Student Number/Name: ____________________

KAMBALA

MATHEMATICS

YEAR 11 PRELIMINARY EXAMINATION

SEPTEMBER 2006

Time Allowed: 2 hours
Reading Time: 5 minutes

MARK: /84

INSTRUCTIONS

- This examination contains 6 questions of equal value. Marks allocated in each part of a question are shown.
- Answer all questions on the writing paper provided. Start each question on a NEW page.
- Approved scientific calculators and drawing templates may be used.
- Show all necessary working.
- Marks may not be awarded for careless or badly arranged work.

Question 1 *(Start a new page)* **Marks**

(a) Expand and simplify $12x+(1-4x)^2$ 2

(b) Factorise fully $4a^2b-12a^3b^2$ 2

(c) Express $\dfrac{3}{4+\sqrt{2}}$ with a rational denominator in simplest form. 2

(d) Simplify $\dfrac{x+1}{3}-\dfrac{1-x}{7}$ 3

(e) Factorise fully $x^3+5x^2-3x-15$ 2

(f) The two shorter sides of a right-angled triangle have sides of lengths $2+\sqrt{3}$ and $2-\sqrt{3}$. Find the exact length of the hypotenuse. 3

Question 2 *(Start a new page)* **Marks**

(a) Solve $x^2=16x$ 2

(b) If $f(x)=x^2+1$:

(i) Evaluate $f(-5)$. 1

(ii) For what value(s) of x is $f(x)=5$? 2

(c) Determine if the function below is odd, even or neither: 2

$$f(x)=\frac{3x}{2+x^2}$$

Question 2 continued

(d) Solve $|2x+5|=14$ **2**

(e) Find the value of x if $\sin(x+28)^\circ = \cos x^\circ$. **2**

(f) Express $1.3\dot{8}$ as a fraction in simplest form. **2**

(g) Write 0.0004020 in scientific notation. **1**

Question 3 *(Start a new page)*

Marks

(a) Find the interior angle sum of a pentagon. **2**

(b) Solve $3^x = \frac{1}{27}$ **1**

(c) State the domain of: $(x+5)^2 + y^2 = 16$ **1**

(d) Show that $\sin^2\theta + \tan^2\theta = \sec^2\theta - \cos^2\theta$. **2**

(e) Find the values of k for which $3x^2 - 4x + k = 0$ has no real roots. **2**

(f) (i) A point $P(x,y)$ moves so that its distance from the point $A(1, 5)$ is twice its distance from the point $B(4,-1)$. Show that the locus of P is: **2**

$$x^2 + y^2 - 10x + 6y + 14 = 0$$

(ii) Describe geometrically the locus of P. **1**

(g) Solve $(5\sin\theta + 2)(2\cos\theta - 5) = 0$ for $0^\circ \le \theta \le 360^\circ$. Answer to the nearest minute. **3**

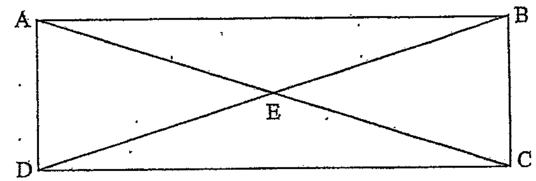
Question 4 *(Start a new page)*

Marks

(a) If $2x^2 + 3x - 5 \equiv A(x+1)^2 + B(x+1) + C$, find A, B and C. **3**

(b) Find the perpendicular distance of the point (5, 2) from the line $2x - y + 3 = 0$. **2**

(c) In the diagram below, ABCD is a rectangle. The diagonals AC and BD intersect at E.



(i) Show that triangles ACD and BDC are congruent. **3**

(ii) Using part (i), prove that the diagonals are equal. **1**

(d) A parabola has vertex (0, 4) and focus (0, 1).

