

KAMBALA

Extension 1 Mathematics

Year 11 Preliminary Course

Assessment Task #2 – Trigonometry

June, 2002

Syllabus Topics to be covered in this task :
5.1 – 5.5
E5.6 – E5.9

Syllabus Outcomes to be addressed in this task :
P3, P4, P5
PE1, PE2, PE6

- Time allowed is 45 minutes
- There are 3 questions, each worth 12 marks
- The mark value of each part is indicated in [...] next to that part
- Start each question on a new page
- A trigonometric Formula Sheet is enclosed

Question 1 : (Start a new page)

[12 marks]

- (a) Find the exact values of :
- (i) $\sin 210^\circ$ [1]
- (ii) $\sec 315^\circ$ [2]
- (iii) $\tan \frac{2\pi}{3}$ [2]
- (b) (i) Sketch the graph of $y = \cos x$ for $0 \leq x \leq 2\pi$. [2]
- (ii) State the period and amplitude of $y = \cos x$. [2]
- (c) Solve the equation $4 \cos^2 \alpha - 3 = 0$ for $0^\circ \leq \alpha \leq 360^\circ$. [3]

Question 2 : (Start a new page)

[12 marks]

- (a) Simplify :
- (i) $\sin 3x \cos 2x - \cos 3x \sin 2x$. [2]
- (ii) $\frac{2 \tan 15^\circ}{1 - \tan^2 15^\circ}$ [2]
- (b) Prove that $\tan \theta - \frac{\sin^3 \theta}{\cos \theta} = \sin \theta \cos \theta$. [3]
- (c) If γ is obtuse, and $\tan \gamma = -\frac{2}{3}$, find the exact value of :
- (i) $\cos \gamma$. [2]
- (ii) $\cos 2\gamma$ [3]

Question 3: (Start a new page.)

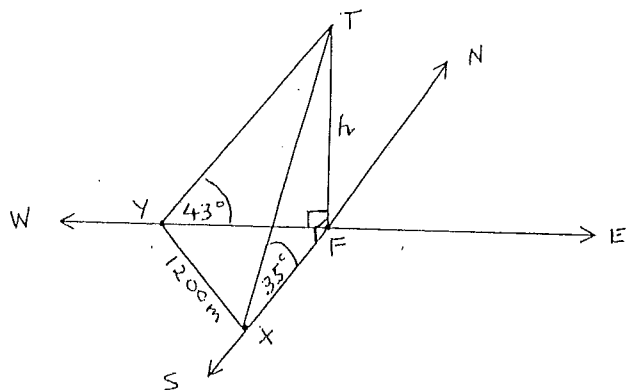
- (a) Given the expansion $\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$ show [3]
that the exact value of $\cos 15^\circ = \frac{\sqrt{3} + 1}{2\sqrt{2}}$.

- (b) Point X is due south and point Y is due west of the foot F of a mountain TF of height h . From X and Y, the angle of elevation of the top of the mountain T are 35° and 43° respectively.

- (i) Show that $XF = h \tan 55^\circ$ and $YF = h \tan 47^\circ$. [2]

- (ii) If X and Y are 1200 metres apart, show that the height h of the mountain is given by the formula: [3]

$$h = \frac{1200}{\sqrt{(\tan^2 55^\circ + \tan^2 47^\circ)}}$$

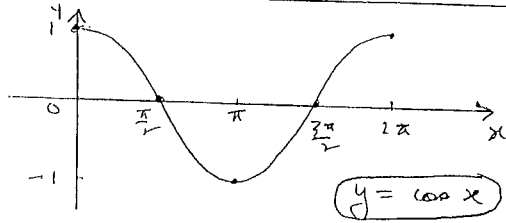


- (c) In $\triangle ABC$, $\angle BAC = 60^\circ$. Prove that $a^2 - b^2 = c(c - b)$. [4]

End of Task

Year 11 EXTENSION 1 MATHS - SOLUTIONS

ASSESSMENT TASK #2 - TRIGONOMETRY, 13/6/02

Qn	Solutions	Marks	Comments
5	(i) $\sin 210^\circ = \sin(180+30^\circ)$ $= -\sin 30^\circ$ S/A T/C $= \boxed{-1/2}$	✓	
	(ii) $\sec 315^\circ = \sec(360-45^\circ)$ $= \sec 45^\circ$ $= \frac{1}{\cos 45^\circ}$ S/A T/C $= \boxed{\frac{2}{1}}$	✓	
	(iii) $\tan \frac{2\pi}{3} = \tan 120^\circ$ $= \tan(180-60^\circ)$ S/A T/C $= -\tan 60^\circ$ $= \boxed{-\sqrt{3}}$	✓	
4	(i) 	✓	1 - graph
	(ii) $\boxed{p = 2\pi}$ $\boxed{A = 1}$	✓	1 - axes
3	$4\cos^2 x - 3 = 0$ $\cos^2 x = \frac{3}{4}$ S/A T/C $\cos x = \pm \frac{\sqrt{3}}{2}$ $\therefore x = 30^\circ, 150^\circ$ $210^\circ, 330^\circ$	✓	1 for $\cos x = \pm \frac{\sqrt{3}}{2}$
		✓	-1 for only Q1/4
		✓	-2 for $x = 30$ only
		✓	-1 for correct answer but $\cos x = \frac{\sqrt{3}}{2}$

