

Student Number: \_\_\_\_\_

**KAMBALA**

## Mathematics Extension 1 Preliminary Examination

September 2006

*Time Allowed: 2 hours plus 5 minutes reading time***Outcomes assessed:**

- P2** provides reasoning to support conclusions which are appropriate to the context  
**P3** performs routine arithmetic and algebraic manipulation involving surds and simple rational expressions  
**P4** chooses and applies appropriate arithmetic, algebraic, graphical and trigonometric techniques  
**P5** understands the concept of a function and the relationship between a function and its graph  
**P6** relates the derivative of a function to the slope of its graph  
**P7** determines the derivative of a function through routine application of the rules of differentiation  
**P8** understands and uses the language and notation of calculus  
**H5** applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems  
**H6** uses the derivative to determine the features of the graph of a function  
**H7** uses the features of a graph to deduce information about the derivative  
**PE2** uses multi-step deductive reasoning in a variety of contexts  
**PE3** solves problems involving inequalities  
**PE5** determines derivatives which require the application of more than one rule of differentiation  
**PE6** makes comprehensive use of mathematical language, diagrams and notation for communicating in a wide variety of situations  
**HE2** uses inductive reasoning in the construction of proofs

**INSTRUCTIONS**

- This examination contains 6 questions of 12 marks each.
- Marks for each part question are shown.
- Answer all questions on the writing paper provided. Start each question on a new page.
- Approved calculators may be used.
- Show all necessary working. Marks may not be awarded for careless or badly arranged work.

**STANDARD INTEGRALS**

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1, \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left( x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

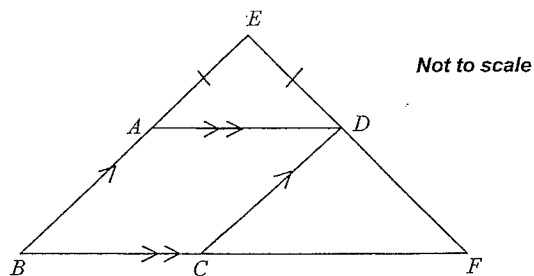
$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left( x + \sqrt{x^2 + a^2} \right)$$

NOTE :  $\ln x = \log_e x, \quad x > 0$

**Question 1 (12 marks)**  
Start a new page.

Marks

- (a) In the diagram,  $ABCD$  is a parallelogram and  $BA$  is produced to  $E$  so that  $AE = ED$ .  $ED$  is produced to cut  $BC$  produced at  $F$ .



Copy the diagram onto your answer page and prove that  $\triangle DCF$  is isosceles. 2

- (b) What is the value of  $x^3 - 8y^3$  if  $x - 2y = 4$  and  $x^2 + 2xy + 4y^2 = -6$ . 2

- (c) The point  $(3, -4)$  divides the interval joining  $A(1, 2)$  and  $B(x, y)$  internally in the ratio  $4 : 3$ . 3

Find the values of  $x$  and  $y$ .

- (d) Find the second derivative of  $f(x) = x^4 - 5x^{-3}$  2

- (e) Express  $2x^2 + 8x - 3$  in the form  $a(x + b)^2 + c$  and hence, or otherwise, find the minimum value of  $2x^2 + 8x - 3$ . 3

**Question 2 (12 marks)**  
Start a new page.

Marks

- (a) If  $x = 2 \sec \theta$  and  $y = 2 \tan \theta$ , express  $x^2 - y^2$  in simplest form. 2

- (b) Given  $P(x) = x(x+1)^2(x-2)(x-3)$   $x^2 \times x^3 \times x^4 \times x^5$

- (i) State the leading term and the constant term of  $P(x)$ . 2

- (ii) Write down the zeros of  $P(x)$ . 1

- (iii) Hence make a neat sketch of  $y = P(x)$ , showing any intercepts. 1

- (iv) Solve  $P(x) > 0$  for all  $x$ . 1

- (c)  $A$  and  $B$  are the fixed points  $(1, 1)$  and  $(-3, 5)$  respectively.  $P$  is a variable point  $(x, y)$ . Suppose that  $P$  moves so that  $PA^2 + PB^2 = 24$ . Show that the locus of  $P$  is a circle and find its centre and radius. 3

