Student Number: _____

Class Teacher (*circle*): DL (GP) CG MC

KAMBALA

YEAR 11 MATHEMATICS

Preliminary Assessment Task 3

July 2010

Time allowed: 45 minutes

Trigonometric Ratios and Parabola, including Locus

- There are two parts to this task.
- The mark value for each part of each question is indicated next to that part.
- Answer each question in the spaces provided on the question paper. For multiple-choice questions circle the answer(s) of your choice.

PART A – Trigonometry

In Question 1 circle the answer (or answers) of your choice.

1. The exact value of $\cos 120°$ is: **1**

(A) $\frac{-\sqrt{3}}{2}$ (B) $\frac{-1}{2}$ $\frac{1}{2}$ (D) $\frac{\sqrt{3}}{2}$

2. Consider the statement: $\sin(90° - \theta) = \sin\theta$. **1**

Is the statement: sometimes true; always true; never true?

Give a reason for your answer or examples to justify your answer.

3. Given $\sin A < 0$ and $\cos A < 0$.

(i) In what quadrant of the unit circle is the angle A? **1**

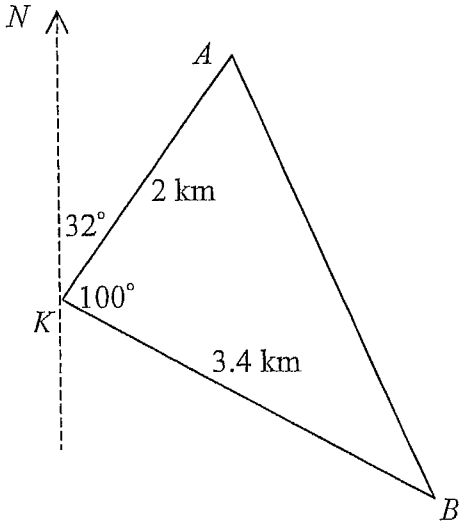
(ii) What is the range of possible values for the size of angle A? **1**

(iii) Is $\cot A$ positive or negative? **1**

4. Find all values of x in the domain $0° \leq x \leq 360°$, for which $\sin^2 x = \frac{3}{4}$. **3**

5. Two students leave Kambala, K, heading for their homes, A and B respectively.

(i) Using the diagram fill in the missing information in the sentences below. **2**

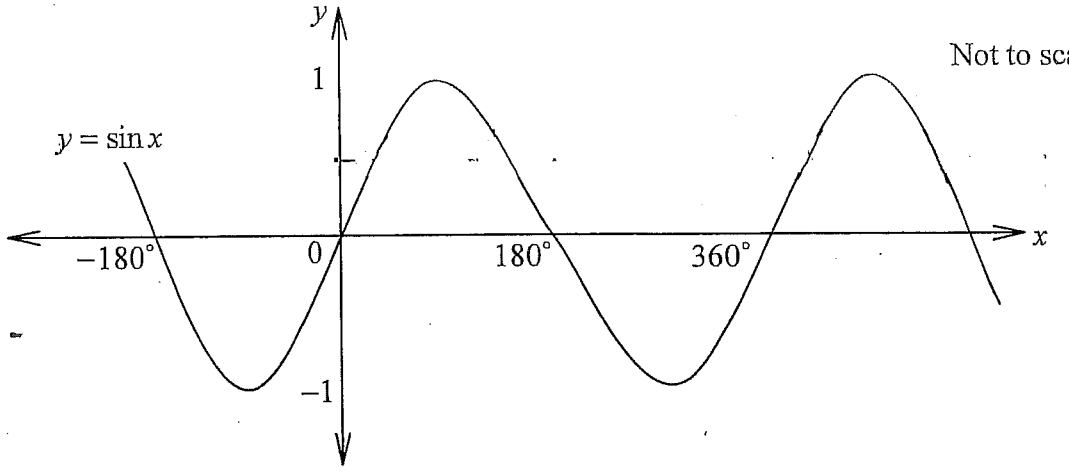


Not to scale

One student walks 2 kilometres on a bearing of _____. The other walks for 3.4 kilometres on a bearing of _____.

