KAMBALA GIRLS' SCHOOL

EXTENSION 1
MATHEMATICS

YEAR 11 PRELIMINARY HSC EXAMINATION

SEPTEMBER 2002

General Instructions

- Reading time - 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- An Extension 1 Trigonometry formula sheet is provided
- All necessary working should be shown in every question

Total Marks - 72

Attempt Questions 1 - 6

All questions are of equal value

Question 1: *Start a new page* *12 marks*

(a) Show that $\frac{1}{3-\sqrt{2}}+\frac{1}{3+\sqrt{2}}$ is a rational number [2]

(b) Solve $5^{3x-2}=\frac{1}{125}$ [2]

(c) Find the **exact** value of $\cos 75°$ [2]

(d) Given $P(1, -2)$ and $Q(5, 6)$, find the point R such that PQ is divided externally in the ratio 2 : 1 [3]

(e) Find the acute angle between the lines $x-3y+4=0$ and $x+2y-7=0$ [3]

Question 2: *Start a new page* *12 marks*

(a) Solve for x: $|2x-2|<6-2x$ [3]

(b) What is the natural range of the relation $\frac{x^2}{16}+\frac{y^2}{9}=1$? [3]

(c) Find the values of x that satisfy the inequality $\frac{3x-4}{x+2}\leq 1$ [3]

(d) What is the shortest distance between the lines $4y-3x-3=0$ and $3x-4y+1=0$? [3]

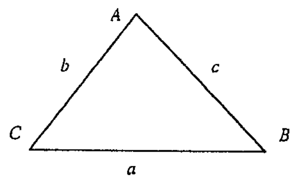
(c) The function $f(x)$ is defined by

$$f(x)=\begin{cases}\tan x & \text{for } 0\le x\le \frac{\pi}{4}\\ 1 & \text{for } \frac{\pi}{4}\le x\le \frac{\pi}{2}\\ \sin x & \text{for } \frac{\pi}{2}\le x\le \pi\end{cases}$$

Sketch $f(x)$ for $0\le x\le \pi$ [3]

(d) Show that the area of ΔABC can be given by $Area=\dfrac{a^2\sin B\sin C}{2\sin A}$ [3]

(Hint: Use the Sine Rule)



Question 5: *Start a new page* *12 marks*

(a) The second term of a geometric series is 2 and its limiting sum is 8. [3]
Determine the values of a and r for this geometric series.

(b) As a New Year's resolution Verity decides to save for her first car.
She invests \$2000 at the beginning of each year in a fund that pays 6% per annum compounded annually.
She starts in January 2002 and makes her last investment in January 2010.

(i) What would be the value of her first investment at the end of 2010? [2]

(i) She needs to have at least \$24 000 at the end of December 2010. [3]
Will she have enough money? Justify your answer.

Question 3: *Start a new page* *12 marks*

(a) If $\dfrac{x}{y}=\dfrac{2}{3}$ evaluate $\dfrac{x^2-y^2}{x^2+y^2}$ [2]

(b) The line l passes through the point of intersection P of the lines $3x-y+1=0$ and $x+4y-4=0$ as well as the point $(-2,1)$.
Show that line l is parallel to the x axis and give its equation. [3]

(c) (i) What value(s) of x are excluded from the domain of the function? $f(x)=\dfrac{|x-2|}{x-2}$ [1]

(ii) Sketch the function $f(x)=\dfrac{|x-2|}{x-2}$ for $-1\le x\le 3$ [2]

(d) Solve $\left(x+\dfrac{1}{x}\right)^2+\left(x+\dfrac{1}{x}\right)-6=0$ [4]

Question 4: *Start a new page* *12 marks*

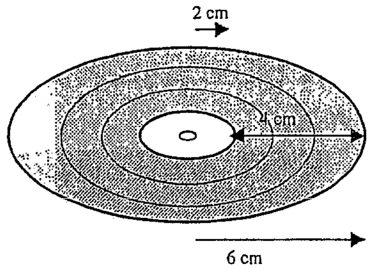
(a) Simplify the expression $\frac{1}{2}\sin 2x\tan x$ [1]

(b) (i) Show that $\sin x+\cos 2x=1+\sin x-2\sin^2 x$ [1]

(ii) Hence or otherwise solve:
$\sin x+\cos 2x=0$ for $0\le x\le \pi$ [4]

(c) The section of a CD used to store information starts 2cm from the centre of the CD and extends 4 cms to the edge of the disk, as in the diagram. As the CD spins, a laser moves along concentric circle 'tracks' on the information section of the disk.

