15242112

ESTO TESTIS SOL

KAMBALA

Extension 2 Mathematics

YEAR 12 HALF-YEARLY EXAMINATION

April 2005

Time Allowed: 2 hours
Reading Time: 5 minutes

INSTRUCTIONS

- This examination contains 5 questions of 15 marks each. Marks for each part question are shown.
- Answer all questions on the writing paper provided. Start each question on a new page.
- Calculators may be used.
- Show all necessary working.
- Marks may be deducted for careless or badly arranged work.

Question 1 *(Start a new page.)* **15 marks**

(a) Given $z_1 = \dfrac{7+4i}{3-2i}$

(i) Express z_1 in the form $x+iy$ where x and y are real **2**

(ii) On an Argand diagram sketch the locus of the point representing the complex number z such that $|z - z_1| = \sqrt{5}$ **2**

(iii) Find the greatest value of $|z|$ subject to the condition in (ii) **1**

(b) The complex number z and its conjugate $\overline{z}$ both satisfy the equation

$$z\overline{z} + 2z = \frac{1}{4} + i.$$

By equating the real and imaginary parts or otherwise, find the possible values of z **3**

(c) The complex number ω is given by $\omega = \sqrt{3} - i$.

(i) On the same Argand diagram plot ω and ω^4 **2**

(ii) Show that ω^4 can be represented by the complex number $ki\omega$, where k is real. Hence find the value of k **2**

(d) z is the complex number $z = x + iy$. On a suitably labelled Argand diagram sketch the region of the complex plane which satisfies both the conditions

$$|z-2-i| = 2 \text{ and } \frac{\pi}{4} \le Arg z \le \frac{\pi}{3}$$ **3**

$\frac{\sin\theta}{\cos\theta}$

$8i =$

$\tan\theta$

$\frac{\cos\theta}{\sin\theta}$

$2 \; \text{cis}\left(\frac{-\pi}{6}\right)$

$8i \left(2\,\text{cis}\left(\frac{-\pi}{6}\right)\right)$

$= 8 \times 2.\; i\,\text{cis}\left(\frac{-\pi}{6}\right)$

$= 16$

Question 2 *(Start a new page.)* **15 marks**

(a) Find $\int \sin^3 2x dx$ 3

(b) Use integration by parts to find $\int xe^{2x} dx$ 3

(c) Find $\int \operatorname{cosec} x dx$ using the substitution $t = \tan\frac{x}{2}$ 3

(d) Given that $I_n = \int_0^1 x^n (1-x)^{\frac{1}{2}} dx$

find $I_{n-1} = \int_0^1 x^{n-1} (1-x)^{\frac{1}{2}}$

(i) Show that $I_n = \dfrac{2n}{2n+3} I_{n-1}$

(Note that $\sqrt{(1-x)^3} = (1-x)\sqrt{1-x}$) 4

(ii) Hence or otherwise find $I_2 = \int_0^1 x^2 (1-x)^{\frac{1}{2}} dx$ 2

2:05

$1 + \cancel{x^2} + 1 - \cancel{x^2}$

$\frac{4\pi}{6}$

$(ax+b)^n \to \dfrac{(ax+b)^{n+1}}{a(n+1)}$

$\cos^2 x = \frac{1}{2}(1+\cos 2x)$

$-\frac{2}{3}$

Question 3 (*Start a new page.*) **15 marks**

(a) The complex number $z = x + iy$ is such that $|z - i| = \operatorname{Im} z$

(i) Show that the locus of z has cartesian equation $y = \dfrac{x^2 + 1}{2}$ **2**

(ii) Sketch this locus and the vector $\vec{z}$ which has the smallest positive argument α **2**

(iii) What is the size of α?
(you may like to look at the gradient of the tangent to the curve through the origin) **2**

(b) Solve the inequality $|1 - 2x| \leq 1 - |x|$ **3**

2 : .25

(c) (i) Use the t results or otherwise to show that $\sqrt{\dfrac{1 + \cos x}{1 - \cos x}} = \cot \dfrac{x}{2}$

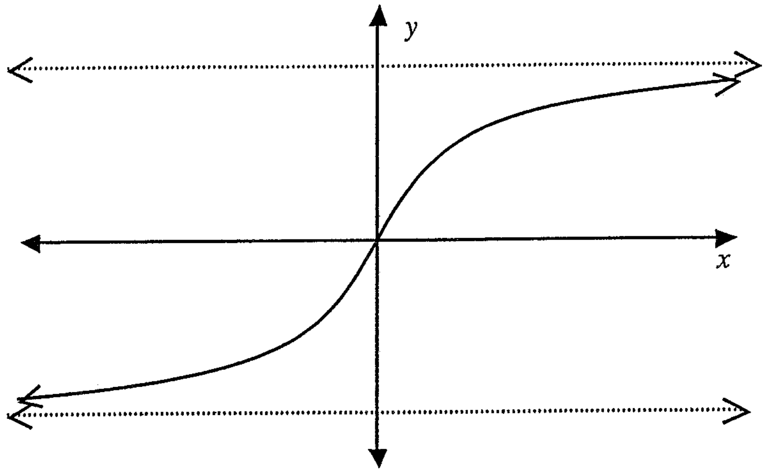
Hence show that $\cot \dfrac{3\pi}{8} = \sqrt{2} - 1$ **4**

