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100%

KAMBALA

MATHEMATICS

Assessment Task #3

Monday 31st May 2004

Time allowed: *70 minutes*

INSTRUCTIONS

- There are 4 questions of equal value.
- Marks for each part of a question are indicated
- All questions should be attempted.
- All necessary working should be shown
- Start each question on a new page
- Approved scientific calculators and drawing templates may be used

Question 1 (START A NEW PAGE) 12 MARKS

(a) Evaluate e^{-3} (to 3 significant figures) [1]

(b) Evaluate $\log_2 16 - \log_3 \frac{1}{9}$ [2]

(c) Simplify $3\log_a(x-y) - \log_a(x+y)$ [2]

(d) Differentiate

(i) $y = e^{2x-3}$ [1]

(ii) $y = \frac{\ln x}{e^x}$ [2]

(iii) $y = \left(e^{-x} + e^x\right)^3$ [2]

(e) Find $\int 5e^{2x+1}dx$ [2]

Question 2 (START A NEW PAGE) 12 MARKS

(a) Find the **equation** of the tangent to the curve $f(x) = x\log_e x$ at the point where $x = 1$. Leave your answer in general form. [3]

(b) If $f(x) = \log_e\left(\frac{x}{2-x}\right)$ find $f'(x)$ [2]

(c) What is the domain of the function $y = \log_e(x-2)$? [1]

(d) Find the volume of the solid of revolution formed when the curve $y = e^{-\frac{1}{2}x}$ from $x = 0$ to $x = 1$ is rotated about the x axis.
Leave your answer in exact form. [3]

(e) Given that $\int \frac{1}{x}dx = \ln x + c$
Show that the **exact** area under the curve $y = \frac{1}{2x}$ between $x = 1$ and $x = 2$ is $\ln\sqrt{2}$ units^2 [3]

Question 3 (START A NEW PAGE) **12 MARKS**

(a) (i) Show that $\int_1^k 2xe^{x^2-1}dx = e^{k^2-1} - 1$ [2]

(ii) Hence find the values of k for which $\int_1^k 2xe^{x^2-1}dx = e^7 - 1$ [2]
(in simplest exact form)

Question 3 continues over the page

(b) Verity, Constance and Prudence are asked to approximate the area under the curve $f(x) = xe^x$ from $x = 1$ to $x = 3$.

(i) Verity decides she will use the Trapezoidal rule with three function values to approximate the area. Show her working and answer, leaving your answer in terms of *e*. [3]

(ii) Constance uses Simpson's Rule once to approximate the answer. Show her working and answer, leaving your answer in terms of *e*. [3]

(iii) Prudence decides to work out the difference between her friends' two values. What is the difference between the two values? (to 2 decimal places) [2]

Question 4 (START A NEW PAGE) 12 MARKS

The function $f(x)$ is given by $f(x) = xe^x$

(a) Show that $f'(x) = e^x(x+1)$ [2]

(b) Find any stationary point(s) and determine their nature [2]

(c) Show that $f(x)$ has a point of inflexion at $x = -2$ [2]

(d) What happens to the curve as x get large? (ie what happens as $x \to +\infty$ and $x \to -\infty$) [2]

(e) Sketch $f(x)$ for $-3 \le x \le 1$ [3]

(f) On your sketch, shade in the region where the inequalities $y \ge xe^x$ and $x \ge 0$ **both** apply [1]