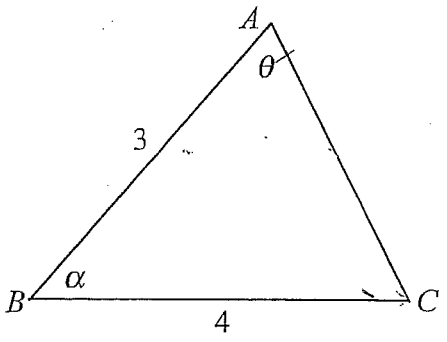
(ii) Find the distance between the students' homes. Answer to 1 decimal place. **2**

6. Consider the graph of $y = \sin x$ drawn below. On the same graph draw in the line $y = \frac{1}{2}$ and state how many solutions exist for the equation $\sin x = \frac{1}{2}$ in the domain $0° \le x \le 360°$. **2**



Not to scale

7. Consider the triangle below.



Not to scale

(i) Is $\theta = 90^\circ$ possible? Justify your answer. **2**

(ii) If $\theta = 68^\circ$, find α to the nearest minute. **2**

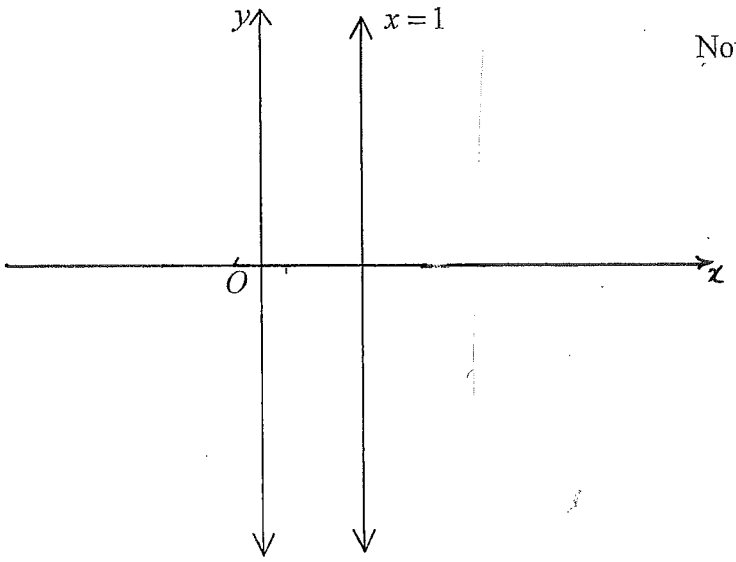
PART B – Locus

In Question 1 circle the answer (or answers) of your choice.

1. The locus of a point that moves so that it is always 3 units from the x-axis is: **2**

(A) $x=3$ (B) $y=3$ (C) $x=-3$ (D) $y=-3$

2. Sketch the locus of all points that are more than 2 units from the line $x=1$. **2**



Not to scale

3. The equation of a locus is $x^2+6x+y^2-4y+5=0$. Elizabeth described the locus as a circle with centre $(-3, 2)$ and radius $2\sqrt{2}$. Is she correct? Justify your answer. **2**

4. For the information below decide whether the locus described represents a line, a parabola or a circle. (You do not have to find the locus.)

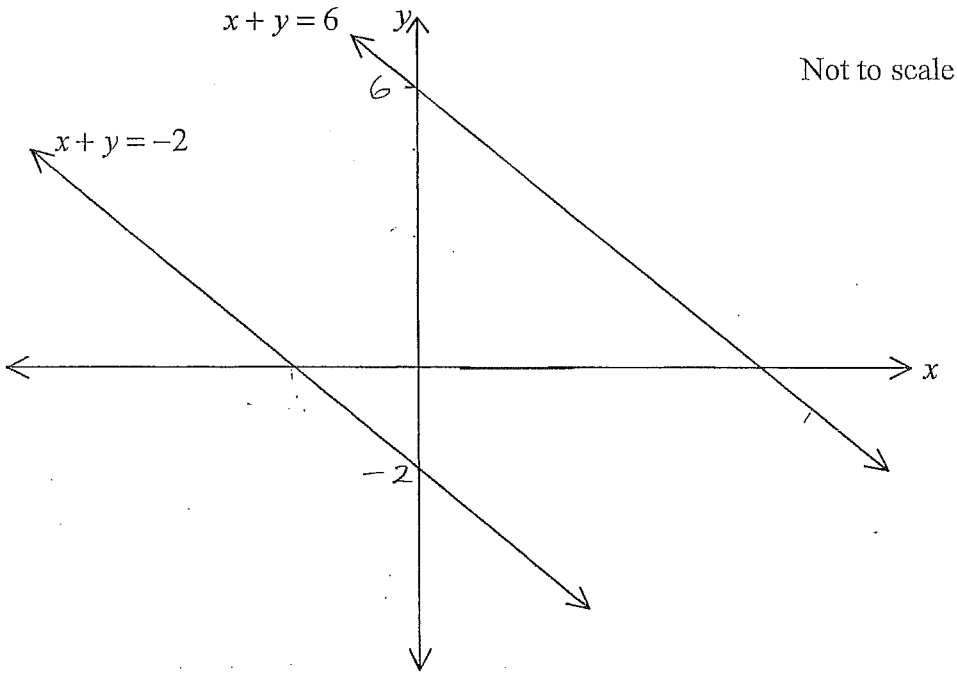
The locus of a point $P(x, y)$ that moves so that it is:

(i) equidistant from $A(4,-2)$ and $B(-3,5)$ is a _____. **1**

(ii) always 5 units from $A(2,-1)$ is a _____. **1**

(iii) equidistant from a fixed point and a fixed line is a _____ **1**

5. On the axes below, draw in the locus of a point that moves so that it is equidistant from $x+y=-2$ and $x+y=6$ and state the equation of this locus. Clearly label any intercepts. **2**



Not to scale

6. A parabola has equation $x^2 = 8y$. Draw a sketch of the parabola, clearly indicating its vertex, focus and directrix. **2**

7. A parabola has focus $(-1, 4)$ and focal length 3 units.

Given the above information, are the following possible? For those which are possible, hence, write down the equation of the parabola formed.

(i) **2**

(ii) Axis of symmetry $y = 4$ and vertex $(2, 4)$. **2**

End of Assessment

Student Number: _____

Class Teacher (*circle*): DL (GP) CG MC

KAMBALA

YEAR 11 MATHEMATICS

Preliminary Assessment Task 3

July 2010

Time allowed: 45 minutes

17/18 17/17

97%

Rank: = 1

Trigonometric Ratios and Parabola, including Locus

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PART A – Trigonometry

In Question 1 circle the answer (or answers) of your choice.

1. The exact value of $\cos 120°$ is: **1**

(A) $\frac{-\sqrt{3}}{2}$ (B) $\frac{-1}{2}$ ✓ $\frac{1}{2}$ (D) $\frac{\sqrt{3}}{2}$

2. Consider the statement: $\sin(90° - \theta) = \sin\theta$. **1**

$\sin(90-45) = \sin 45$

Is the statement: sometimes true; always true; never true?

Give a reason for your answer or examples to justify your answer.

Never true ✗ $\sin(90-\theta) = \cos\theta$

let $\theta = 30$

e.g. $\sin(90-30) = \sin 30$

$\sin 60 \neq \sin 30$

$\frac{\sqrt{3}}{2} \neq \frac{1}{2}$

But, $\sin(90-30) = \cos 30$

$\sin 60 = \cos 30$

$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

3. Given $\sin A < 0$ and $\cos A < 0$.

(i) In what quadrant of the unit circle is the angle A? **1**

3rd ✓

S | A

T | C

(ii) What is the range of possible values for the size of angle A? **1**

$180 < A < 270$ ✓

(iii) Is $\cot A$ positive or negative? **1**

positive ✓

4. Find all values of x in the domain $0° \leq x \leq 360°$, for which $\sin^2 x = \frac{3}{4}$. **3**

$\sin^2 x = \frac{3}{4}$

$\sin x = \pm\frac{\sqrt{3}}{2}$

S | A

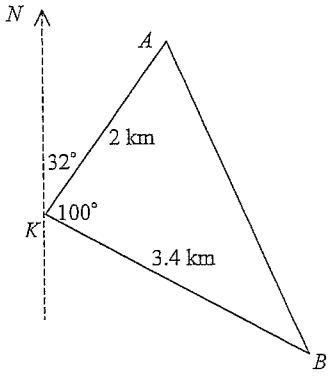
T | C

$x = 60°, 120°, 240°, 300°$ ✓

7

5. Two students leave Kambala, K, heading for their homes, A and B respectively.

(i) Using the diagram fill in the missing information in the sentences below. **2**



Not to scale

One student walks 2 kilometres on a bearing of 032°. The other walks for 3.4 kilometres on a bearing of 132°.