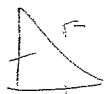
- (d) (i) Show that  $\frac{x^2}{x^2 - 3} = 1 + \frac{3}{x^2 - 3}$ . 1

- (ii) Hence, or otherwise, evaluate  $\lim_{x \rightarrow \infty} \frac{x^2}{x^2 - 3}$ . 1

**Question 3 (12 marks)**  
Start a new page.

Marks

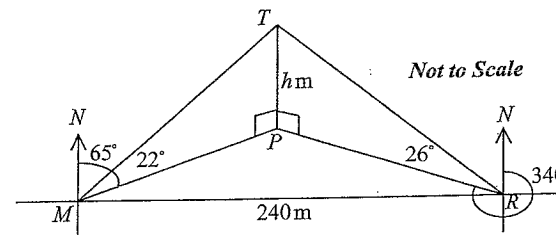
- (a) For the equation  $kx^2 - (k-1)x - (k+6) = 0$  find the value of  $k$  given one root is two times the reciprocal of the other root. 2
- (b) Differentiate the following with respect to  $x$ .
- (i)  $\frac{x^2 - 7}{x}$  2
- (ii)  $(4x + 3)^5$  2
- (iii)  $\frac{2x - 1}{x^2 + 1}$  2
- (c) (i) Sketch  $y = \cos 2x$  for  $0^\circ \leq x \leq 360^\circ$ . 1
- (ii) On the same graph sketch  $y = \frac{1}{2}$ . 1
- (iii) Solve  $\cos 2x = \frac{1}{2}$  for  $0^\circ \leq x \leq 360^\circ$ . 2



**Question 4 (12 marks)**  
Start a new page.

Marks

- (a) Solve  $(x^2 + x) + \frac{12}{x^2 + x} - 8 = 0$  for  $x$ . 3
- (b) Find the value of  $x$  on the curve  $y = \sqrt{x}$  where the gradient of the curve is 2. 2
- (c) Janice walks along a straight road. At point  $M$  she notices a tower  $PT$  on a bearing of  $065^\circ$  with an angle of elevation to the top of the tower of  $22^\circ$ . After walking 240 m to  $R$ , the tower is on a bearing of  $340^\circ$ , with an angle of elevation to the top of the tower of  $26^\circ$ .



- (i) Show that  $\angle RPM = 85^\circ$ . 1
- (ii) Show that  $MP = \frac{h}{\tan 22^\circ}$ . 1
- (iii) Find the height of the tower correct to the nearest metre. 3
- (d) (i) Show that  $\frac{1}{\sqrt{x} + \sqrt{c}} = \frac{\sqrt{x} - \sqrt{c}}{x - c}$ . 1
- (ii) Hence find  $\lim_{x \rightarrow c} \frac{\sqrt{x} - \sqrt{c}}{x - c}$ . 1

$(x-c)(\sqrt{x} + \sqrt{c})$

Marks

**Question 5 (12 marks)**  
Start a new page.

- (a) Consider the function  $f(x) = x^3 - 6x^2$ .
- (i) Find all stationary points and determine their nature. 3
- (ii) Sketch the curve  $y = f(x)$ , clearly showing the above features and the  $x$  and  $y$  intercepts. 2
- (b) Consider the polynomial  $P(x) = x^3 - x^2 - 5x - 3$ .
- (i) Show that  $(x+1)$  is a factor of  $P(x)$ . 1
- (ii) Hence, or otherwise, factorise  $P(x)$ . 2
- (c) Prove by Mathematical Induction that  $2 + 5 + 8 + \dots + (3n-1) = \frac{n}{2}(3n+1)$  for all positive integral  $n$ . 4

