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KAMBALA

Extension 1 Mathematics

Year 11 Preliminary Course

Assessment Task #2 – Trigonometry

June, 2002

Syllabus Topics to be covered in this task :	5.1 – 5.5 E5.6 – E5.9
Syllabus Outcomes to be addressed in this task :	P3, P4, P5 PE1, PE2, PE6

- Time allowed is 45 minutes
- There are 3 questions, each worth 12 marks
- The mark value of each part is indicated in [...] next to that part
- Start each question on a new page
- A trigonometric Formula Sheet is enclosed

Question 1 : (Start a new page) [12 marks]

(a) Find the exact values of :

(i) $\sin 210°$ [1]

(ii) $\sec 315°$ [2]

(iii) $\tan \frac{2\pi}{3}$ [2]

(b) (i) Sketch the graph of $y = \cos x$ for $0 \le x \le 2\pi$. [2]

(ii) State the period and amplitude of $y = \cos x$. [2]

(c) Solve the equation $4\cos^2\alpha - 3 = 0$ for $0° \le \alpha \le 360°$. [3]

Question 2 : (Start a new page) [12 marks]

(a) Simplify :

(i) $\sin 3x \cos 2x - \cos 3x \sin 2x$. [2]

(ii) $\dfrac{2\tan 15°}{1-\tan^2 15°}$ [2]

(b) Prove that $\tan\theta - \dfrac{\sin^3\theta}{\cos\theta} = \sin\theta.\cos\theta$. [3]

(c) If γ is obtuse, and $\tan\gamma = -\dfrac{2}{3}$, find the exact value of :

(i) $\cos\gamma$. [2]

(ii) $\cos 2\gamma$ [3]

Question 3 : (Start a new page)

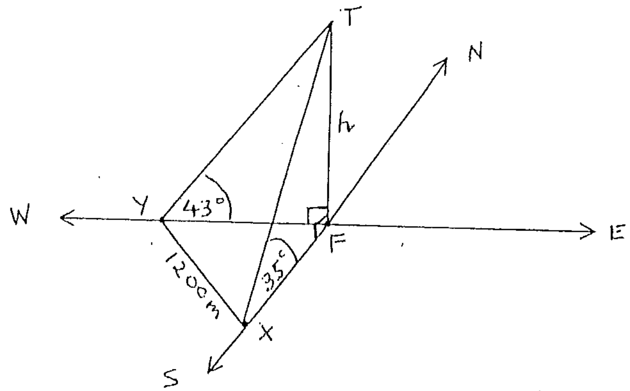
(a) Given the expansion $\cos(\alpha-\beta)=\cos\alpha.\cos\beta+\sin\alpha.\sin\beta$ show that the exact value of $\cos 15° = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$. [3]

(b) Point X is due south and point Y is due west of the foot F of a mountain TF of height h. From X and Y, the angle of elevation of the top of the mountain T are 35° and 43° respectively.

(i) Show that $XF = h.\tan 55°$ and $YF = h.\tan 47°$. [2]

(ii) If X and Y are 1200 metres apart, show that the height h of the mountain is given by the formula : [3]

$$h=\frac{1200}{\sqrt{(\tan^2 55° + \tan^2 47°)}}$$

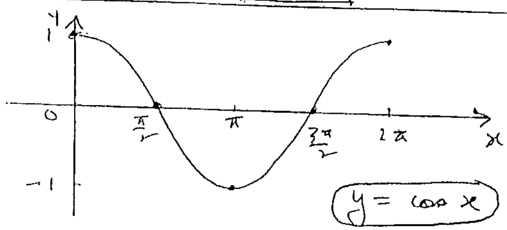


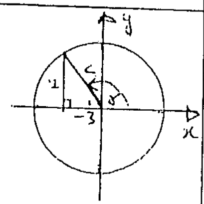
(c) In Δ ABC, $\angle$ BAC = 60°. Prove that $a^2-b^2=c(c-b)$. [4]