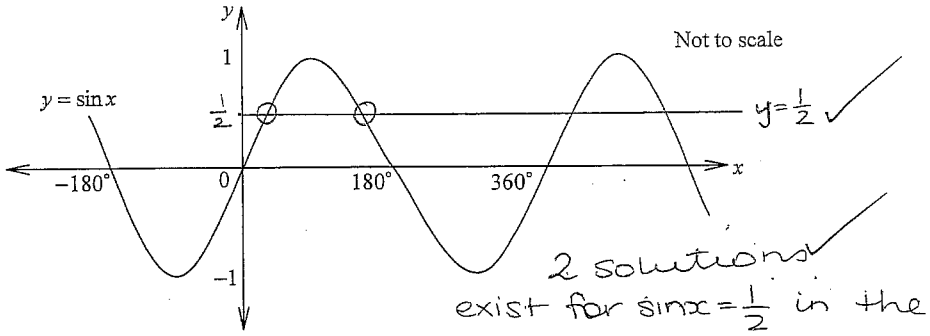
(ii) Find the distance between the students' homes. Answer to 1 decimal place. **2**

$AB^2 = (2)^2 + (3.4)^2 - 2(2)(3.4) \times \cos 100$

$AB = \sqrt{\text{ans}}$

$= 4.2$ km (1 dp.)

6. Consider the graph of $y = \sin x$ drawn below. On the same graph draw in the line $y = \frac{1}{2}$ and state how many solutions exist for the equation $\sin x = \frac{1}{2}$ in the domain $0° \le x \le 360°$. **2**

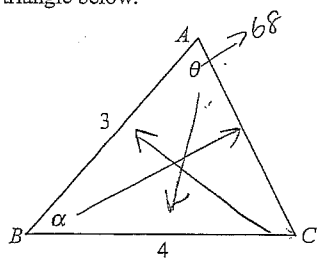


Not to scale

2 solutions exist for $\sin x = \frac{1}{2}$ in the domain $0° \le x \le 360°$.

6

7. Consider the triangle below.



Not to scale

(i) Is $\theta = 90°$ possible? Justify your answer. **2**

Yes $\theta = 90°$ is possible as there are no other defined angles and the sides when used in Pythagoras' theorum still produce a valid answer for the unknown side.

(ii) If $\theta = 68°$, find α to the nearest minute. **2**

$\frac{\sin 68}{4} = \frac{\sin ACB}{3}$

$\sin ACB = \frac{3 \sin 68}{4}$

$\angle ACB = \sin^{-1}(\text{ans.})$

$= 44° 3' \ldots$

$\therefore \alpha = 180 - 44° 3' \ldots - 68$

$= 67° 57'$ (nearest minute)

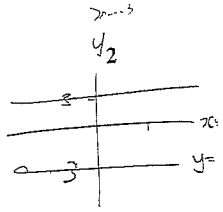
4

PART B – Locus

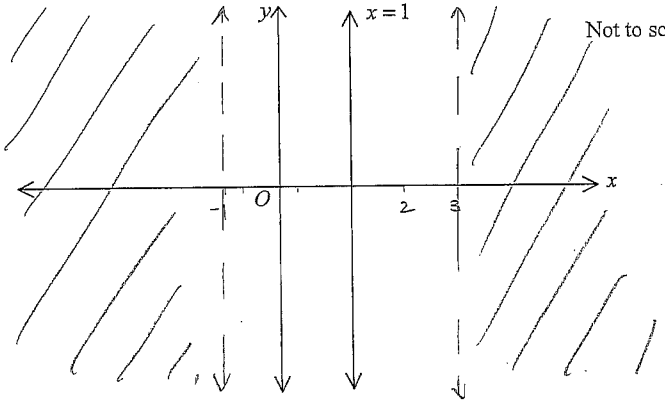
In Question 1 circle the answer (or answers) of your choice.

1. The locus of a point that moves so that it is always 3 units from the x-axis is:

(A) $x=3$ (B) $y=3$ (C) $x=-3$ (D) $y=-3$



2. Sketch the locus of all points that are more than 2 units from the line $x=1$. **2**



Not to scale

– all point >2units from x=1

3. The equation of a locus is $x^2+6x+y^2-4y+5=0$. Elizabeth described the locus as a circle with centre $(-3,2)$ and radius $2\sqrt{2}$. Is she correct? Justify your answer. **2**

$x^2+6x+y^2-4y+5=0$

$x^2+6x+9+y^2-4y+4=-5+9+4$

$(x+3)^2+(y-2)^2=8$

Centre $(-3,2)$

Radius: $\sqrt{8}=2\sqrt{2}$

Yes she is correct as when you complete the square of the locus the equation of a circle with C(-3,2) and radius $2\sqrt{2}$ is formed.

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4. For the information below decide whether the locus described represents a line, a parabola or a circle. (You do not have to find the locus.)