Qn	Solutions	Marks	Comments
2	(a) (i) $\sin 3x \cdot \cos 2x - \cos 3x \cdot \sin 2x$ $= \sin(3x-2x)$ $= \boxed{\sin x}$	✓	
2	(ii) $\frac{2 \tan 15^\circ}{1 - \tan^2 15^\circ} = \tan 30^\circ$ $= \boxed{\frac{1}{\sqrt{3}}}$	✓	-1/2 for wrong exact value -1/2 for build tan 30
3	(b) $\tan \theta - \frac{\sin^3 \theta}{\cos \theta} = \sin \theta \cdot \cos \theta$ LHS = $\frac{\sin \theta}{\cos \theta} - \frac{\sin^3 \theta}{\cos \theta}$ $= \frac{\sin \theta (1 - \sin^2 \theta)}{\cos \theta}$ $= \frac{\sin \theta \cdot \cos^2 \theta}{\cos \theta}$ $= \sin \theta \cdot \cos \theta$ $= \text{RHS}$	✓	
2	(c) γ obtuse $\tan \gamma = -\frac{2}{3}$ $c^2 = a^2 + b^2$ $" = 9 + 4$ $a = \sqrt{13}$	✓	
2	(i) $\cos x = \frac{A}{H} = \frac{-3}{\sqrt{13}}$	✓	-1/2 for $\pm \frac{3}{\sqrt{13}}$
3	(ii) $\cos 2x = 2\cos^2 x - 1$ $" = 2\left(\frac{-3}{\sqrt{13}}\right)^2 - 1$ $" = 2\left(\frac{9}{13}\right) - 1$ $" = \frac{18}{13} - 1$ $\cos 2x = \boxed{\frac{5}{13}}$	✓	-1 for $\frac{3}{\sqrt{13}}$ with no investigation

Qn	Solutions	Marks	Comments
3(a)	$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$ $\cos(60^\circ - 45^\circ) = \cos 60^\circ \cdot \cos 45^\circ + \sin 60^\circ \cdot \sin 45^\circ \quad \checkmark$ $\cos 15^\circ = \frac{1}{2} \cdot \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} \quad \checkmark \checkmark$ $= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$ $\cos 15^\circ = \frac{\sqrt{3} + 1}{2\sqrt{2}}$		
(b)	<p>The diagram shows a triangle with a horizontal base of 1200 m. A vertical height h is drawn from the top vertex to the base. The base is divided into segments x and y. The angle between the height and the left side is 47 degrees, and the angle between the height and the right side is 55 degrees. The total angle at the top vertex is 43 degrees. The base is labeled with points Y, X, F, X, Y from left to right. The height is labeled h. The base segments are labeled x and y. The total base length is 1200 m.</p>		
(i)	<p>In ΔTXF, $\tan 55^\circ = \frac{XF}{h}$</p> $\therefore XF = h \tan 55^\circ \quad \checkmark$ <p>In ΔTYF, $\tan 47^\circ = \frac{YF}{h}$</p> $\therefore YF = h \tan 47^\circ \quad \checkmark$		
(ii)	<p>In ΔXYF, $c^2 = a^2 + b^2$</p> $\therefore 1200^2 = x^2 + y^2 \quad \checkmark$ <p>ii. $1200^2 = h^2 \tan^2 55^\circ + h^2 \tan^2 47^\circ \quad \checkmark$</p> $1200^2 = h^2 (\tan^2 55^\circ + \tan^2 47^\circ) \quad \checkmark$ $\therefore h = \frac{1200}{\sqrt{\tan^2 55^\circ + \tan^2 47^\circ}} \quad \checkmark$		

Qn	Solutions	Marks	Comments
3(c)	<p>The diagram shows a triangle with vertices A, B, and C. The angle at vertex A is 60 degrees. The side opposite to A is labeled a. The side opposite to B is labeled b. The side opposite to C is labeled c.</p>		
	<p>Using the cosine rule,</p> $a^2 = b^2 + c^2 - 2bc \cos A \quad \checkmark$ $a^2 = b^2 + c^2 - 2bc \cos 60^\circ \quad \checkmark$ $a^2 = b^2 + c^2 - 2bc \left(\frac{1}{2}\right) \quad \checkmark$ $a^2 = b^2 + c^2 - bc \quad \checkmark$ $\therefore a^2 - b^2 = c^2 - bc \quad \checkmark$ $a^2 - b^2 = c(c - b) \quad \checkmark$		
4			