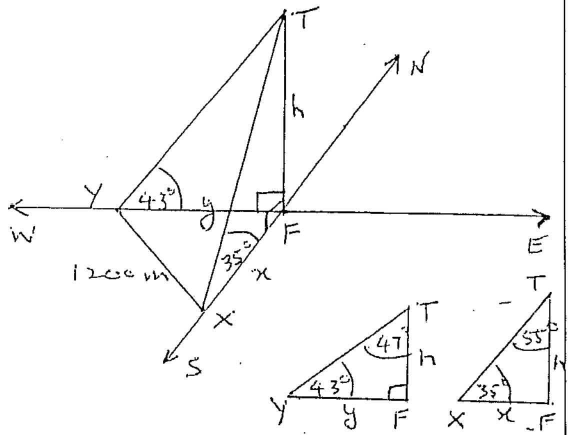
End of Task

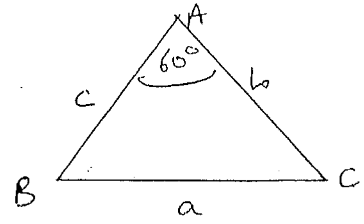
Year 11 Extension 1 Maths — Solutions

Assessment Task #2, Trigonometry, 13/6/02

Qn	Solutions	Marks	Comments
1(a)(i) 5	$\sin 210^\circ = \sin(180+30^\circ)$ $= -\sin 30^\circ$ $= \boxed{-1/2}$ S | A, T | C ✓ (ii) $\sec 315^\circ = \sec(360-45^\circ)$ $= \sec 45^\circ$ $= \frac{1}{\cos 45^\circ}$ S | A, T | C ✓ $= \boxed{\frac{\sqrt{2}}{1}}$ (iii) $\tan \frac{2\pi}{3} = \tan 120^\circ$ $= \tan(180-60^\circ)$ ✓ S | A, T | C $= -\tan 60^\circ$ $= \boxed{-\sqrt{3}}$	✓ ✓ ✓ ✓ ✓	
(b) 4	(i)  (ii) $\boxed{P = 2\pi}$ $\boxed{A = 1}$	✓ ✓ ✓ ✓	1 – graph 1 – axes
(c) 3	$4\cos^2\alpha - 3 = 0$ $\cos^2\alpha = \frac{3}{4}$ $\cos\alpha = \pm\frac{\sqrt{3}}{2}$ S | A, T | C ✓ ✓ ✓ ✓ $\therefore \alpha = 30^\circ, 150^\circ, 210^\circ, 330^\circ$	✓ ✓ ✓	1 for $\cos\alpha = \pm\frac{\sqrt{3}}{2}$ –1 for only Q1/4 –2 for $\alpha = 30$ only –1 for correct answer but $\cos\alpha = \frac{\sqrt{3}}{2}$

Qn	Solutions	Marks	Comments
2(a)(i) 2	$\sin 3x . \cos 2x - \cos 3x . \sin 2x$ $= \sin(3x - 2x)$ $= \sin x$	✓ ✓	
(ii) 2	$\frac{2\tan 15^\circ}{1-\tan^2 15^\circ} = \tan 30^\circ$ $= \boxed{\frac{1}{\sqrt{3}}}$	✓ ✓	$-\frac{1}{2}$ for wrong exact value $-\frac{1}{2}$ for build $\tan 30^\circ$
(b) 3	$\tan\theta - \frac{\sin^3\theta}{\cos\theta} = \sin\theta . \cos\theta$ $LHS = \frac{\sin\theta}{\cos\theta} - \frac{\sin^3\theta}{\cos\theta}$ $= \frac{\sin\theta(1-\sin^2\theta)}{\cos\theta}$ $= \frac{\sin\theta . \cos^{\cancel{2}}\theta}{\cancel{\cos\theta}}$ $= \sin\theta . \cos\theta$ $= RHS$	✓ ✓ ✓	
(c) 2	γ obtuse $\tan\gamma = -\frac{2}{3}$  $c^2 = a^2 + b^2$ $\text{"} = 9 + 4$ $c = \sqrt{13}$ (i) $\cos\gamma = \frac{A}{H} = \frac{-3}{\sqrt{13}}$	✓ ✓	$-\frac{1}{2}$ for $\pm\frac{3}{\sqrt{13}}$ -1 for $\frac{3}{\sqrt{13}}$ with no investig^n.
3	(ii) $\cos 2\gamma = 2\cos^2\gamma - 1$ $\text{"} = 2\left(\frac{-3}{\sqrt{13}}\right)^2 - 1$ $\text{"} = 2\left(\frac{9}{13}\right) - 1$ $\text{"} = \frac{18}{13} - 1$ $\cos 2\gamma = \frac{5}{13}$	✓ ✓ ✓	

Qn	Solutions	Marks	Comments
3(a)	$\cos(\alpha-\beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$		
	$\cos(60^\circ - 45^\circ) = \cos 60^\circ.\cos 45^\circ + \sin 60^\circ.\sin 45^\circ$	✓	
	$\cos 15^\circ = \frac{1}{2}.\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}.\frac{1}{\sqrt{2}}$	✓✓	
	$\quad '' \quad = \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$		
	$\boxed{\cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}}$		
(b)			
(i)	In ΔTXF, $\tan 55^\circ = \frac{XF}{h}$		
2	$\therefore \boxed{XF = h\tan 55^\circ}$	✓	
	In ΔTYF, $\tan 47^\circ = \frac{YF}{h}$		
	$\therefore \boxed{YF = h\tan 47^\circ}$	✓	
(ii)	In ΔXYF, $c^2 = a^2 + b^2$	✓	
	ie $1200^2 = x^2 + y^2$		
3	ie $1200^2 = h^2\tan^2 55 + h^2\tan^2 47^\circ$	✓	
	$1200^2 = h^2(\tan^2 55 + \tan^2 47^\circ)$		
	$\therefore h = \frac{1200}{\sqrt{\tan^2 55 + \tan^2 47^\circ}}$	✓	

Qn	Solutions	Marks	Comments
3(c)			
	Using the cosine rule,		
	$a^2 = b^2 + c^2 - 2bc\cos A$	✓	
4	$\quad '' = b^2 + c^2 - 2bc\cos 60^\circ$	✓	
	$\quad '' = b^2 + c^2 - 2bc\left(\frac{1}{2}\right)$		
	$c^2 = b^2 + c^2 - bc$	✓	
	$\therefore a^2 - b^2 = c^2 - bc$		
	$\boxed{a^2 - b^2 = c(c-b)}$	✓	