Marks

**Question 6 (12 marks)**  
Start a new page.

- (a) A box manufacturer wishes to construct a closed rectangular box with height,  $h$  centimetres and whose length is twice its width. Let the width be  $x$  cm. The surface area of the box is  $60 \text{ cm}^2$ .
- (i) Show that  $h = \frac{30 - 2x^2}{3x}$ . 2
- (ii) Show that the volume,  $V \text{ cm}^3$ , of the box is given by 1
- $$V = 20x - \frac{4}{3}x^3$$
- (iii) Hence find the dimensions of the box in order to maximise its volume. 3
- (b) (i) The quadratic equation  $ax^2 + bx + c = 0$  has 2 unequal roots. What condition must the discriminant satisfy for this statement to be true? 1
- (ii) Consider the circle  $x^2 + y^2 - 2x - 14y + 25 = 0$ . 2  
The line  $y = mx$  intersects the circle in 2 distinct points.  
Show that  $(1 + 7m)^2 - 25(1 + m^2) > 0$ .
- (iii) For what values of  $m$  is the line  $y = mx$  a tangent to the circle? 3

**END OF EXAMINATION**

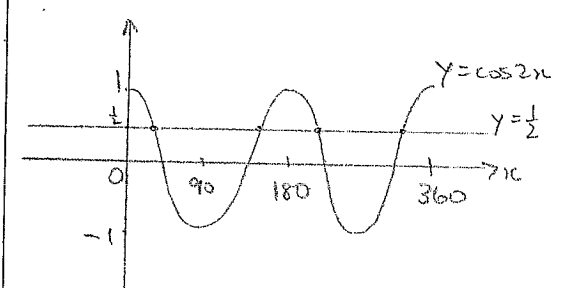
Qn	Solutions	Marks	Comments+Criteria
1	<p>(a) <math>\angle EAD = \angle ABC</math> (<math>AD \parallel BF</math>, Corresp. <math>\angle s =</math>)</p> <p><math>\angle ABC = \angle DCF</math> (<math>AB \parallel DC</math>, Corresp. <math>\angle s =</math>)</p> <p><math>\angle EDA = \angle DFC</math> (<math>AD \parallel BF</math>, Corresp. <math>\angle s =</math>)</p> <p><math>\therefore \angle DCF = \angle DFC</math></p> <p>Since base <math>\angle s =</math> in <math>\triangle DCF</math>, then</p> <p><math>\triangle DCF</math> is isosceles</p>	1  1	statements  reasons
	<p>(b) <math>x - 2y = 4</math></p> <p><math>x^2 + 2xy + 4y^2 = -6</math></p> <p><math>x^3 - 8y^3 = (x)^3 - (2y)^3</math></p> <p><math>= (x - 2y)(x^2 + 2xy + 4y^2)</math></p> <p><math>= 4x - 6</math></p> <p><math>= -24</math></p>	1  1	
	<p>(c) <math>A(1, 2)</math> <math>(3, -4)</math> <math>B(x, y)</math></p> <p><math>x_1, y_1</math> <math>x_2, y_2</math> <math>x_2, y_2</math></p> <p>ratio <math>4 : 3</math></p> <p><math>m : n</math></p> <p><math>\left( \frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)</math></p> <p><math>= \left( \frac{4x + 3 \cdot 1}{7}, \frac{4y + 3 \cdot 2}{7} \right)</math></p> <p><math>\frac{4x + 3}{7} = 3, \quad \frac{4y + 6}{7} = -4</math></p> <p><math>4x + 3 = 21, \quad 4y + 6 = -28</math></p> <p><math>4x = 18, \quad 4y = -34</math></p> <p><math>x = \frac{18}{4}</math> <math>y = -\frac{34}{4}</math></p> <p><math>= \frac{9}{2}, \quad = -\frac{17}{2}</math></p> <p><math>x = 4\frac{1}{2}, \quad y = -8\frac{1}{2}</math></p>	1	correct use of formula
		2	1mk for each soln

