KAMBALA

Year 11 Mathematics

Preliminary Assessment Task 2

MAY 2005

Time Allowed: 1.5 hours working time, plus 5 minutes for reading at the beginning.

INSTRUCTIONS

- This examination contains 5 questions of 12 marks each. Marks for each question are shown.
- Answer all questions on the writing paper provided. Start each question on a new page.
- Calculators should be used.
- Show all necessary working.
- Marks may be deducted for careless or badly arranged work.

Question One — Marks

a) What is the value of $|8-12m|$ when $m=3$? — 1

b) Factorise $4w^2-9$ — 1

c) Evaluate $\sqrt{\dfrac{22.4-16.38}{5.72+3.05}}$ correct to 2 decimal places. — 2

d) Write $4^4+4^4+4^4+4^4$ in the form a^b — 1

e) Simplify $\sqrt{27}+\sqrt{3}-\sqrt{18}$ — 2

f) State whether the equation $|u+2|=|u|+2$ is true for all values of u. If it is not always true, give a counter example. — 2

g) Solve $x^2=10x-21$ — 2

h) Simplify $\cos(-\alpha)$, if α is an acute angle. — 1

Question Two (begin a new page)

a) For the function defined by: — 2

$$f(x)=\begin{cases}-1 \text{ for } x<0\\ x^2 \text{ for } 0\le x\le 2\\ 2x \text{ for } x>2\end{cases}$$

Find the value of $f(1)-f(-1)$.

b) Find the exact value of: — 2

i) $\cos 135^\circ$

ii) $\operatorname{cosec} 60^\circ$

c) Given that $A=\dfrac{1}{2}$, $B=\dfrac{-2}{3}$ and $C=\dfrac{9}{16}$, evaluate $\dfrac{C^{\frac{1}{2}}}{A^2B}$ — 2

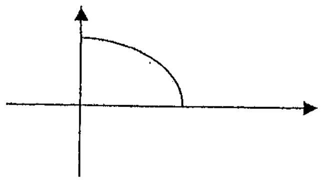
Question Two continues on the next page

6.02

Question Two continued — Marks

d) Find a quadratic equation with roots $\frac{1}{2}$ and $\frac{-2}{3}$. **3**
Give your answer in general form.

e) The diagram shows part of the graph of the function $y = f(x)$. **1**
You are told that it is an odd function.



Copy the diagram and complete the graph of the function.

f) Express $0.1\dot{2}$ as a simplified fraction. **2**

Question Three (begin a new page)

a) A triangular airfield is bordered by three straight roads of length 3.2 km, 3.8 km and 4.6 km. **4**

i) Find the largest angle in the triangle
ii) Find the area of the airfield, correct to the nearest hectare

b) Write down one other formula that can be derived from the Pythagorean Identity $\sin^2\theta + \cos^2\theta = 1$ **1**

c) Draw a neat sketch of the function $y = \sqrt{4 - x^2}$. **4**
Include any intercepts with the x and y-axes and state the domain and range of the function.

d) The domain of the cubic function $y = x^3$ is limited to $-3 \le x \le 2$. **1**
What is the range of the function with this domain?

e) Solve $\cos 2\theta = \frac{1}{\sqrt{2}}$ in the domain $0^\circ \le \theta \le 360^\circ$ **2**

Question Four begins on the next page.

Question Four (begin a new page) — Marks

a) When Libby wanted to find the value of $\tan 90^\circ$, she used her calculator, but it gave her the answer of "error." **2**

i) Explain why this is the case.
ii) State the value of $\cot 90^\circ$ and explain how you arrived at your answer.

b) i) Solve $|2k - 1| \le \frac{1}{2}$ **3**

ii) Graph the solution to $|2k - 1| \le \frac{1}{2}$ on a number line.

c) Solve $\sqrt{3}\tan\theta = -1$ for the domain $-180^\circ \le \theta \le 180^\circ$ **2**

d) i) Show that $\tan\alpha + \cot\alpha = \frac{1}{\sin\alpha\cos\alpha}$ **5**

ii) Hence, or otherwise, solve:
$\frac{1 + \cot\alpha}{\operatorname{cosec}\alpha} - \frac{\sec\alpha}{\tan\alpha + \cot\alpha} = -1$ for $0^\circ \le \alpha \le 360^\circ$

Question Five (begin a new page)

a) Show that $\frac{8}{3 - 2\sqrt{3}} + \frac{16}{6 + 4\sqrt{3}}$ is a rational number. **3**

b) i) Simplify $\frac{1}{x+2} - \frac{1}{x}$ **3**

ii) Hence, explain why the expression $\frac{1}{x+2} - \frac{1}{x}$ is always negative if $x > 0$.

c) By first completing the square, find the centre and radius of the circle with equation $x^2 - 6x + y^2 + 8y = 39$. **3**

d) i) Simplify $\dfrac{1 + \frac{x}{y}}{1 + \frac{y}{x}}$ **3**

ii) Use this result to simplify $\dfrac{1 + \frac{\sqrt{3}}{2}}{1 + \frac{2}{\sqrt{3}}}$