(i) What is the focal length? **1**

(ii) What is the equation of the directrix? **1**

(iii) Find the equation of the parabola. **1**

(iv) Hence sketch the parabola, showing clearly the vertex, focus and directrix. **2**

Question 5 *(Start a new page)* **Marks**

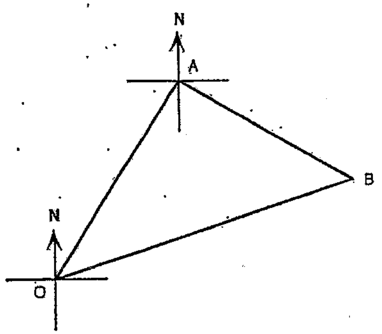
(a) If α and β are the roots of the quadratic equation $2x^2-7x+12=0$, find:

(i) $\alpha+\beta$. 1

(ii) $\alpha\beta$ 1

(iii) $\alpha^2+\beta^2$ 2

(b) A ship starts from O and sails 80 kilometres on a bearing of 035° to A. It then changes course and sails 55 kilometres on a bearing of 110° to B.



(i) Copy the diagram, marking on it the information supplied.

(ii) Show that $\angle OAB = 105°$. 1

(iii) Calculate the distance of B from O, correct to 1 decimal place. 2

(c) For the points A(2, 1), B(-1, 4) and C(-6, -3), find:

(i) the exact distance between A and C. 2

(ii) the gradient of the line passing through A and B. 1

(iii) the mid-point of the interval AB. 2

(iv) the equation of the perpendicular bisector of AB. 2

Question 6 *(Start a new page)* **Marks**

(a) (i) Solve simultaneously: 3

$$\left.\begin{array}{l} y=2x-10 \\ x^2+y^2=25 \end{array}\right\}$$

(ii) Hence sketch the graphs of the equations from part (i), showing their point(s) of intersection clearly. 2

(iii) On your graph, **clearly shade** the region bounded by $y \geq 2x-10$ **and** $x^2+y^2 \leq 25$. 1

(b) In ΔABC, $\angle ABC$ is a right angle. $\angle BCA = 60°$ and side $AB = 4$ cm.

(i) Draw a diagram showing this information.

(ii) Find the length of BC in exact form. 2

(iii) Hence find the area of ΔABC in exact form. 1

(c) Find the angle of inclination with the positive direction of the x-axis of the line $6x+6y-1=0$. 2

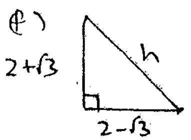
(d) A function is defined by the rule:

$$f(x)=\begin{cases} x^2+4 & x \geq 0 \\ \dfrac{1}{x} & x<0 \end{cases}$$

(i) Find $f(k^2)$ 1

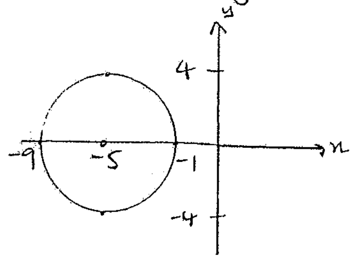
(ii) Sketch $y=f(x)$ showing all its features. 2

END OF EXAMINATION

Qn	Solutions	Marks	Comments+Criteria
1	(a) $12x + (1-4x)^2$ $= 12x + 1 - 8x + 16x^2$ $= 16x^2 + 4x + 1$	 1 1	 correct expansion
	(b) $4a^2b - 12a^3b^2$ $= 4a^2b\,(1 - 3ab)$	2 OR 1	HCF + correct (1−3ab) some common factor + correct ( )
	(c) $\frac{3}{4+\sqrt{2}} \times \frac{4-\sqrt{2}}{4-\sqrt{2}}$ $= \frac{12-3\sqrt{2}}{16-2}$ $= \frac{12-3\sqrt{2}}{14}$	1 1	correct conjugate
	(d) $\frac{x+1}{3} - \frac{1-x}{7}$ $= \frac{7(x+1)}{21} - \frac{3(1-x)}{21}$ $= \frac{7x+7-3+3x}{21}$ $= \frac{10x+4}{21}$	 1 1 1	
	(e) $x^3 + 5x^2 - 3x - 15$ $= x^2(x+5) - 3(x+5)$ $= (x^2-3)(x+5)$	 1 1	
	(f)  $h^2 = (2+\sqrt{3})^2 + (2-\sqrt{3})^2$ $= 4 + 4\sqrt{3} + 3 + 4 - 4\sqrt{3} + 3$ $h^2 = 14$ $h = \sqrt{14}$	1 1 1	