(ii) Hence show that $\tan \dfrac{3\pi}{8} - \cot \dfrac{3\pi}{8} = 2$ **2**

$\frac{6\pi}{8} = \frac{3\pi}{4}$

Question 4 (*Start a new page.*) **15 marks**

(a) The diagram shows the graph of $f(x) = \dfrac{e^x - 1}{e^x + 1}$



Without using calculus, use the graph of $f(x)$ to sketch, on separate axes:

(i) $y = \dfrac{e^{-x} - 1}{e^{-x} + 1}$ **1**

(ii) $y^2 = \dfrac{e^x - 1}{e^x + 1}$ **2**

(iii) $y = \dfrac{e^x + 1}{e^x - 1}$ **2**

(b) Consider the function given by $f(x) = \dfrac{1 - |x|}{|x|}$

$x \to 0$

(i) Find whether $f(x)$ is an odd function, an even function or neither. **1**

(ii) Sketch $f(x)$ **2**

(iii) Hence or otherwise solve $f(x) \geq 1$ **1**

(iv) Sketch $y = \dfrac{1}{f(x)}$ **2**

(v) Hence or otherwise solve $\dfrac{1}{f(x)} \leq 1$ **2**

(vi) Sketch $y = e^{f(x)}$ **2**

0.36791
0.3678

Question 5 *(Start a new page.)* **15 marks**

(a) A conic has x-intercept a and eccentricity $\frac{\sqrt{3}}{2}$.

Give the equation(s) of all such conics and provide a simple sketch for each **2**

(b) A hyperbola H has equation $\frac{x^2}{16} - \frac{y^2}{9} = 1$

(i) Sketch H showing the coordinates of its foci, its directrices and its asymptotes **4**

(ii) A point $P(4\sec\theta, 3\tan\theta)$ lies on H.
Show that the product of the lengths of the perpendiculars from P to the asymptotes of H is independent of the position of P. **3**

(c) A hyperbola with eccentricity e is given by the equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

(i) Show that the normal to this hyperbola at $P(a\sec\theta, b\tan\theta)$ is given by $ax\sin\theta + by = (a^2 + b^2)\tan\theta$ **3**

This normal meets the x-axis at G. PN is the perpendicular from P to the x-axis and O is the origin.

(ii) Prove that $OG = e^2 ON$ **3**

$|a| \times |b|$

$\frac{\sin\theta}{\cos\theta} \div \frac{1}{\cos\theta}$

$= \frac{\sin\theta}{\cancel{\cos\theta}} \times \frac{\cancel{\cos\theta}}{1}$

END OF EXAMINATION

1.45

Q1

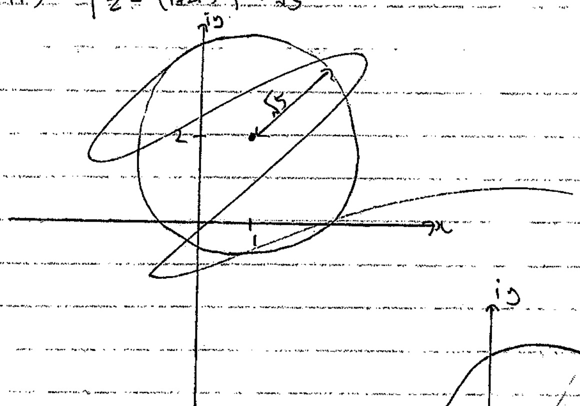
a) i) $z_1 = \frac{(7+4i)(3+2i)}{9-4}$

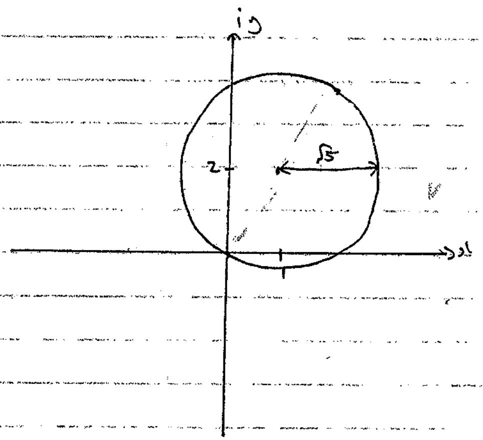
$= \frac{21+14i+12i-8}{13}$

$= \frac{13}{13} + \frac{26i}{13}$

$= 1 + 2i$

ii) $|z-(1+2i)| = \sqrt{5}$





iii) Greatest value of $|z| = 2\sqrt{5}$

b) $z\bar{z} + 2z = \frac{1}{4} + i$

$x^2+y^2 + 2x + 2iy = \frac{1}{4} + i$

$x^2+y^2+2x - \frac{1}{4} + 2iy - i = 0$

Equating Re:/Im..