There are 20 of these tracks for every *millimetre* of the radius of the information section of the disk.



(i) Show that the nth concentric track from the centre has length

$$2\pi\left(2+\frac{(n-1)}{200}\right)\text{ cm}$$ [2]

(ii) Find the total distance of information 'track' on a typical CD [2]

Question 6: *Start a new page* *12 marks*

(a) (i) The set of pronumerals *a, b, c* form an *arithmetic* sequence. Show that (-1,2) is a point on the line $ax + by = c$ [2]

(ii) Constance notices something unusual when she writes a pair of equations

$$ax + by = c$$
$$dx + ey = f$$

and the two sets of pronumerals *a, b, c* and *d, e, f* are both arithmetic sequences.

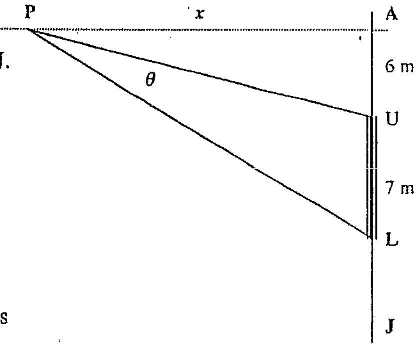
When she solves them simultaneously, she gets the same solution no matter what sets of numbers in arithmetic progression she uses.

Explain why this is so and find the solution. [2]

(b) Prudence is a hockey player and is moving along a path that is perpendicular to the goal line **AJ**. Her path intersects the goal line at **A**, 6 metres from the nearest goal post at **U**.

The goal posts U and L are 7 metres apart.

Suppose θ is the angle subtended by the goal posts when Prudence is at **P**, x metres from the goal line.



Show that $\tan\theta = \dfrac{ax}{x^2+b}$ where a and b are constants

(Hint: $\tan(\alpha-\beta) = \ldots\ldots\ldots$) [3]

(c) The series $2^{5x} + 2^{3x} + 2^{x} + \ldots\ldots\ldots\ldots\ldots + 2^{(11-6k)x}$ has n terms

(i) The sequence $5x, 3x, x, \ldots\ldots\ldots\ldots (11-6k)x$ also has n terms and can be seen as an arithmetic progression.

Show that $n = 3k - 2$ [2]

(ii) Show that the sum of the series $2^{5x} + 2^{3x} + 2^{x} + \ldots\ldots\ldots\ldots\ldots + 2^{(11-6k)x}$ can be given by

$$S_n = \frac{2^{7x}\left(1-2^{2x(2-3k)}\right)}{2^{2x}-1}$$ [3]

End of examination

FORMULAE SHEET

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\text{for} \quad t = \tan \frac{A}{2} \qquad \begin{cases} \sin A = \dfrac{2t}{1+t^2} \\ \cos A = \dfrac{1-t^2}{1+t^2} \\ \tan A = \dfrac{2t}{1-t^2} \end{cases}$$