Qn	Solutions	Marks	Comments+Criteria
2	(a) $x^2 = 16x$ $x^2 - 16x = 0$ $x(x-16) = 0$ $x = 0,\quad x = 16$	 1 1	 Correct factorisation both solutions
	(b) $f(x) = x^2 + 1$ (i) $f(-5) = (-5)^2 + 1$ $= 25 + 1$ $= 26$	 1	
	(ii) $5 = x^2 + 1$ $x^2 = 4$ $x = \pm 2$	 1 1	 both solns
	(c) $f(x) = \frac{3x}{2+x^2}$ $-f(x) = \frac{-3x}{2+x^2}$ $f(-x) = \frac{3(-x)}{2+(-x)^2}$ $= \frac{-3x}{2+x^2}$ ie $f(-x) = -f(x)$ ∴ odd	 1 1	 Correct substitution and proof
	(d) $\lvert 2x+5 \rvert = 14$ $2x+5 = 14$ or $2x+5 = -14$ $2x = 9$ $2x = -19$ $x = \frac{9}{2}$ $x = -\frac{19}{2}$ ∴ $x = 4\frac{1}{2}$ $x = -9\frac{1}{2}$		

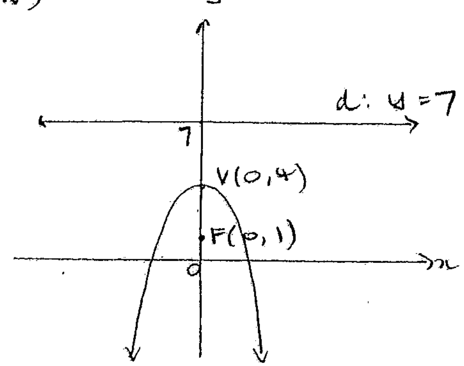
Qn	Solutions	Marks	Comments+Criteria
2 ctd	(e) $\sin(x+28) = \cos x$ $= \sin(90-x)$ $x+28 = 90-x$ $2x = 62$ $\therefore x = 31°$	1 1	
	(f) $1\cdot3\dot{8}$ Let $x = 1\cdot38888\ldots$ ① $\therefore 10x = 13\cdot88888\ldots$ ② ② − ①: $9x = 12\cdot5$ $x = \frac{12\cdot5}{9}$ $x = \frac{125}{90}$ $= 1\frac{35}{90}$ $x = 1\frac{7}{18}$	1 1	process of simult. eqns fraction in simplest form
	(g) $0\cdot0004020$ $= 4\cdot02 \times 10^{-4}$	1	

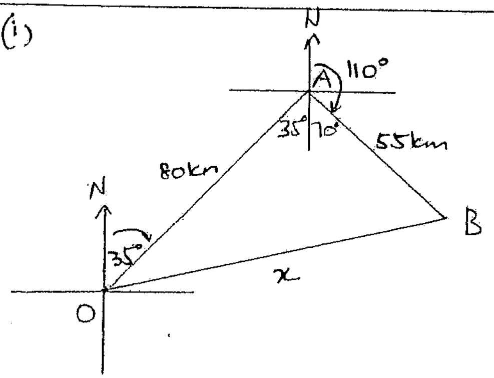
Qn	Solutions	Marks	Comments+Criteria
3	(a) pentagon has $n = 5$. $\therefore$ interior angle sum $= (n-2)\times180°$ $= (5-2)\times180$ $= 3\times180$ $= 540°$	1 1	
	(b) $3^x = \frac{1}{27}$ $= \frac{1}{3^3}$ $= 3^{-3}$ $\therefore x = -3$	1	
	(c) $(x+5)^2 + y^2 = 16$  $C(-5,0)$ $r = 4$ $D: -9 \le x \le -1$	1	
	(d) Show $\sin^2\theta + \tan^2\theta = \sec^2\theta - \cos^2\theta$ LHS $= \sin^2\theta + \tan^2\theta$ $= (1-\cos^2\theta) + (\sec^2\theta - 1)$ $= \sec^2\theta - \cos^2\theta$ $=$ RHS	1 1	either identity correct simplification, proof