Qn	Solutions	Marks	Comments+Criteria
1	<p>(d) <math>f(x) = x^4 - 5x^3</math></p> <p><math>f'(x) = 4x^3 + 15x^2</math></p> <p><math>f''(x) = 12x^2 - 60x</math></p>	1  1	
	<p>(e) <math>2x^2 + 8x - 3</math></p> <p><math>= 2(x^2 + 4x) - 3</math></p> <p><math>= 2(x^2 + 4x + 4) - 3 - 8</math></p> <p><math>= 2(x + 2)^2 - 11</math> <math>\therefore</math> min. value is <math>-11</math>.</p> <p>OR <math>x = \frac{-b}{2a}</math></p> <p><math>= \frac{-8}{2 \cdot 2}</math></p> <p><math>= \frac{-8}{4}</math></p> <p><math>x = -2</math></p> <p>@ <math>x = -2, 2 \times (-2)^2 + 8 \times -2 - 3</math></p> <p><math>= 8 - 16 - 3</math></p> <p><math>= -11</math></p> <p><math>\therefore</math> min. value is <math>-11</math>.</p> <p>OR Let <math>y = 2x^2 + 8x - 3</math></p> <p><math>\frac{dy}{dx} = 4x + 8</math></p> <p>Stat pt @ <math>4x + 8 = 0</math></p> <p><math>4x = -8</math></p> <p><math>x = -2</math></p> <p><math>\frac{d^2y}{dx^2} = 4 &gt; 0 \therefore</math> min at <math>x = -2</math>.</p> <p>ie <math>y = -11</math>.</p>	1  1  1  1  1  1	

Qn	Solutions	Marks	Comments+Criteria
2	$(a) \ x = 2\sec\theta, \ y = 2\tan\theta$ $x^2 - y^2 = (2\sec\theta)^2 - (2\tan\theta)^2$ $= 4\sec^2\theta - 4\tan^2\theta$ $= 4(\sec^2\theta - (\sec^2\theta - 1))$ $= 4 \cdot (0 + 1)$ $= 4$ $\text{or } x^2 - y^2 = 4\sec^2\theta - 4\tan^2\theta$ $= 4(\tan^2 + 1) - 4\tan^2\theta$ $= 4\tan^2\theta + 4 - 4\tan^2\theta$ $= 4$	1	
	$(b) \ P(x) = x(x+1)^2(x-2)(x-3)$ <p>(i) leading term is <math>x \cdot x^2 \cdot x \cdot x</math>  <math>= x^5</math></p> <p>constant is <math>0 \cdot 1^2 \cdot -2 \cdot -3 = 0</math></p>	1 1	
	<p>(ii) zeros are <math>0, -1, 2, 3, -1</math></p>	1	
	<p>(iii)</p>	1	
	<p>(iv) <math>P(x) &gt; 0</math> for <math>0 &lt; x &lt; 2, x &gt; 3</math></p>	1	

Qn	Solutions	Marks	Comments+Criteria
2	$(c) \ A(1,1) \ P(x,y) \ B(-3,5)$ $PA^2 + PB^2 = 24$ $PA^2 = (x-1)^2 + (y-1)^2$ $= x^2 - 2x + 1 + y^2 - 2y + 1$ $= x^2 - 2x + y^2 - 2y + 2$ $PB^2 = (x+3)^2 + (y-5)^2$ $= x^2 + 6x + 9 + y^2 - 10y + 25$ $= x^2 + 6x + y^2 - 10y + 34$ $PA^2 + PB^2 = 2x^2 + 4x + 2y^2 - 12y + 36$ $2(x^2 + 2x + y^2 - 6y + 18) = 24$ $x^2 + 2x + y^2 - 6y = -6$ $(x+1)^2 + (y-3)^2 = -6 + 1 + 9$ $= 4$ <p><math>\therefore</math> The locus of P is a circle,  centre <math>(-1, 3)</math> and radius 2 units.</p>	1 1 1	
	<p>(d) (i) <math>RHS = 1 + \frac{3}{x^2-3}</math></p> $= \frac{x^2-3+3}{x^2-3}$ $= \frac{x^2}{x^2-3}$ $= LHS$	1	correct proof
	<p>(ii) <math>\lim_{x \rightarrow \infty} \frac{x^2}{x^2-3} = \lim_{x \rightarrow \infty} \left[ 1 + \frac{3}{x^2-3} \right]</math></p> $= 1 + 0$ $= 1$	1	correct proof