END OF TEST

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Qn	Solutions	Marks	Comments+Criteria
1(a)	$e^{-3} = \frac{1}{e^3} = 0.049787$ $\doteqdot 0.0498$ (3sf)	✓ (1)	$\frac{1}{2}$ off if not to 3sf
(b)	$\log_2 16 - \log_3 \frac{1}{9}$ $= 4 - (-2)$ $= 6$	✓✓ (2)	1 mark each
(c)	$3\log_a(x-y) - \log_a(x+y)$ $= \log_a(x-y)^3 - \log_a(x+y)$ $= \log_a \frac{(x-y)^3}{(x+y)}$	✓ ✓ (2)	
(d)	(i) $y = e^{2x-3}$ $y' = 2e^{2x-3}$	✓ 1	
	(ii) $y = \frac{\ln x}{e^x} = \frac{u}{v}$ $y' = \frac{e^x \cdot \frac{1}{x} - \ln x \cdot e^x}{(e^x)^2}$ $= \frac{\frac{e^x}{x} - e^x \ln x}{e^{2x}} \left[= \frac{e^x\left(\frac{1}{x} - \ln x\right)}{e^{2x}}\right]$	✓✓ 2	1 for recognition of quotient and a start on the process $\rightarrow \frac{\frac{1}{x} - \ln x}{e^x}$ or $\frac{1 - x\ln x}{x e^x}$
	(iii) $y = (e^{-x} + e^x)^3$ $y' = 3(e^{-x}+e^x)^2 \cdot (-e^{-x} + e^x)$ $= 3(e^{-x}+e^x)^2(e^x - e^{-x})$	✓✓ 2 (5)	
(e)	$\int 5e^{2x+1}\,dx$ $= 5\int e^{2x+1}\,dx$ $= \frac{5}{2}e^{2x+1} + C$	✓✓ (2)	For 2 mks must be $\frac{5}{2}$ not $\frac{1}{2}5$

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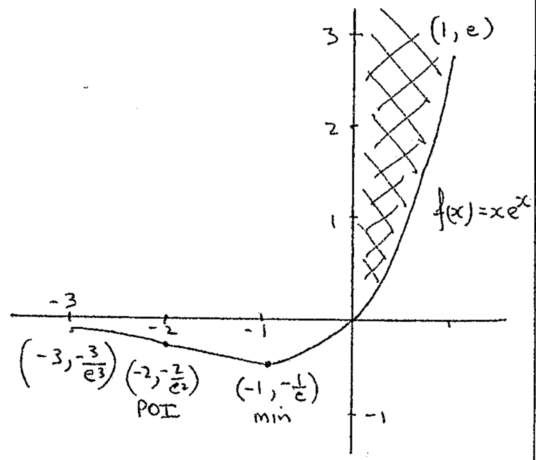
Qn	Solutions	Marks	Comments+Criteria
2(a)	$f(x) = x\ln x$ $f'(x) = x \cdot \frac{1}{x} + \ln x \cdot 1$ $= 1 + \ln x$ $f'(1) = m_{tang} = 1 + \ln 1$ $= 1$ pt is $x = 1$ $y = 1.\ln 1$ ie $(1,0)$ $= 0$ $\therefore$ eqn is $y - 0 = 1(x-1)$ ie $y = x - 1$	✓ ✓ ✓ (3)	$1\frac{1}{2}/3$ Incorrect $f'(1)$ followed through correctly
(b)	$f(x) = \log_e \frac{x}{2-x}$ $= \log_e x - \log_e(2-x)$ $f'(x) = \frac{1}{x} - \frac{-1}{2-x}$ $= \frac{1}{x} + \frac{1}{2-x}$	✓ ✓ (2)	
(c)	Domain: $x - 2 > 0$ $x > 2$	✓ (1)	$x \neq 2$ $x < 2$ ($\frac{1}{2}$)
(d)	$V = \pi\int_a^b y^2\,dx$ $y = e^{-\frac{1}{2}x}$ $y^2 = e^{-x}$ $= \pi\int_0^1 e^{-x}\,dx$ $= \pi\left[-e^{-x}\right]_0^1$ $= \pi(-e^{-1} + e^0)$ $= \pi\left(1 - \frac{1}{e}\right)$ unit^3	✓ ✓ ✓ (3)	$1\frac{1}{2}/3$ or $2/3$ (Incorrect but correctly completed)