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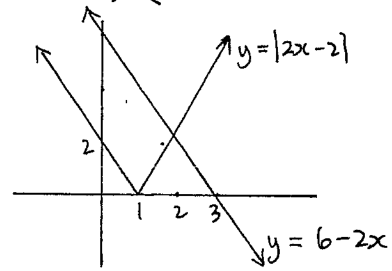
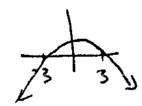
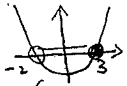
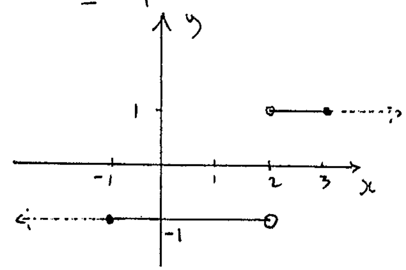
Qn	Solutions	Marks	Comments
1(a)	$\frac{1}{3-\sqrt{2}} + \frac{1}{3+\sqrt{2}} = \frac{3+\sqrt{2}+3-\sqrt{2}}{3^2-(\sqrt{2})^2}$ $= \frac{6}{7}$	✓ ✓	
(b)	$5^{3x-2} = \frac{1}{125} = 5^{-3}$ $3x-2 = -3$ $x = -\frac{1}{3}$	✓ ✓	
(c)	$\cos 75^\circ = \cos(45^\circ + 30^\circ)$ $= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$ $= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$ $= \frac{\sqrt{3}-1}{2\sqrt{2}}$	✓ ✓	
(d)	$(x_1, y_1) \equiv (1, -2)$ $(x_2\ y_2) \equiv (5, 6)$ $m:n = 2:-1$ $R = \left(\frac{2\cdot5-1\cdot1}{2-1}, \frac{2\cdot6-1\cdot-2}{2-1}\right)$ $= (9, 14)$	✓✓ ✓	
(e)	$x-3y+4=0 \quad m_1 = \frac{1}{3}$ $x+2y-7=0 \quad m_2 = -\frac{1}{2}$ $\tan\theta = \left\lvert \frac{\frac{1}{3} - -\frac{1}{2}}{1-\frac{1}{3}\cdot\frac{1}{2}} \right\rvert$ $= 1$ $\theta = 45^\circ$	✓ ✓ ✓	

Qn	Solutions	Marks	Comments
2(a)	$\lvert 2x-2 \rvert < 6-2x$  Solve $2x-2 = 6-2x$ $x = 2$ ie soln is $x < 2$ (or equivalent case method)		
(b)	$\frac{x^2}{16} + \frac{y^2}{9} = 1$ gives $9x^2 = 144 - 16y^2$ $x^2 = \frac{1}{9}(144-16y^2)$ $x = \frac{1}{3}\sqrt{144-16y^2}$ ie $144 - 16y^2 \geq 0$ $(12-4y)(12+4y) \geq 0$  $-3 \leq y \leq 3$ is the range.		
(c)	$\frac{3x-4}{x+2} \leq 1 \quad (x \neq -2)$ $(3x-4)(x+2) \leq (x+2)^2$ $3x^2+2x-8 \leq x^2+4x+4$ $2x^2-2x-12 \leq 0$ $x^2-x-6 \leq 0$		

Qn	Solutions	Marks	Comments
2(c)	$(x-3)(x+2) \leq 0$  $\therefore -2 < x \leq 3 \quad (x \neq -2)$		
(d)	point on $4y-3x-3=0$ is $(-1,0)$ $\therefore d = \dfrac{\lvert 3.-1-0+1 \rvert}{\sqrt{3^2+4^2}}$ $= \dfrac{2}{5}$ units		
3(a)	$\dfrac{x}{y} = \dfrac{2}{3}$ $\dfrac{x^2-y^2}{x^2+y^2}$ $= \dfrac{\frac{x^2}{y^2} - \frac{y^2}{y^2}}{\frac{x^2}{y^2} + \frac{y^2}{y^2}}$ $= \dfrac{\frac{4}{9}-1}{\frac{4}{9}+1} = \dfrac{-\frac{5}{9}}{\frac{13}{9}}$ $= -\dfrac{5}{13}$		
(b)	$3x-y+1 + k(x+4y-4)=0$ thru $(-2,1)$ ie $-6-1+1+k(-2+4-4)=0$ $-6-2k=0$ $k=-3$ $\therefore 3x-y+1-3(x+4y-4)=0$ $-13y+13=0$ $y=1$ which is $\parallel$ to x axis		