Qn	Solutions	Marks	Comments+Criteria
3	$(a) kx^2 - (k-1)x - (k+6) = 0$ Let the roots be $\alpha, \frac{2}{\alpha}$ . $\alpha + \frac{2}{\alpha} = \frac{-b}{a}$ $\quad = \frac{k-1}{k}$ $\alpha \cdot \frac{2}{\alpha} = \frac{c}{a}$ $\therefore 2 = \frac{-(k+6)}{k}$ $2k = -k-6$ $3k = -6$ $k = -2$	1 1	
	$(b) (i) \frac{d}{dx} \left[ \frac{x^2-7}{x} \right]$ $= \frac{d}{dx} [x - 7x^{-1}]$ $= 1 + 7x^{-2}$ $= 1 + \frac{7}{x^2}$	1 1	$\frac{d}{dx} \left[ \frac{x^2-7}{x} \right]$ $= \frac{2x(x) - 1(x^2-7)}{x^2}$ $= \frac{2x^2 - x^2 + 7}{x^2}$ $= \frac{x^2 + 7}{x^2}$
	$(ii) \frac{d}{dx} [(4x+3)^5]$ $= 5(4x+3)^4 \times 4$ $= 20(4x+3)^4$	1 1	
	$(ii) \frac{d}{dx} \left[ \frac{2x-1}{x^2+1} \right]$ $= \frac{2(x^2+1) - 2x(2x-1)}{(x^2+1)^2}$ $= \frac{2x^2+2-4x^2+2x}{(x^2+1)^2}$ $= \frac{-2x^2+2x+2}{(x^2+1)^2}$ $= \frac{-2(x^2-x-1)}{(x^2+1)^2}$	1 1 1	

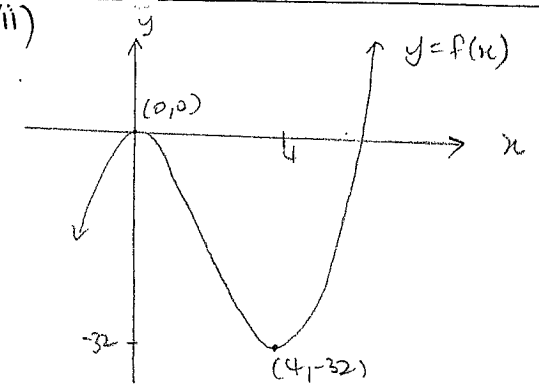
Qn	Solutions	Marks	Comments+Criteria
3	$(c) (i) y = \cos 2x, 0 \leq x \leq 360^\circ$ 	1	
	$(ii) \text{Sketch } y = \frac{1}{2}$	1	
	$(iii) \cos 2x = \frac{1}{2}, 0 \leq x \leq 360^\circ$ basic angle, $2x = 60^\circ$ $2x = 60^\circ, 300^\circ, 420^\circ, 660^\circ$ $x = 30^\circ, 150^\circ, 210^\circ, 330^\circ$	1 1	

Qn	Solutions	Marks	Comments+Criteria
4	<p>(a) <math>(x^2+x) + \frac{12}{x^2+x} - 8 = 0</math></p> <p>Let <math>m = x^2+x</math></p> <p><math>m + \frac{12}{m} - 8 = 0</math></p> <p><math>m^2 - 8m + 12 = 0</math></p> <p><math>(m-6)(m-2) = 0</math></p> <p><math>\therefore x^2+x-6=0</math> or <math>x^2+x-2=0</math></p> <p><math>(x+3)(x-2)=0</math>      <math>(x+2)(x-1)=0</math></p> <p><math>x = -3, 2</math>              <math>x = -2, 1</math></p>	1	
	<p>(b) <math>y = \sqrt{x}</math>      <math>y' = 2</math>      <math>x = ?</math></p> <p><math>y = x^{\frac{1}{2}}</math></p> <p><math>\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}</math></p> <p><math>= \frac{1}{2\sqrt{x}}</math></p> <p>when gradient is 2, <math>\frac{1}{2\sqrt{x}} = 2</math></p> <p><math>2\sqrt{x} = \frac{1}{2}</math></p> <p><math>\sqrt{x} = \frac{1}{4}</math></p> <p><math>x = \frac{1}{16}</math></p>	1	