End of Examination

Preliminary Year Mathematics
Half-Yearly Examination 2005
SOLUTIONS

Question One

a) $|8-12m|$
$= |8-12(3)|$
$= |8-36|$
$= |-28|$
$= 28$

b) $4w^2 - 9$
$= (2w-3)(2w+3)$

c) $\sqrt{\dfrac{22\cdot4-16\cdot38}{5\cdot72+3\cdot05}}$

$= \sqrt{\dfrac{6\cdot02}{8\cdot77}}$

$\doteqdot \sqrt{0\cdot686431014}$
$= 0\cdot83$ (to 2 dec. pl.)

d) $4^4 + 4^4 + 4^4 + 4^4$
$= 4 \times 4^4$
$= 4^5$

e) $\sqrt{27} + \sqrt{3} - \sqrt{18}$
$= \sqrt{9\times3} + \sqrt{3} - \sqrt{9\times2}$
$= 3\sqrt{3} + \sqrt{3} - 3\sqrt{2}$
$= 4\sqrt{3} - 3\sqrt{2}$

f) $|u+2| = |u| + 2$
This statement is not always true
e.g. $|-3+2| \neq |-3| + 2$
since $|-1| \neq 3+2$

g) $x^2 = 10x - 21$
$x^2 - 10x + 21 = 0$
$(x-7)(x-3) = 0$
$\therefore x = 3, 7$

h) $\cos(-\alpha)$
$= \cos(360° - \alpha)$
$= \cos\alpha$

Question Two

a) $f(1) - f(-1)$
$= (1)^2 - (-1)$
$= 1 + 1$
$= 2$

b)(i) $\cos 135°$
$= -\cos 45°$
$= -\frac{1}{\sqrt{2}}$

(ii) $\operatorname{cosec} 60°$
$= \dfrac{1}{\sin 60°}$

$= \dfrac{1}{\frac{\sqrt{3}}{2}}$

$= 2$

c) $\dfrac{C^{\frac{1}{2}}}{A^2 B}$

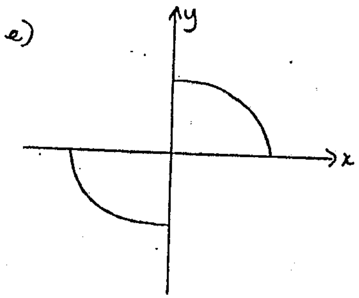
$= \dfrac{\sqrt{\frac{9}{16}}}{(\frac{1}{2})^2(-\frac{2}{3})}$

$= \dfrac{\frac{3}{4}}{\frac{1}{4}\cdot -\frac{2}{3}}$

$= -4\frac{1}{2}$

d) $(x-\frac{1}{2})(x+\frac{2}{3}) = 0$
$\therefore (2x-1)(3x+2) = 0$
$\therefore 6x^2 + x - 2 = 0$

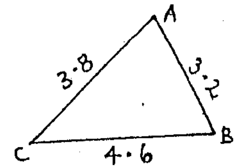
e)



f) $0\cdot1\dot{2}$
let $x = 0\cdot1222\ldots$
$\therefore 10x = 1\cdot2222\ldots$
$\therefore 9x = 1\cdot1$
$\therefore x = \frac{11}{90}$

Question Three

a)



i) The largest angle is opposite largest side
$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\cos A = \dfrac{(3\cdot8)^2 + (3\cdot2)^2 - (4\cdot6)^2}{2(3\cdot8)(3\cdot2)}$

$= \dfrac{3\cdot52}{24\cdot32}$

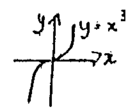
$\therefore \cos A \doteqdot 0\cdot144736842$
$\therefore A = 81°41'$ (to nearest minute)

(ii) $A = \frac{1}{2}bc\sin A$
$b = 3\cdot8\text{km} = 3800\text{m}$, $c = 3\cdot2\text{km} = 320$
$\therefore$ Area $= \frac{1}{2}(3800)(3200)\sin 81°41'$
$\doteqdot 6016061\cdot235\ \text{m}^2$
$= 602$ ha (to nearest ha)

b) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$
or $\tan^2\theta + 1 = \sec^2\theta$

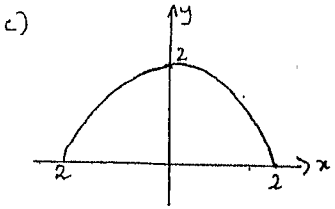
c) P.T.O.

d) $y = x^3$



When $x = -3$, $y = -27$
When $x = 2$, $y = 8$

c)



Domain is $\{x: -2 \leq x \leq 2\}$

Range is $\{y: 0 \leq y \leq 2\}$

e) $\cos 2\theta = \frac{1}{\sqrt{2}}, \quad 0^\circ \leq \theta \leq 360^\circ$

$2\theta = 45^\circ, (360^\circ - 45^\circ), (360^\circ + 45^\circ), (360^\circ + 360^\circ - 45^\circ)$