Qn	Solutions	Marks	Comments+Criteria
3 ctd	(e) $3x^2 - 4x + k = 0$ No real roots if $\Delta < 0$ ie $b^2 - 4ac < 0$ $(-4)^2 - 4 \cdot 3 \cdot k < 0$ $16 - 12k < 0$ $-12k < -16$ $k > \frac{16}{12}$ $k > \frac{4}{3}$ $k > 1\frac{1}{3}$	1 1	
	(f)(i) $A(1,5)$ $P(x,y)$ $B(4,-1)$ $AP = 2PB$ $AP^2 = 4PB^2$ $(x-1)^2 + (y-5)^2 = 4[(x-4)^2 + (y+1)^2]$ $x^2 - 2x + 1 + y^2 - 10y + 25 = 4[x^2 - 8x + 16 + y^2 + 2y + 1]$ $x^2 - 2x + y^2 - 10y + 26 = 4x^2 - 32x + 64 + 4y^2 + 8y + 4$ $3x^2 + 3y^2 - 30x + 18y + 42 = 0$ $x^2 + y^2 - 10x + 6y + 14 = 0$	1 1	
	(ii) $x^2 - 10x + y^2 + 6y = -14$ $(x-5)^2 + (y+3)^2 = -14 + 25 + 9$ $= 20$ The locus of P is a circle $C(5,-3)$ and radius $\sqrt{20} = 2\sqrt{5}$ units	1	

Qn	Solutions	Marks	Comments+Criteria
3 ctd	(g) $(5\sin\theta + 2)(2\cos\theta - 5) = 0$ for $0 \le \theta \le 360°$ $\sin\theta = \frac{-2}{5}$ or $\cos\theta = \frac{5}{2} = 2\frac{1}{2}$ basic angle, $\theta = 23°35'$ No solutions ~~S~~ | ~~A~~ ; T | C $\therefore \theta = 180 + 23°35'$ and $360 - 23°35'$ $\theta = 203°35', 336°25'$	1 1 1	$\cos\theta$ no solns solving $\theta = 23°35'$ correct solns

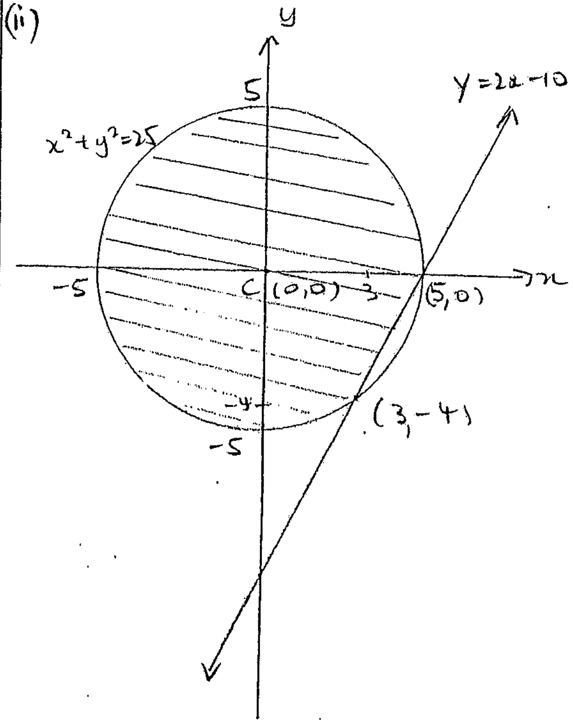
Qn	Solutions	Marks	Comments+Criteria
4	(a) $2x^2+3x-5 \equiv A(x+1)^2 + B(x+1)+C$ $= A(x^2+2x+1) + Bx+B+C$ $= Ax^2 + 2Ax + A + Bx + B + C$	1	
	$2x^2+3x-5 = Ax^2 + (2A+B)x + (A+B+C)$ ie $\boxed{A=2}$, $2A+B=3$ $4+B=3$ $\therefore \boxed{B=-1}$ $A+B+C=-5$ $2-1+C=-5$ $\boxed{C=-6}$	2	correctly equating coefficients
	(b) $(5, 2)$ $\quad 2x-y+3=0$ $x \;\; y \quad a=2,\ b=-1,\ c=3$ perp. $d = \dfrac{\lvert ax+by+c \rvert}{\sqrt{a^2+b^2}}$ $= \dfrac{\lvert 2\times5 + -1\times2 + 3 \rvert}{\sqrt{2^2+(-1)^2}}$	1	
	$= \dfrac{\lvert 10-2+3 \rvert}{\sqrt{5}}$ $= \dfrac{\lvert 11 \rvert}{\sqrt{5}}$ $\therefore d = \dfrac{11}{\sqrt{5}}$ or $\dfrac{11\sqrt{5}}{5}$ units	1	