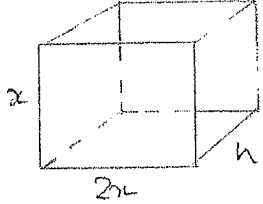
Qn	Solutions	Marks	Comments+Criteria
4	<p>(c) (i) <math>\angle RPM = 180 - (25 + 70)</math></p> <p><math>= 180 - 95</math></p> <p><math>= 85^\circ</math> (angle sum of <math>\Delta</math>)</p>	1	
	<p>(ii) <math>\tan 22 = \frac{h}{MP}</math></p> <p><math>\therefore MP = \frac{h}{\tan 22}</math></p>	1	
	<p>(iii) <math>\tan 26 = \frac{h}{PR}</math></p> <p><math>\therefore PR = \frac{h}{\tan 26}</math></p> <p><math>240^2 = PM^2 + PR^2 - 2PM \cdot PR \cos 85</math></p> <p><math>240^2 = \frac{h^2}{\tan^2 22} + \frac{h^2}{\tan^2 26} - 2 \cdot \frac{h}{\tan 22} \cdot \frac{h}{\tan 26} \cdot \cos 85</math>      ①</p> <p><math>240^2 = \frac{h^2(\tan^2 26 + \tan^2 22 - 2 \cos 85 \tan 22 \tan 26)}{\tan^2 22 \tan^2 26}</math></p> <p><math>h^2 = \frac{240^2 \tan^2 22 \tan^2 26}{\tan^2 26 + \tan^2 22 - 2 \cos 85 \tan 22 \tan 26}</math></p> <p><math>= 6098.319035</math></p> <p><math>h = 78.0917...</math></p> <p><math>h = 78m</math></p>	1	① working
	<p>(a) (i) <math>\frac{1}{\sqrt{x+c}} \times \frac{\sqrt{x}-\sqrt{c}}{\sqrt{x}-\sqrt{c}}</math></p> <p><math>= \frac{\sqrt{x}-\sqrt{c}}{x-c}</math></p>	1	working and answer
	<p>(ii) <math>\lim_{x \rightarrow c} \frac{\sqrt{x}-\sqrt{c}}{x-c} = \lim_{x \rightarrow c} \frac{1}{\sqrt{x+c}}</math></p> <p><math>= \frac{1}{\sqrt{c+c}}</math></p> <p><math>= \frac{1}{2\sqrt{c}}</math></p>	1	correct proof



Qn	Solutions	Marks	Comments+Criteria																
5	<p>(a) <math>f(x) = x^3 - 6x^2</math></p> <p>(i) <math>f'(x) = 3x^2 - 12x</math></p> <p>stat pts at <math>f'(x) = 0</math></p> <p>i.e. <math>3x(x-4) = 0</math></p> <p><math>x = 0, x = 4.</math></p> <p><math>y = 0 \quad y = -32</math></p> <p>(0,0):</p> <table style="margin-left: 20px;"> <tr><td><math>x</math></td><td>-1</td><td>0</td><td>1</td></tr> <tr><td><math>f(x)</math></td><td>15</td><td>0</td><td>-9</td></tr> </table> <p style="margin-left: 40px;">/ — \</p> <p><math>\therefore</math> max at (0,0)</p> <p>(4,-32):</p> <table style="margin-left: 20px;"> <tr><td><math>x</math></td><td>3</td><td>4</td><td>5</td></tr> <tr><td><math>f'(x)</math></td><td>-9</td><td>0</td><td>15</td></tr> </table> <p style="margin-left: 40px;">\ — /</p> <p><math>\therefore</math> min. turning pt at (4,-32)</p> <p>OR</p> <p><math>f''(x) = 6x - 12</math></p> <p>(0,0): <math>f''(0) = 0 - 12</math> <math>= -12 &lt; 0 \curvearrowright</math></p> <p><math>\therefore</math> max at (0,0)</p> <p>(4,-32): <math>f''(4) = 24 - 12</math> <math>= 12 &gt; 0 \curvearrowright</math></p> <p><math>\therefore</math> min at (4,-32).</p>	$x$	-1	0	1	$f(x)$	15	0	-9	$x$	3	4	5	$f'(x)$	-9	0	15	1  1  1  OR 1  1	           testing           testing
$x$	-1	0	1																
$f(x)$	15	0	-9																
$x$	3	4	5																
$f'(x)$	-9	0	15																