$2\theta = 45^\circ, 315^\circ, 405^\circ, 675^\circ$

$\therefore \theta = 22^\circ 30', 157^\circ 30', 202^\circ 30', 337^\circ 30'$

Question Four

a) i) $\tan 90^\circ$ is undefined because an angle of 90° makes a vertical line with the positive x-axis.
but $\tan\theta = \frac{\text{rise}}{\text{run}}$, and a vertical line has no run (ie run is zero)
$\therefore \tan 90^\circ = \frac{\text{rise}}{0}$
and we cannot have zero in the denominator of a fraction
$\therefore \tan 90^\circ$ is undefined, so Libby's calculator gave the answer of "error"

(ii) $\tan 90^\circ = \frac{\text{rise}}{0}$

$\therefore \cot 90^\circ = \frac{0}{\text{rise}}$

$\therefore \cot 90^\circ = 0$

Since $\cot\theta$ is the reciprocal of $\tan\theta$, $\cot 90^\circ = 0$

b) i) $|2k-1| \leq \frac{1}{2}$

$\therefore +(2k-1) \leq \frac{1}{2}$ or $-(2k-1) \leq \frac{1}{2}$

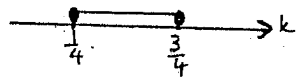
$2k-1 \leq \frac{1}{2}$ $\qquad -2k+1 \leq \frac{1}{2}$

$2k \leq 1\frac{1}{2}$ $\qquad -2k \leq -\frac{1}{2}$

$4k \leq 3$ $\qquad -4k \leq -1$

$k \leq \frac{3}{4}$ $\qquad k \geq \frac{1}{4}$

ii)



c) $\sqrt{3}\tan\theta = -1 \qquad -180^\circ \leq \theta \leq 180^\circ$

$\therefore \tan\theta = -\frac{1}{\sqrt{3}}$

$\therefore \theta = (180^\circ - 30^\circ), (360^\circ - 30^\circ - 360^\circ)$

$\therefore \theta = 150^\circ, -30^\circ$

d) i) RTP: $\tan\alpha + \cot\alpha = \frac{1}{\sin\alpha\cos\alpha}$

LHS: $\tan\alpha + \cot\alpha$

$= \frac{\sin\alpha}{\cos\alpha} + \frac{\cos\alpha}{\sin\alpha}$

$= \frac{\sin^2\alpha + \cos^2\alpha}{\sin\alpha\cos\alpha}$

$= \frac{1}{\sin\alpha\cos\alpha}$ (by Pythagorean identity)

$= \text{RHS}$

$\therefore \tan\alpha + \cot\alpha = \frac{1}{\sin\alpha\cos\alpha}$ as required

ii) $\frac{1+\cot\alpha}{\operatorname{cosec}\alpha} - \frac{\sec\alpha}{\tan\alpha + \cot\alpha} = -1$

$\left(1 + \frac{\cos\alpha}{\sin\alpha}\right).\sin\alpha - (\sin\alpha\cos\alpha).\frac{1}{\cos\alpha} = -1$

$\sin\alpha + \cos\alpha - \sin\alpha = -1$

$\therefore \cos\alpha = -1$

$\therefore \alpha = 180^\circ$

Question Five

a) $\frac{8}{3-2\sqrt{3}} + \frac{16}{6+4\sqrt{3}}$

$= \frac{8}{3-2\sqrt{3}} \times \frac{3+2\sqrt{3}}{3+2\sqrt{3}} + \frac{16}{6+4\sqrt{3}} \times \frac{6-4\sqrt{3}}{6-4\sqrt{3}}$

$= \frac{24+16\sqrt{3}}{9-4(3)} + \frac{96-64\sqrt{3}}{36-16(3)}$

$= \frac{24+16\sqrt{3}}{-3} + \frac{96-64\sqrt{3}}{-12}$

$\therefore \frac{96+64\sqrt{3}+96-64\sqrt{3}}{-12}$

$= \frac{192}{-12}$

$= -16$, a rational number

b) i) $\frac{1}{x+2} - \frac{1}{x}$

$= \frac{x-(x+2)}{x(x+2)}$

$= \frac{-2}{x(x+2)}$

ii) If $x > 0$, then $x(x+2) > 0$, $\therefore$ the denominator will always be a positive number.
Since the numerator is negative and the denominator is positive, the expression $\frac{1}{x+2} - \frac{1}{x}$ will always be negative if $x > 0$

c) $x^2 - 6x + y^2 + 8y = 39$

$x^2 - 6x + 9 + y^2 + 8y + 16 = 39 + 9 + 16$

$(x-3)^2 + (y+4)^2 = 64$

$\therefore$ centre is $(3,-4)$, radius is 8

d) i) $\frac{1+\frac{x}{y}}{1+\frac{y}{x}}$

$= \frac{\frac{y+x}{y}}{\frac{x+y}{x}}$

$= \frac{y+x}{y} \times \frac{x}{x+y}$

$= \frac{x}{y}$

ii) $\frac{1+\frac{\sqrt{3}}{2}}{1+\frac{2}{\sqrt{3}}}$ $\qquad x = \sqrt{3}, y = 2$

$= \frac{\sqrt{3}}{2}$