Qn	Solutions	Marks	Comments+Criteria
4	(c) (i) Show $\triangle ACD \equiv \triangle BDC$ In $\triangle ACD$ and $\triangle BDC$: (i) $\angle ADC = \angle BCD = 90°$ (prop of rectangle ABCD)	1	correct statements
	(ii) $AD = BC$ (")	1	correct reasons
	(iii) CD shared $\therefore \triangle ACD \equiv \triangle BDC$ (SAS test)	1	
	(ii) Since $\triangle ACD \equiv \triangle BDC$, then $AC = BD$ ie diagonals are equal.	1	
	(d) $V(0,4)$ $\quad F(0,1)$ (i) focal length is 3 units	1	
	(ii) directrix : $y=7$	1	
	(iii) $(x-h)^2 = -4a(y-k)$ $x^2 = -12(y-4)$	1	
	(iv) 	1 1	correct shape of ∩ and directrix correct labelling of key features

Qn	Solutions	Marks	Comments+Criteria
5	(a) $2x^2 - 7x + 12 = 0$		
	(i) $\alpha + \beta = -\frac{b}{a}$ $= \frac{7}{2} = 3\frac{1}{2}$	1	
	(ii) $\alpha\beta = \frac{c}{a}$ $= \frac{12}{2}$ $= 6$	1	
	(iii) $\alpha^2 + \beta^2$ $= (\alpha+\beta)^2 - 2\alpha\beta$ $= (\frac{7}{2})^2 - 2.6$ $= \frac{49}{4} - 12$ $= 12\frac{1}{4} - 12$ $= \frac{1}{4}$	1 1	
	(b) (i) 		
	(ii) $\angle OAB = 35 + 70$ $= 105°$	1	correct working + answer
	(iii) $BO^2 = 80^2 + 55^2 - 2.80.55.\cos 105°$ $= 11702 \cdot 6076$ $BO = 108 \cdot 1785912$	1 1	correct subs. into correct formula

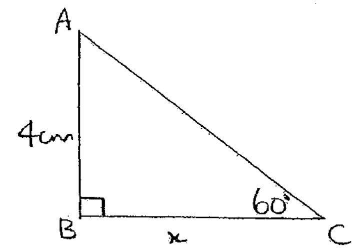
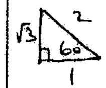
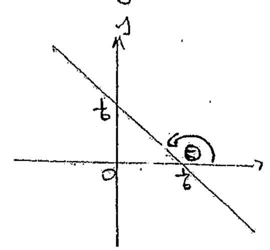
$= 108 \cdot \underline{2}$ km as required