Qn	Solutions	Marks	Comments+Criteria
5 det	<p>(ii)</p> 	1  1	Curve correct labelling of features
	<p>(b) <math>P(x) = x^3 - x^2 - 5x - 3</math></p> <p>(i) <math>P(-1) = (-1)^3 - (-1)^2 - 5(-1) - 3</math> <math>= -1 - 1 + 5 - 3</math> <math>= 0</math></p> <p><math>\therefore (x+1)</math> is a factor of <math>P(x).</math></p>	1	
	<p>(ii)</p> $  \begin{array}{r}  x^2 - 2x - 3 \\  x+1 \overline{) x^3 - x^2 - 5x - 3} \\  \underline{x^3 + x^2} \phantom{- 3} \\  -2x^2 - 5x \phantom{- 3} \\  \underline{-2x^2 - 2x} \phantom{- 3} \\  -3x - 3 \\  \underline{-3x - 3} \\  0  \end{array}  $ <p><math>\therefore P(x) = (x+1)(x^2 - 2x - 3)</math> <math>= (x+1)(x-3)(x+1)</math></p>	1  1	long division

Qn	Solutions	Marks	Comments+Criteria
5	<p>(a) Prove <math>2+5+8+\dots+(3n-1) = \frac{1}{2}(3n+1)</math></p> <p><u>Step 1</u> <math>n=1</math>  LHS = 2      RHS = <math>\frac{1}{2}(3+1)</math>  <math>= \frac{1}{2} \times 4</math>  <math>= 2</math>  <math>= \text{LHS}</math></p> <p><math>\therefore</math> true for <math>n=1</math></p> <p><u>Step 2</u> Assume true for <math>n=k</math>.  ie <math>2+5+8+\dots+(3k-1) = \frac{k}{2}(3k+1)</math> (A)</p> <p><u>Step 3</u> Prove true for <math>n=k+1</math>  ie Prove <math>2+5+8+\dots+(3k-1)+(3k+2)</math>  <math>= \frac{k+1}{2}(3k+4)</math></p> <p>LHS = <math>\underbrace{2+5+8+\dots+(3k-1)}_{\frac{k}{2}(3k+1)} + (3k+2)</math>  <math>= \frac{k}{2}(3k+1) + (3k+2)</math> }</p> <p><math>= \frac{1}{2} [k(3k+1) + 2(3k+2)]</math>  </p> <p><math>= \frac{1}{2} [3k^2+k+6k+4]</math></p> <p><math>= \frac{1}{2} [3k^2+7k+4]</math></p> <p><math>= \frac{1}{2} [(3k+4)(k+1)]</math></p> <p><math>= \frac{k+1}{2}(3k+4)</math>  </p> <p><math>= \text{RHS}</math></p> <p><u>Step 4</u></p>		