$2y - 1 = 0$

$2y = 1$

$y = \frac{1}{2}$

$x^2+y^2+2x - \frac{1}{4} = 0$

$x^2 + \frac{1}{4} + 2x - \frac{1}{4} = 0$

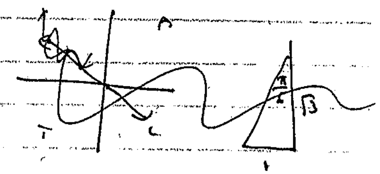
$x(x+2) = 0$

$x = 0$ or -2

c) i) $w = \sqrt{3} - i$

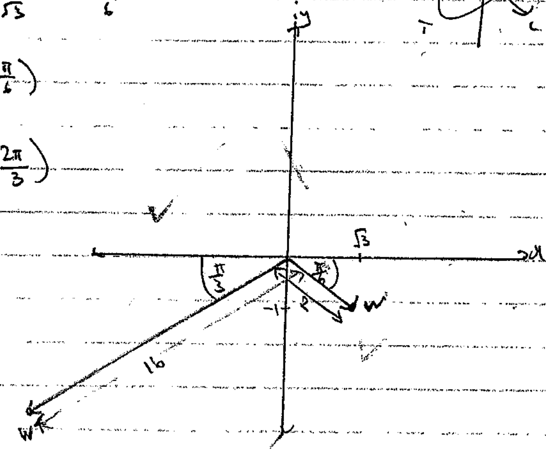
$r = \sqrt{3+1} = 2$

$\theta = \tan^{-1}\frac{-1}{\sqrt{3}} = -\frac{\pi}{6}$



$w = 2\,\text{cis}\left(-\frac{\pi}{6}\right)$

$w^4 = 16\,\text{cis}\left(-\frac{2\pi}{3}\right)$



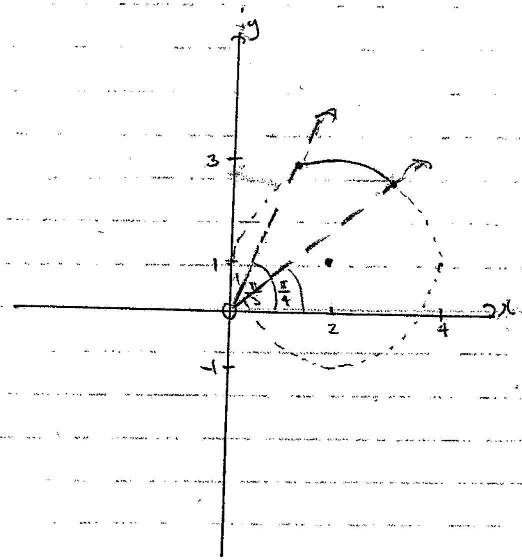
ii) $\angle wOw^4 = \pi - \left(\frac{\pi}{3}+\frac{\pi}{6}\right)$

$= \frac{\pi}{2}$ (if O is the origin)

So w^4 can be obtained by multiplying w by $\text{cis}\frac{\pi}{2} = i$ and a real number k ie $w^4 = kiw$ (as multiplying by $\text{cis}\,\theta$ rotates the vector θ radians anticlockwise)

$k = 8$

d)



$|z - (2+i)| = 2$

$\frac{\pi}{4} \le \text{Arg}\, z \le \frac{\pi}{3}$

Q2

a) ~~$\int \sin^2 2x \sin$~~

$I = \int \sin^2 2x \cdot \sin 2x \, dx$

$I = \int (1-\cos^2 2x)\sin 2x \, dx$

$\int \cos x$ $\quad \int \sin x \to -\cos x$

Let $u = \cos 2x$

$\frac{du}{dx} = -2\sin 2x$

$\therefore I = -\frac{1}{2}\int (1-u^2)\frac{du}{dx}\,\cancel{dx}$

$= -\frac{1}{2}\left[u - \frac{u^3}{3}\right]$

$= -\frac{u}{2} + \frac{u^3}{6}$

$= \frac{-\cos 2x}{2} + \frac{\cos^3 2x}{6} + c$

b) $I = \int x e^{2x}\, dx$

Let $u = x$ $\quad$ Let $\frac{dv}{dx} = e^{2x}$

$\frac{du}{dx} = 1$

~~$du = dx$~~

Let $\frac{dv}{dx} = x$

Let $u = e^{2x}$

$\frac{du}{dx} = 2e^{2x}$ $\quad v = \frac{x^2}{2}$

$du = 2e^{2x}\,dx$

$I = e^{2x}\cdot\frac{x^2}{2} - \int \frac{x^2}{\cancel{2}}\cdot 2e^{2x}\, dx$

$= e^{2x}\cdot\frac{x^2}{2} -$

b) $I = \int x e^{2x}\, dx$

let $u = x$, $du = dx$

let $\frac{dv}{dx} = e^{2x}$

$v = \int e^{2x}\, dx$

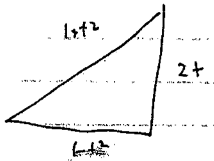
$= \frac{1}{2}\int 2e^{2x}\, dx$

$= \frac{1}{2}e^{2x}$

$I = \frac{xe^{2x}}{2} - \frac{1}{2}\int e^{2x}\, dx$

$= \frac{xe^{2x}}{2} - \frac{1}{4}e^{2x} + c$

c) let $t = \tan\frac{x}{2}$



$\cos x = \frac{1-t^2}{1+t^2}$ $\sin x = \frac{2t}{1+t^2}$, $\operatorname{cosec} x = \frac{1+t^2}{2t}$