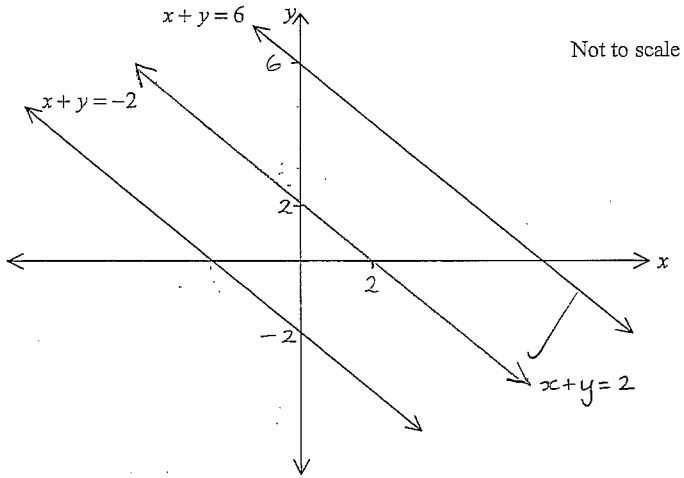
The locus of a point $P(x,y)$ that moves so that it is:

(i) equidistant from $A(4,-2)$ and $B(-3,5)$ is a line. **1**

(ii) always 5 units from $A(2,-1)$ is a circle. **1**

(iii) equidistant from a fixed point and a fixed line is a parabola **1**

5. On the axes below, draw in the locus of a point that moves so that it is equidistant from $x+y=-2$ and $x+y=6$ and state the equation of this locus. Clearly label any intercepts. **2**



Not to scale

$P(x,y)$

A: $x+y+2=0$

B: $x+y-6=0$

$PA=PB$

$PA=|x+y+2|$

$PB=|x+y-6|$

Case ①

$x+y+2=x+y-6$

$\therefore$ no solution

Case ②

$x+y+2=-x-y+6$

$\frac{2x+2y-4=0}{2}$

$x+y-2=0$

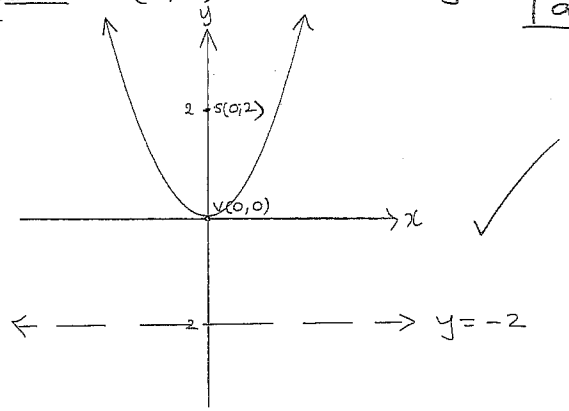
$y=-x+2$

5

6. A parabola has equation $x^2 = 8y$. Draw a sketch of the parabola, clearly indicating its vertex, focus and directrix. 2

V(0,0) focal length: $4a = 8$, $a = 2$

S(0,2) D: $y = -2$



7. A parabola has focus $(-1, 4)$ and focal length 3 units.

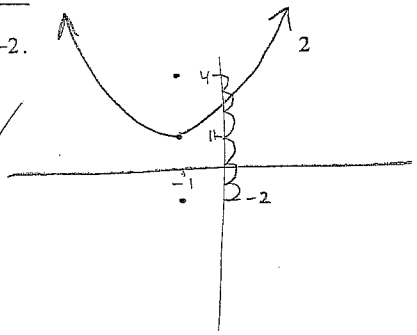
Given the above information, are the following possible? For those which are possible, hence, write down the equation of the parabola formed.

(i) A vertex of $(-1, 1)$ and equation of directrix $y = -2$. 2

S(-1, 4) focal length: 3

Form: $(x-h)^2 = 4a(y-k)$

$(x+1)^2 = 12(y-1)$

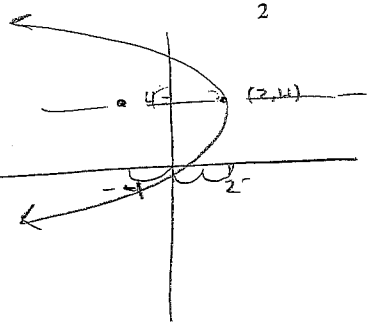


(ii) Axis of symmetry $y = 4$ and vertex $(2, 4)$. 2

focal length: 3

Form: $(y-k)^2 = -4a(x-h)$

$(y-4)^2 = -12(x-2)$



End of Assessment

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