^2 + 3(x+h)$ $= x^2 + 2xh + h^2 + 3x + 3h$	1	
	$f'(x) = \lim\limits_{h \to 0} \dfrac{\cancel{x^2} + 2xh + h^2 + \cancel{3x} + 3h - \cancel{x^2} - \cancel{3x}}{h}$	1	
	$= \lim\limits_{h \to 0} \dfrac{\cancel{h}(2x + h + 3)}{\cancel{h}}$ $= 2x + 3$	1	
b)	(i) m_{DC} $D(\overset{x_1}{-3}, \overset{y_1}{-1})$ $C(\overset{x_2}{6}, \overset{y_2}{2})$ $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2 - -1}{6 - -3} = \dfrac{3}{9} = \dfrac{1}{3}$	1	Parts i)–iii) were generally well done.
	(ii) $\underset{(0,a)}{A}$ $\underset{(3,4)}{B}$ $\dfrac{1}{3} = \dfrac{4-a}{3-0}$ $\therefore$ $\dfrac{1}{3} = \dfrac{4-a}{3}$ $4 - a = 1$ $4 - 1 = a = 3$	1	
	(iii) $y - y_1 = m(x - x_1)$ $y + 1 = \frac{1}{3}(x + 3)$ $3y + \cancel{3} = x + \cancel{3}$ $x - 3y = 0$	1	

Qn	Solutions	Marks	Comments
(iv)	$B(3,4)$ line DC $a = 1,\ b = -3,\ c = 0$ $d = \dfrac{\lvert ax_1 + by_1 + c \rvert}{\sqrt{a^2 + b^2}} = \dfrac{\lvert 1 \times 3 - 3 \times 4 + 0 \rvert}{\sqrt{1^2 + 3^2}}$	1	
	$= \dfrac{\lvert 3 - 12 \rvert}{\sqrt{10}} = \dfrac{9}{\sqrt{10}} \cdot \dfrac{\sqrt{10}}{\sqrt{10}}$ $= \dfrac{9\sqrt{10}}{10}$	1	Some students (wrongly) used only the horizontal distance.
(v)	$DC = \sqrt{(6+3)^2 + (2+1)^2}$ $= \sqrt{81 + 9} = \sqrt{90}$ $= 3\sqrt{10}$	1	
	$AB = \sqrt{(3-0)^2 + (4-3)^2}$ $= \sqrt{9 + 1} = \sqrt{10}$	1	
	$\therefore DC = 3AB$		
(vi)	Area of trapezium $=$ $\frac{1}{2} h (a + b)$ $= \frac{1}{2} \times \dfrac{9\sqrt{10}}{10} \times (\sqrt{10} + 3\sqrt{10})$	1	Some students did not know the area of a trapezium
	$= \dfrac{9\sqrt{10}}{20} \times 4\sqrt{10} = \dfrac{360}{20}$ $= 18\ u^2$	1	

Question 4

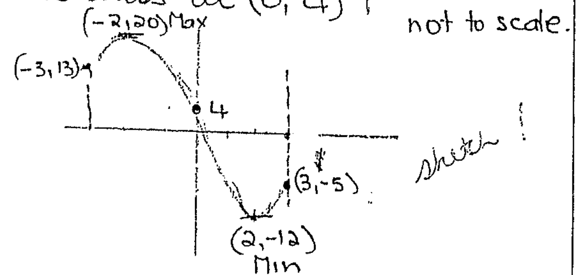
Qn	Solutions	Marks	Comments
(a)	$\tan\theta = \frac{1}{\sqrt{3}}$ S A T C Q1 + Q3 $\theta = 30^\circ$ $180 + \theta = 210^\circ$	1	
(b)	$\lvert x-2 \rvert = 2x-1$		
	$x-2 = 2x-1$ or $x-2 = -(2x-1)$ $-2+1 = 2x-x$ or $x-2 = -2x+1$ $\underline{-1 = x}$ $3x = 3$	1	
	$\underline{x = 1}$	1	
	check: $x=-1$ LHS $= \lvert -1-2 \rvert = 3$ RHS $= 2\times -1 - 1 = -3$ No! not a solution $x = 1$ LHS $= \lvert 1-2 \rvert = 1$ ✓ RHS $= 2\times 1 - 1 = 1$		Many students did not check their solutions to find that $x=-1$ is not really a solution to the original equation.
	$x = 1$ is a solution	1	
(c)	$9^x = \frac{\sqrt{3}}{3}$		
	$3^{2x} = \frac{3^{\frac{1}{2}}}{3}$ $(3^{\frac{1}{2}-1})$	1	
	$3^{2x} = 3^{-\frac{1}{2}}$ $\therefore 2x = -\frac{1}{2}$ $x = -\frac{1}{4}$	1	