$\frac{dt}{dx} = 2\sec^2\frac{x}{2}$ $\int \sec^2 x \to \tan x$

$\frac{dx}{dt} = \frac{1}{2}\cos^2\frac{x}{2}$

$dx = \frac{1}{2}\cos^2\frac{x}{2}\, dt$ $\cos^2\frac{x}{2} = \frac{1}{2}(1+\cos x)$

$dx = \frac{1}{2}\cdot\frac{1}{2}(1+\cos x)\cdot dt$

$= \frac{1}{4}(1+\cos x)\, dt$

$= \frac{1}{4}\left(1+\frac{1-t^2}{1+t^2}\right) dt$

$= \frac{1}{4}\left(\frac{2}{1+t^2}\right) dt$

$= \frac{1}{2(1+t^2)}\, dt$

Q2

$\therefore I = \int \operatorname{cosec} x\, dx$

$= \int \frac{\cancel{1+t^2}}{2t}\cdot\frac{1}{2\cancel{(1+t^2)}}\, dt$

$= \int \frac{1}{4t}\, dt$

$= \frac{1}{4}\int\frac{1}{t}\, dt$

$= \frac{1}{4}\ln t$

$= \frac{1}{4}\ln\left|\tan\frac{x}{2}\right| + c$

d) i) $I_n = \int_0^1 x^n (1-x)^{\frac{1}{2}}\, dx$

let $u = x^n$, $\frac{du}{dx} = nx^{n-1}$, $du = nx^{n-1}\, dx$

$\frac{dv}{dx} = (1-x)^{\frac{1}{2}}$

$v = \frac{(1-x)^{\frac{3}{2}}}{-1\left(\frac{3}{2}\right)}$

$= \frac{-2(1-x)\sqrt{1-x}}{3}$

$I_n = \left[\frac{-2(1-x)\sqrt{1-x}}{3}\cdot x^n\right]_0^1 - \int_0^1 -2\frac{(1-x)\sqrt{1-x}}{3}\cdot nx^{n-1}\, dx$

$= \cancel{0 - \frac{-2(1)(0)}{3}}\ 0 + \frac{2n}{3}\int_0^1 (1-x)^{\frac{3}{2}}\cdot x^{n-1}\, dx$

$I_n = \frac{2n}{3}\underbrace{\int_0^1 (1-x)^{\frac{3}{2}}\cdot x^{n-1}\, dx}_{I_2}$

$= \frac{2n}{3}\int_0^1 (1-x)(1-x)^{\frac{1}{2}}x^{n-1}\, dx$

Q2

let $I_2 = \int_0^1 (1-x)^{\frac{3}{2}} \cdot x^{n-1}\, dx$

let $u = (1-x)^{\frac{3}{2}}$ $\quad \frac{dv}{dx} = x^{n-1}$

$\frac{du}{dx} = \frac{3}{2}(1-x)^{\frac{1}{2}}$ $\quad v = \frac{x^n}{n}$

$$I_2 = \left[\frac{x^n (1-x)^{\frac{3}{2}}}{n}\right]_0^1 - \frac{1}{n}\int_0^1 x^n \cdot \frac{3}{2}(1-x)^{\frac{1}{2}}\, dx$$

$$= \left[\frac{x^n (1-x)^{\frac{3}{2}}}{n}\right]_0^1 - \frac{3}{2n}\int_0^1 x^n (1-x)^{\frac{1}{2}}\, dx$$

$$= \left[\frac{x^n (1-x)^{\frac{3}{2}}}{n}\right]_0^1 - \frac{3}{2n} I_n$$

$= -\frac{3}{2n} I_n$

$\therefore \cancel{I_n = \frac{2}{3} + \frac{2n}{3} \times -\frac{3}{2n} I_n}$

$\cancel{I_n = \frac{2}{3}n - I_n}$

$\rightarrow 0 - I = -\frac{3}{2n} I_n$

$\therefore I_n = \frac{2n}{3} \times -\frac{3}{2n} I_n$

$I_n = -I_n$

Q2

d) ii) $I_2 = \frac{4}{4+3}(I_1)$

$I_1 = \int_0^1 x(1-x)\, dx$

$= \int_0^1 x - x^2\, dx$

$= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$

$= \frac{1}{2} - \frac{1}{3}$

$= \frac{1}{6}$

$\therefore I_2 = \frac{1}{6} \times \frac{4}{7}$

$= \frac{2}{21}$

Q3.