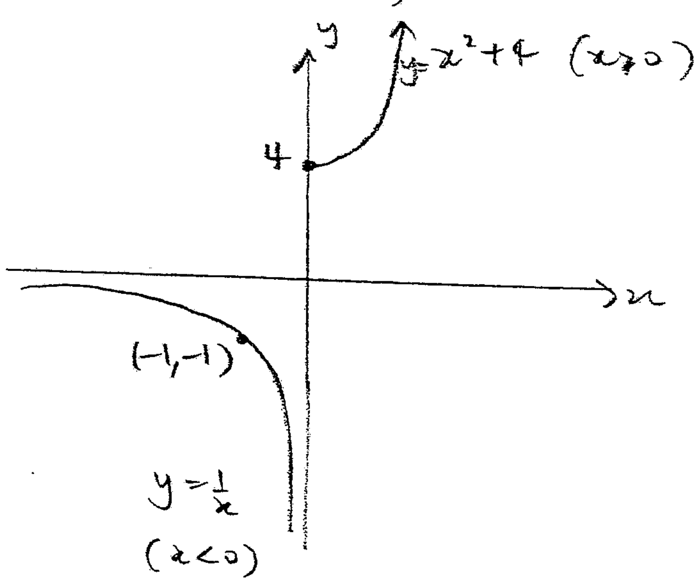
Qn	Solutions	Marks	Comments+Criteria
5 ctd	(c) $A(2,1)$ $B(-1,4)$ $C(-6,-3)$		
	(i) $d_{AC} = \sqrt{(2+6)^2 + (1+3)^2}$ $= \sqrt{8^2 + (4)^2}$ $= \sqrt{64+16}$ $= \sqrt{80}$ units $= \sqrt{16 \times 5}$ $d_{AC} = 4\sqrt{5}$ u	1 1	
	(ii) $m_{AB} = \frac{4-1}{-1-2}$ $= \frac{3}{-3}$ $= -1$	1	
	(iii) midpt of AB $= \left(\frac{2-1}{2}, \frac{1+4}{2}\right)$ $= \left(\frac{1}{2}, \frac{5}{2}\right)$ $= \left(\frac{1}{2}, 2\frac{1}{2}\right)$	1 1	
	(iv) perp. bisector of AB has $m = 1$ and passes through $(\frac{1}{2}, 2\frac{1}{2})$ $y - \frac{5}{2} = 1\left(x - \frac{1}{2}\right)$ $y = x + 2$ OR $x - y + 2 = 0$	1 1	either $y = mx + b$ or general form

Qn	Solutions	Marks	Comments+Criteria
6	(a) (i) $y = 2x - 10$ ① $x^2 + y^2 = 25$ ② ① into ②: $x^2 + (2x-10)^2 = 25$	1	
	$x^2 + 4x^2 - 40x + 100 = 25$ $5x^2 - 40x + 75 = 0$ $x^2 - 8x + 15 = 0$	1	
	$(x-3)(x-5) = 0$ $x = 3$ or $x = 5$ $y = -4$ $\quad$ $y = 0$	1	
	$(3,-4)$ and $(5,0)$		
	(ii) 	1 1	Correct circle and line correct labelling of points of intersection

(iii) Shade $y \geqslant 2x - 10 \cap x^2 + y^2 \leq 25$. ① mk

$(0,0)$: $0 \geqslant 0 - 10$ $\qquad$ $0 + 0 \leq 25$

Qn	Solutions	Marks	Comments+Criteria
6 ctd	(b) (i) 		
	(ii) $\tan 60 = \frac{4}{x}$	1	
	 $\sqrt{3} = \frac{4}{x}$ $\therefore x = \frac{4}{\sqrt{3}}$	1	
	$= \frac{4\sqrt{3}}{3}$		
	(iii) A of $\triangle ABC = \frac{1}{2}bh$ $= \frac{1}{2} \cdot \frac{4}{\sqrt{3}} \cdot 4$ $= \frac{16}{2\sqrt{3}}$ $= \frac{8}{\sqrt{3}}$ or $\frac{8\sqrt{3}}{3}\ u^2$	1	
	(c) $6x + 6y - 1 = 0$ $6y = -6x + 1$ $y = -x + \frac{1}{6}$		
	 $m = \tan\theta$ $\tan\theta = -1$	1	
	$\therefore \theta = 135°$	1	

Qn	Solutions	Marks	Comments+Criteria
6 ctd	(d) $f(x) = \begin{cases} x^2+4 & x \geq 0 \\ \frac{1}{x} & x < 0 \end{cases}$		
	(i) $f(k^2) = (k^2)^2 + 4$ $= k^4 + 4$	1	
	(ii) Sketch $y = f(x)$ 	1 1	correct shape of curves correct labelling eg $(-1,-1)$ and closed circle $x=4$