Qn	Solutions	Marks	Comments+Criteria								
6	<p>(a) (i)</p>  <p><math>SA = 60 \text{ cm}^2</math></p> <p><math>SA: 2(2x^2 + 2xh + xh) = 60</math>  </p> <p><math>2x^2 + 3xh = 30</math></p> <p><math>3xh = 30 - 2x^2</math></p> <p><math>h = \frac{30 - 2x^2}{3x}</math>  </p>										
	<p>(ii) <math>V = x \cdot 2x \cdot h</math></p> <p><math>= 2x^2 \cdot \left[ \frac{30 - 2x^2}{3x} \right]</math></p> <p><math>= \frac{2x(30 - 2x^2)}{3}</math></p> <p><math>= \frac{60x}{3} - \frac{4x^3}{3}</math></p> <p><math>V = 20x - \frac{4x^3}{3}</math>  </p>										
	<p>(iii) <math>V' = 20 - 4x^2</math></p> <p>stat pts at <math>V'=0</math>, ie <math>20 - 4x^2 = 0</math></p> <p><math>4x^2 = 20</math></p> <p><math>x^2 = 5</math></p> <p><math>x &gt; 0 \therefore x = \sqrt{5}</math> only  </p> <p><math>x = \sqrt{5}</math>:</p> <table border="1" data-bbox="1422 1220 1668 1332"> <tr> <td><math>x</math></td> <td>2</td> <td><math>\sqrt{5}</math></td> <td>3</td> </tr> <tr> <td><math>V'</math></td> <td>4</td> <td>0</td> <td>-16</td> </tr> </table> <p>  <math>\therefore</math> max.  </p> <p>OR <math>V'' = -8x</math></p> <p>@ <math>x = \sqrt{5}</math>, <math>V'' = -8\sqrt{5} &lt; 0</math> <math>\checkmark</math> max</p>	$x$	2	$\sqrt{5}$	3	$V'$	4	0	-16		
$x$	2	$\sqrt{5}$	3								
$V'$	4	0	-16								

Qn	Solutions	Marks	Comments+Criteria
6 ctd	$x = \sqrt{5} \therefore$ width is $\sqrt{5}$ cm. $2x = 2\sqrt{5} \therefore$ length is $2\sqrt{5}$ cm $h = \frac{30 - 2x^2}{3x}$ $= \frac{30 - 2.5}{3\sqrt{5}}$ $= \frac{20}{3\sqrt{5}} \text{ cm}$ ie $h = \frac{20\sqrt{5}}{15}$ $= \frac{4\sqrt{5}}{3} \text{ cm}$	1	for $2x, h$
	(b)(i) $\Delta > 0$	1	
	(ii) $x^2 + y^2 - 2x - 14y + 25 = 0$ } ① $y = mx$ } ② ② into ①: $x^2 + (mx)^2 - 2x - 14 \cdot mx + 25 = 0$ $x^2 + m^2x^2 - 2x - 14mx + 25 = 0$ $x^2(1+m^2) - 2x(1+7m) + 25 = 0$ Line intersects twice, $\therefore \Delta > 0$ ie $b^2 - 4ac > 0$ $[2(1+7m)]^2 - 4(1+m^2) \cdot 25 > 0$ $4(1+7m)^2 - 4 \cdot 25(1+m^2) > 0$ ie $(1+7m)^2 - 25(1+m^2) > 0$	1	
	(iii) $y = mx$ a tangent if $\Delta = 0$ . ie $(1+7m)^2 - 25(1+m^2) = 0$	1	

Qn	Solutions	Marks	Comments+Criteria
6 ctd	$1 + 14m + 49m^2 - 25 - 25m^2 = 0$ $24m^2 + 14m - 24 = 0$ $12m^2 + 7m - 12 = 0$ $m = \frac{-7 \pm \sqrt{49 - 4 \cdot 12 \cdot 12}}{2 \cdot 12}$ $= \frac{-7 \pm \sqrt{625}}{24}$ $= \frac{-7 \pm 25}{24}$ $m = \frac{-7 + 25}{24} \quad \vee \quad m = \frac{-7 - 25}{24}$ $m = \frac{18}{24} \quad \quad \quad m = \frac{-32}{24}$ $= \frac{6}{8} \quad \quad \quad = \frac{-4}{3}$ $m = \frac{3}{4} \quad \quad \quad \vee \quad m = -\frac{1}{3}$	1	