a) i) $|z - i| = \operatorname{Im} z$

$y = |x + iy - i|$

$= |x + i(y-1)|$

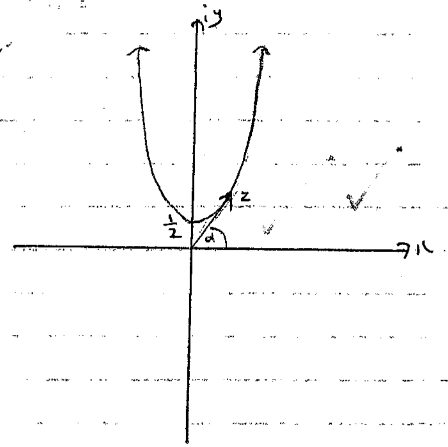
$y = \sqrt{x^2 + (y-1)^2}$

$\cancel{y^2} = x^2 + \cancel{y^2} - 2y + 1$

$2y = x^2 + 1$

$y = \dfrac{x^2+1}{2}$

ii)



$\frac{x^2}{2} + \frac{1}{2}$

iii) Gradient of a tangent to $y = \frac{x^2}{2} + \frac{1}{2}$

$\frac{dy}{dx} = x$

Equation of tangent through origin?

~~Tangent line through (0,0) $(x, \frac{x^2+1}{2})$~~

A tangent to $y = \frac{x^2+1}{2}$ at $P(x_0, y_0)$ has gradient x_0

so $y - y_0 = x_0(x - x_0)$

$y = x_0 x - x_0^2 + y_0$

Q3 a) iii)

If it also goes through $(0,0)$

$0 = -x_0^2 + y_0$

$x_0^2 = y_0$

so $x_0^2 = y$

and $\frac{x_0^2+1}{2} = y$

$\therefore x_0^2 = \frac{x_0^2+1}{2}$

$2x_0^2 = x_0^2 + 1$

$x_0^2 - 1 = 0$

$x_0 = \pm 1$

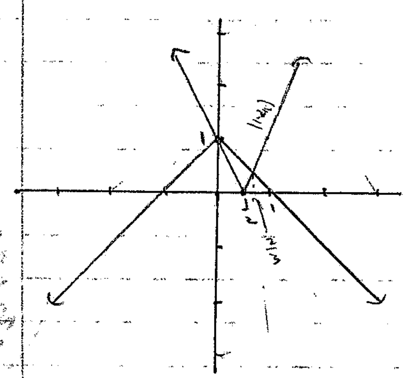
from diagram, $x_0 = 1$

$\therefore y_0 = \frac{1+1}{2} = 1$

$\therefore$ Gradient of tangent $= \frac{1}{1} = 1$

$\therefore \angle\alpha = \frac{\pi}{4}$

b) $|1-2x| \le 1 - |x|$



$|1-2x| = 1 - |x|$

if $x \ge 0$ and $1-2x < 0$

$1 < 2x$

$\frac{1}{2} < x$

then $2x - 1 = 1 - x$

$3x = 2$

$x = \frac{2}{3}$

$\therefore$ Answer: $0 \le x \le \frac{2}{3}$

Q3

c) i) $t = \tan\frac{x}{2}$

$\sin x = \frac{2t}{1+t^2}$ $\quad \cos x = \frac{1-t^2}{1+t^2}$ $\quad$ ~~tanx~~ $\cot x = \frac{1}{t}$

$$LHS = \sqrt{\frac{1+\cos x}{1-\cos x}}$$

$$= \sqrt{\frac{1+\frac{1-t^2}{1+t^2}}{1-\frac{1-t^2}{1+t^2}}}$$

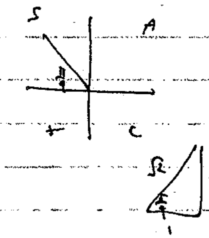
$$= \sqrt{\frac{2}{1+t^2} \div \frac{2t^2}{1+t^2}}$$

$$= \sqrt{\frac{2}{\cancel{1+t^2}} \times \frac{\cancel{1+t^2}}{2t^2}}$$

$$= \sqrt{\frac{1}{t^2}}$$

$$= \frac{1}{t}$$

$$= \cot x = RHS$$



iii) $\therefore \cot\frac{3\pi}{8} = \sqrt{\frac{1+\cos\frac{3\pi}{4}}{1-\cos\frac{3\pi}{4}}}$

$$= \sqrt{\frac{1-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}}$$

$$= \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}} \div \frac{\sqrt{2}+1}{\sqrt{2}}}$$

$$= \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$$

Q3 cont...

c) iii)

$$= \sqrt{\frac{(\sqrt{2}-1)^2}{2-1}}$$

$$= \sqrt{2}-1$$

ii) $\tan\frac{3\pi}{8} = \frac{1}{\cot\frac{3\pi}{8}} = \frac{1}{\sqrt{2}-1}$

$LHS = \cancel{\sqrt{2}-1 - \frac{1}{\sqrt{2}-1}}$ $\quad \frac{1}{\sqrt{2}-1} - (\sqrt{2}-1)$