Qn	Solutions	Marks	Comments
3(c) (i)	$x \neq 2$	✓	
(ii)	$f(x) = \dfrac{\lvert x-2 \rvert}{x-2}$ $f(x) = \dfrac{x-2}{x-2}$ for $x-2>0$ $= 1 \qquad x>2$ $f(x) = \dfrac{-(x-2)}{x-2}$ for $x-2<0$ $= -1 \qquad x<2$  	✓ ✓	
3(d)	$\left(x+\frac{1}{x}\right)^2 + \left(x+\frac{1}{x}\right) - 6 = 0$ $u^2 + u - 6 = 0$ $(u+3)(u-2)=0$ $u = 2, -3$ $\therefore x + \frac{1}{x} = 2$ $x^2-2x+1=0$ $(x-1)^2=0$ $x=1$ + $\quad x+\frac{1}{x} = -3$ $x^2+3x+1=0$ $x = \dfrac{-3 \pm \sqrt{9-4}}{2}$ $= \dfrac{-3 \pm \sqrt{5}}{2}$	✓ ✓ ✓ ✓	

Qn	Solutions	Marks	Comments

4(a)

$\frac{1}{2}\sin 2x \tan x$

$= \frac{1}{2}\cdot 2\sin x\cos x \cdot \frac{\sin x}{\cos x}$ ✓

$= \sin^2 x$

(b)(i)

$\sin x + \cos 2x \quad [= \text{LHS}]$

$= \sin x + 1 - 2\sin^2 x$

$= 1 + \sin x - 2\sin^2 x$ ✓

(ii)

$\sin x + \cos 2x = 0$

$1 + \sin x - 2\sin^2 x = 0$

$2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$\sin x = 1$

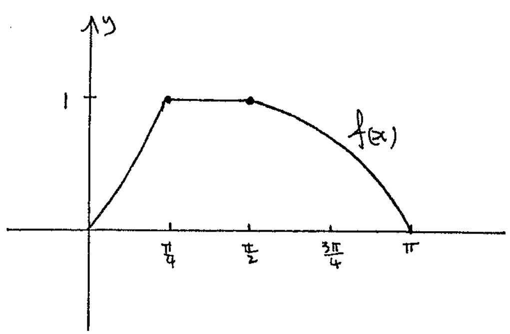
$\therefore x = \frac{\pi}{2}$

$2\sin x + 1 = 0$

$\sin x = -\frac{1}{2}$

no solutions as in Q3/Q4

(c)



Qn	Solutions	Marks	Comments

4 (d)

$A_T = \frac{1}{2}ab\sin C$

and $\frac{b}{\sin B} = \frac{a}{\sin A}$

this gives $b = \frac{a\sin B}{\sin A}$

on subst^n:

$\text{Area} = \frac{1}{2}a\left(\frac{a\sin B}{\sin A}\right).\sin C$

$= \frac{1}{2}a^2\frac{\sin B\sin C}{\sin A}$

$\text{Area} = \frac{a^2\sin B\sin C}{2\sin A}$

QED

5(a)

$T_2 = 2 \qquad S_\infty = 8$

ie $ar = 2$ and $8 = \frac{a}{1-r}$

$\therefore a = \frac{2}{r}$

$8 = \frac{\frac{2}{r}}{1-r}$

$8 - 8r = \frac{2}{r}$

$8r - 8r^2 = 2$

$4r^2 - 4r + 1 = 0$

$(2r-1)^2 = 0$

$r = \frac{1}{2}$

$\therefore a = 4$

(b)(i)

$A_9 = 2000(1.06)^9$ (9 years)