Qn	Solutions	Marks	Comments
(d)	$f'(-1) = 0$ means at $x=-1$ there is a stationary point.		
	$f'(x) > 0$ $x < -1$ means gradient is positive (increasing) when x is less than -1	1	1 off per mistake
	$f'(x) < 0$ $x > -1$ graph decreasing, x is greater than -1	1	Many students merely restated the equation or inequation in words for example, 'when x is less than -1, the derivative is negative.' Students needed to give the geometrical significance eg, when x is less than -1, the curve is decreasing etc.
	$f(2) = f(-4) = 0$ x values of $2, -4$ have y values 0	1	
	$(2,0)$ $(-4,0)$ are roots of eqn. 	1	
		2	

Question 5

Qn	Solutions	Marks	Comments
(a) (i)	$y = (3x^3+4)^5$	1	
	$y' = 5(3x^3+4)^4 \cdot 9x^2$		
	$= 45x^2(3x^3+4)^4$	1	
(ii)	$y = \dfrac{x-1}{x^2}$ $u = x-1$, $\dfrac{du}{dx} = 1$ $v = x^2$, $\dfrac{dv}{dx} = 2x$		
	$\dfrac{dy}{dx} = \dfrac{v.\frac{du}{dx} - u.\frac{dv}{dx}}{v^2}$		
	$= \dfrac{x^2.1 - (x-1)2x}{(x^2)^2}$	1	
	$= \dfrac{x^2 - 2x^2 + 2x}{x^4}$		
	$= \dfrac{-x^2+2x}{x^4}$		
	$= \dfrac{x(2-x)}{x^4} = \dfrac{2-x}{x^3}$	1	
(b)	$y = x^4 + 2x^2 - 5$ $\boxed{x=1,\ y=-2}$	1	
	$y' = 4x^3 + 4x$	1	
	tangent $x = 1$		
	$y' = 4+4 = 8$		
	$\therefore$ normal $m = -\frac{1}{8}$	1	
	$y - y_1 = m(x - x_1)$		
	$y + 2 = -\frac{1}{8}(x-1)$		
	$8y + 16 = -x + 1$		
	$x + 8y + 15 = 0$	1	
(c)	$\lim\limits_{x\to 1} \dfrac{x-1}{x^2-x}$		
	$\lim\limits_{x\to 1} \dfrac{\cancel{x-1}}{x(\cancel{x-1})} = \dfrac{1}{1} = 1$	1	factorise
		1	
(d)	$\log_4 x - \log_4(x-1) = 1$		
	$\log_4\left(\dfrac{x}{x-1}\right) = 1$	1	
	$4^1 = \dfrac{x}{x-1}$		
	$4(x-1) = x$		
	$4x - 4 = x$		
	$3x = 4$		
	$x = \dfrac{4}{3}$	1	

Question 6

Qn	Solutions	Marks	Comments+Criteria
(a)	$y = x^3 - 12x + 4$ $y' = 3x^2 - 12$ Stat Pts occur when $y' = 0$ $0 = 3x^2 - 12$ $0 = 3(x^2 - 4)$ $\therefore x = 2, -2$ $x = 2 \quad y = -12 \qquad (2,-12)$ ✓ $x = -2 \quad y = 20 \qquad (-2,20)$ ✓ Max|Min - nature $y'' = 6x$ $x = 2 \quad y'' > 0 \therefore$ Min $x = -2 \quad y'' < 0 \therefore$ Max ✓ $(2,-12)$ Min 1, $(-2,20)$ Max.		or 1 mark if only found y' and y'' * need to state x and y values for a point. No marks deducted if point and co-ords clearly labelled on part (ii)
	cuts axis at $(0,4)$ 1 not to scale.  $x = 3, \; y = -5$ $x = -3 \quad y = 13$		

Qn	Solutions	Marks	Comments+Criteria
(b)	9, 21, 69 $T_n = 5 + 4^n$ $T_4 = 5 + 4^4 = 261$ $S_n = 5+4^1 + 5+4^2 \ldots + 5+4^n$ $= 5n + [4^1 + 4^2 + 4^3 + \ldots 4^n]$ $= 5n + \dfrac{a(r^n-1)}{r-1}$ $= 5n + \dfrac{4(4^n-1)}{3}$ $= 5n + \dfrac{4(4^n-1)}{3}$	1 3	Alternative answer $(4^1+4^2+\ldots+4^n) + (5+5+5\ldots)$ n terms n terms $= \dfrac{4(4^n-1)}{4-1} + 5n$ $= \dfrac{4^{n+1}-4}{3} + 5n$
(c)	$S_\infty = \dfrac{a}{1-r} \quad \|r\| < 1$ $12 = \dfrac{40}{1-r}$ $1 - r = \dfrac{40}{12}$ $1 - r = 3\frac{1}{3}$ $1 - 3\frac{1}{3} = r = -2\frac{1}{3}$ $\therefore$ can't be limiting sum as $\|r\| < 1$		