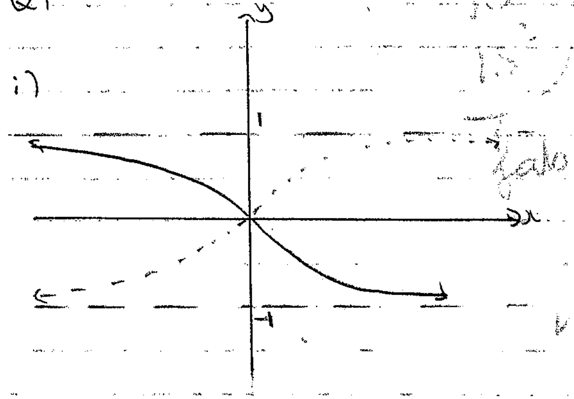
$\cancel{= \sqrt{2}-1 - \frac{\sqrt{2}+1}{2-1}}$

$$= \sqrt{2}+1-\sqrt{2}+1$$

$$= 2 = RHS$$

Q4

a) i)



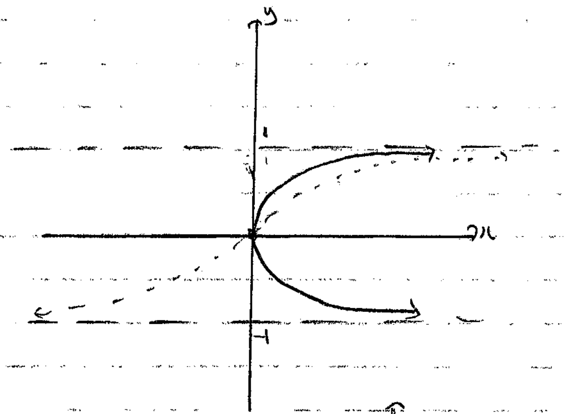
$x \to \infty$

$f(x) \to -1$

$x \to -\infty$

$f(x) \to 1^{+}$

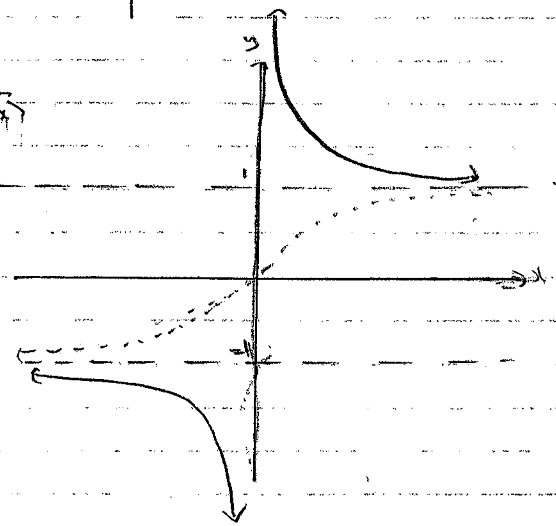
ii)



$y^2 = f(x)$

$y = \pm\sqrt{f(x)}$

iii) $y = \frac{1}{f(x)}$

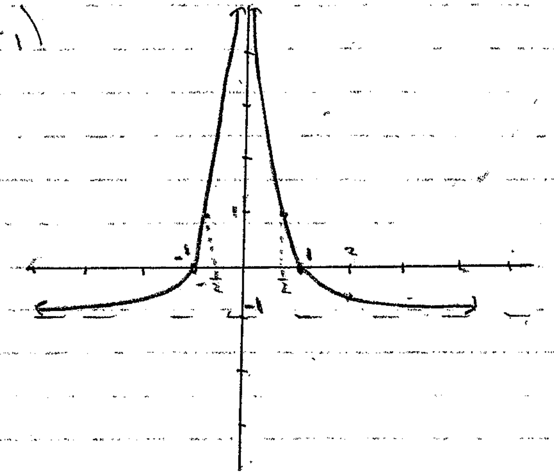


b) i) ? $f(-x) = f(x)$

$\frac{1-|x|}{|x|} = \frac{1-|-x|}{|-x|}$

$\therefore f(x)$ is even

ii)



$x \to 0 \quad y \to \infty$

$x \to \infty$

$f(x) \to \frac{-\infty}{\infty}$

$= -1$

iii) $f(x) \geq 1$?