$= 2000.(1.689..) = \$3378.957...$

(ii) $\text{Total} = 2000\left[1.06 + 1.06^2 + \ldots + 1.06^9\right]$

Qn	Solutions	Marks	Comments
5(b) cont	Total $= 2000\left[\frac{1.06(1.06^9-1)}{0.06}\right]$ $a = 1.06$ $r = 1.06$ $n = 9$ $= 2000 \times 12.18079\ldots$ $\doteqdot \$24\,361.59$ Yes, she will have \$24000		
5(c)(i)	Track 1 is $2 \times 2\pi = 4\pi$ cm at each new track radius increases by $\frac{1}{200}$ cm ie the nth track has $(n-1)$ increases in radius ∴ nth track is $2\pi\left(2 + (n-1)\frac{1}{200}\right)$ cm		
(ii)	Total distance is sum of all track lengths so $S_n = \frac{n}{2}(2a+(n-1)d)$ where $n = 20 \times 40 + 1$ $= 801$ $a = 2.2\pi = 4\pi$ $d = \frac{2\pi}{200} = \frac{\pi}{100}$ $S_{801} = \frac{801}{2}\left(8\pi + 800 \cdot \frac{\pi}{100}\right)$ $= 801 \times 8\pi$ $= 20\,131.325\ldots$ cm $\doteqdot 201.31$ m		

Qn	Solutions	Marks	Comments
6(a)(i)	$(-1,2)$ on $ax+by=c$ means $-a+2b=c$ $b = \frac{a+c}{2}$ which defines an AP ∴ a, b, c in AP OR let $a = a$ $b = a+d$ $c = a+2d$ ∴ $ax+by=c$ becomes $ax+(a+d)y=a+2d$ $a(x+y-1)+d(y-2)=0$ ∴ $x+y-1=0$ and $y-2=0$ ∴ $x=-1 \Longleftarrow y=2$		
(ii)	if a, b, c in AP and $d\ e\ f$ in AP ∴ $(-1,2)$ is on both $ax+by=c$ and $dx+ey=f$ ∴ it must be on both at the same time ie the simultaneous solution $(-1,2)$ is the intersection of the 2 lines unless they are the same line (ie $a=d$, $b=e$ $c=f$)		

Qn	Solutions	Marks	Comments
6(b)	$\tan A\hat{P}L = \dfrac{13}{x}$ $\tan A\hat{P}U = \dfrac{6}{x}$ $\tan U\hat{P}L = \tan\theta$ $= \tan(A\hat{P}L - A\hat{P}U)$ $\tan(A\hat{P}L - A\hat{P}U) = \dfrac{\tan A\hat{P}L - \tan A\hat{P}U}{1 + \tan A\hat{P}L \tan A\hat{P}U}$ $= \dfrac{\frac{13}{x} - \frac{6}{x}}{1 + \frac{13.6}{x^2}}$ $= \dfrac{\frac{7}{x}}{\frac{x^2+78}{x^2}}$ $= \dfrac{7x}{x^2+78}$		
6(c)	(i) $5x, 3x, x, \dots (11-6k)x$ $T_n = a + (n-1)d$ $= 5x + (n-1)(-2x)$ $= 7x - 2nx$ but $T_n = (11-6k)x$ $\therefore 7x - 2nx = (11-6k)x$ $\therefore 7+2n = 11-6k$ $-2n = 4-6k$ $n = 3k-2$ QED		

Qn	Solutions	Marks	Comments
6(c)(ii)	$S_n = \dfrac{a(r^n - 1)}{(r-1)}$ as it is a GP with $a = 2^{5x}$ $r = 2^{-2x}$ $n = 3k-2$ So $S_n = \dfrac{2^{5x}\left((2^{-2x})^{3k-2} - 1\right)}{2^{-2x} - 1}$ $= \dfrac{2^{5x}\left(2^{2x(2-3k)} - 1\right)}{\frac{1}{2^{2x}} - 1}$ $= \dfrac{2^{5x}\left(2^{2x(2-3k)} - 1\right)}{\frac{1-2^{2x}}{2^{2x}}}$ $= \dfrac{2^{7x}\left(1 - 2^{2x(2-3k)}\right)}{2^{2x}-1}$ QED		