$f(x) = 1$

$1 = \frac{1-|x|}{|x|}$

$|x| = 1-|x|$

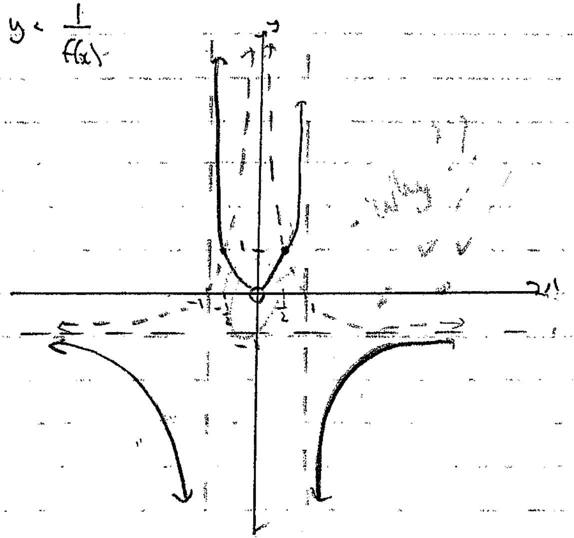
$2|x| = 1$

$|x| = \frac{1}{2}$

$\therefore f(x) \geq 1$ when $-\frac{1}{2} \leq x \leq \frac{1}{2}$, $x \neq 0$

Q4

b) iv) $y = \frac{1}{f(x)}$



$f(x) \to -1^+, y \to \frac{1}{-1^+}$

$= -1^-$

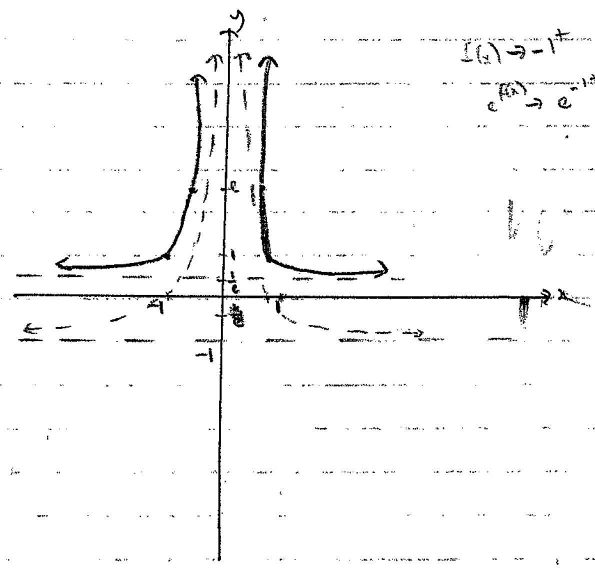
$f(x) \to 0^+, y \to -\infty$

$f(x) \to \infty, y \to 0$

v) $\frac{1}{f(x)} \leq 1$

$x > 1, \quad x < -1 \quad$ or $\quad -\frac{1}{2} \leq x \leq \frac{1}{2}, \ x \neq 0$

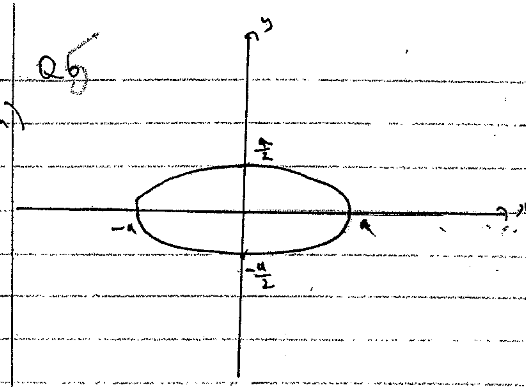
vi) $y = e^{f(x)}$



$f(x) \to -1^+$

$e^{f(x)} \to e^{-1^+}$

Q6 a)



15/15

$b^2 = a^2(1-e^2)$

$b^2 = a^2(1-\frac{3}{4})$

$b^2 = \frac{a^2}{4}$

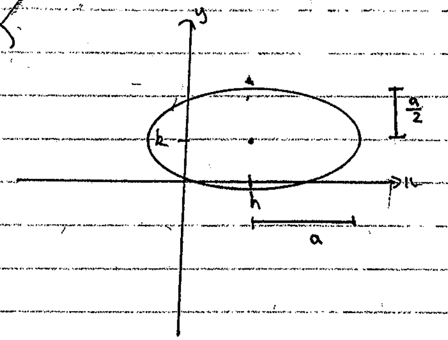
$b = \frac{a}{2}$

Equation: $\frac{x^2}{a^2} + \frac{y^2}{\left(\frac{a^2}{4}\right)} = 1$

Or if the centre is not at $(0,0)$

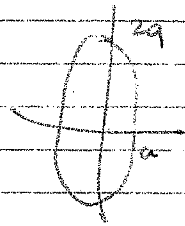
~~or for a hyperbola~~

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{\left(\frac{a^2}{4}\right)} = 1$$



2/2

b)(ii)



$\frac{x^2}{a^2} + \frac{y^2}{(2a)^2} = 1$?

Q65

b) i) $\frac{x^2}{16} - \frac{y^2}{9} = 1$ $a = 4,\ b = 3$

$9 = 16(e^2-1)$

$\frac{9}{16} = e^2 - 1$

$\frac{25}{16} = e^2$

$\frac{5}{4} = e$

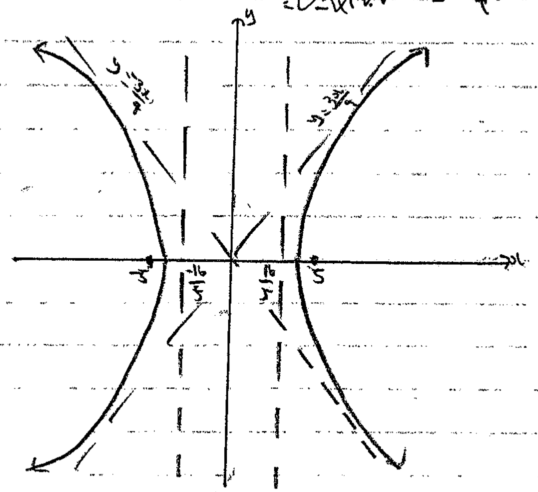
Foci $(5,0)$ $(-5,0)$

Directrices : $x = \pm 4 \div \frac{5}{4}$

$= \pm \frac{16}{5}$

Asymptotes : $y = \pm \frac{b}{a}x$

$= \pm \frac{3}{4}x$



Q6

b ii) Asymptotes : $y = \frac{3}{4}x$ or $y = -\frac{3}{4}x$

$4y - 3x = 0$ or $4y + 3x = 0$

Perpendiculars from P to asymptotes : let lengths = A, B

$A = \frac{|4.3\tan\theta - 3.4\sec\theta|}{\sqrt{25}}$ $B = \frac{|4.3\tan\theta + 3.4\sec\theta|}{\sqrt{25}}$

$AB = \frac{|12\tan\theta - 12\sec\theta|}{5} \times \frac{|12\tan\theta + 12\sec\theta|}{5}$

$= \frac{12.12\,|\tan\theta - \sec\theta|\,|\tan\theta + \sec\theta|}{25}$

$= \frac{144}{25}|\tan^2\theta - \sec^2\theta|$

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$\sec^2\theta - \tan^2\theta = 1$

$|\tan^2\theta - \sec^2\theta| = 1$

$AB = \frac{144}{25}$

Expand ✓

which is independent of position of P

but add this

$\frac{3}{3}$

Q5

c) i) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$\frac{2x}{a^2} - \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{2x}{a^2} \div \frac{2y}{b^2}$$

$$= \frac{\cancel{2}x}{a^2} \times \frac{b^2}{\cancel{2}y}$$

$$= \frac{b^2x}{a^2y}$$

$\therefore$ ~~Att~~ At P, gradient of tangent $= \frac{b^{\cancel{2}} a\sec\theta}{a^{\cancel{2}}\cancel{b}\tan\theta}$

$$= \frac{b\sec\theta}{a\tan\theta}$$

$\therefore m_{normal} = \frac{-a\tan\theta}{b\sec\theta}$

$\therefore$ Equation of normal: $y - b\tan\theta = \frac{-a\tan\theta}{b\sec\theta}(x - a\sec\theta)$

$$y - \frac{b\sin\theta}{\cos\theta} = \frac{-a}{b}\sin\theta\left(x - \frac{a}{\cos\theta}\right)$$

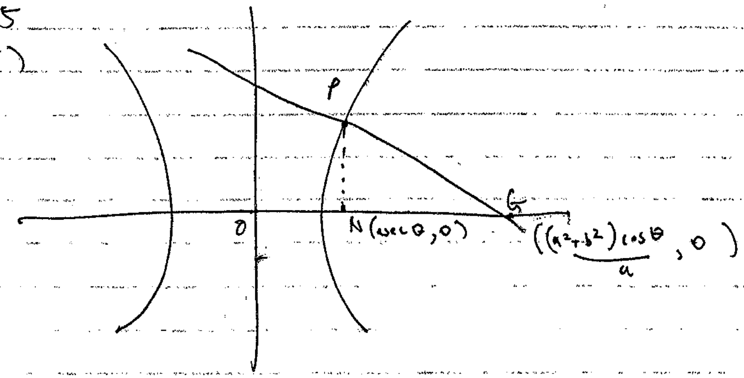
$\cancel{by - \frac{b^2\sin^2\theta}{\cos\theta}}$

$$by - b^2\tan\theta = -ax\sin\theta + a^2\tan\theta$$

$$ax\sin\theta + by = (a^2+b^2)\tan\theta$$

Q5

c) ii)



$ax\sin\theta + by = (a^2+b^2)\tan\theta$

let $y = 0$

$$ax\sin\theta = (a^2+b^2)\tan\theta$$

$$x = \frac{(a^2+b^2)\tan\theta}{a\sin\theta}$$

$$= \frac{(a^2+b^2)\cancel{\cos\theta}}{a\cos\theta}$$

$$OG = \frac{(a^2+b^2)\cancel{\cos\theta}}{a\cos\theta} = \frac{(a^2+b^2)}{a}\sec\theta$$

$$ON = a\sec\theta$$

$$b^2 = a^2(e^2-1)$$

$$\frac{b^2}{a^2} = e^2 - 1$$

$$\frac{b^2+a^2}{a^2} = e^2$$

$$RHS = e^2 ON = \frac{b^2+a^2}{a^2} \times a\sec\theta$$

$$= \frac{(b^2+a^2)}{a}\sec\theta$$

$$= OG = LHS$$