Qn	Solutions	Marks	Comments+Criteria
(e)	$A = \int_1^2 \frac{1}{2x}\,dx$ $= \frac{1}{2}\int_1^2 \frac{1}{x}\,dx$ $= \frac{1}{2}\left[\ln x\right]_1^2$ $= \frac{1}{2}(\ln 2 - \ln 1)$ $= \frac{1}{2}\ln 2 = \ln\sqrt{2}\ u^2$	✓ ✓ ✓	
3(a)	(i) $\int_1^k 2x\,e^{x^2-1}\,dx$ $= \left[e^{x^2-1}\right]_1^k$ $= e^{k^2-1} - e^0$ $= e^{k^2-1} - 1$ (ii) $k^2 - 1 = 7$ $k^2 = 8$ $k = \pm\sqrt{8} = \pm 2\sqrt{2}$	✓ ✓ ✓	Let $u = x^2-1$ $du = 2x\,dx$ $x=k,\ u=k^2-1$ $x=1,\ u=0$ $I = \int_0^{k^2-1} e^u.du$ $=\left[e^u\right]_0^{k^2-1}$ $=e^{k^2-1} - e^0$ $=e^{k^2-1} - 1$ 1 only for $\sqrt{8}$ 2 for $\pm\sqrt{8}$
(b)	$f(x) = x.e^x \quad x=1$ to $x=3$ x: 1, 2, 3 ; $f(x)$: e^1, $2e^2$, $3e^3$ So $A \doteqdot \frac{b-a}{4}\left[f(a) + 2f\left(\frac{a+b}{2}\right) + f(b)\right]$ $= \frac{3-1}{4}\left[e + 4e^2 + 3e^3\right]$	✓ ✓	$h = \frac{b-a}{n} = \frac{3-1}{2} = 1$ $A \doteqdot \frac{1}{2}\left[e + 3e^3 + 2.2e^2\right]$

Qn	Solutions	Marks	Comments+Criteria
	$A \doteqdot \frac{1}{2}(e + 4e^2 + 3e^3)$	✓	
	(ii) $A \doteqdot \frac{b-a}{6}\left(f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right)$ $\doteqdot \frac{3-1}{6}(e + 8e^2 + 3e^3)$ $\doteqdot \frac{1}{3}(e + 8e^2 + 3e^3)$	✓✓ ✓	$A \doteqdot \frac{1}{3}(e + 3e^3 + 4\times 2e^2)$
	(iii) Difference is $\frac{1}{2}(e + 4e^2 + 3e^3) - \frac{1}{3}(e + 8e^2 + 3e^3)$ $= \frac{1}{6}e + \left(2e^2 - \frac{8}{3}e^2\right) + \frac{1}{6}.3e^3$ $= \frac{1}{6}e - \frac{2e^2}{3} + \frac{e^3}{2}$ $= 0.453\ldots - 4.926\ldots + 10.04\ldots$ $= 5.569\ldots \doteqdot 5.57\ u^2$	✓✓	1 for part calculation
4 (a)	$f(x) = xe^x = u.v$ $f'(x) = xe^x + e^x.1$ $= e^x(x+1)$	✓ ✓	
(b)	SP's when $f'(x) = 0$ ie $e^x(x+1) = 0$ $\therefore x = -1$ as $e^x \neq 0$ $f''(x) = e^x(x+1) + e^x$ $= e^x(x+2)$ $f''(-1) = e^{-1}(-1+2) > 0$ $\therefore$ min pt at $x = -1$ $\left(y = \frac{-1}{e}\right)$	✓ ✓	

Qn	Solutions	Marks	Comments+Criteria
	$\therefore$ min pt is $x=-1$ $y = -1e^{-1} = -\frac{1}{e}$ ie $\left(-1, -\frac{1}{e}\right)$		Ignore y coord.
(c)	$f''(x) = e^{x}(x+2)$	✓	if not found before
	$f''(x) = 0$ for POI ie $e^{x}(x+2)=0$ $\therefore x=-2$ $\begin{array}{c\|c\|c\|c} x & -1 & -2 & -3 \\ \hline f''(x) & e^{-1} & 0 & -e^{-3} \\ & + & 0 & - \end{array}$ change in concavity $\therefore \left(-2, -2e^{-2}\right)$ is a POI	✓	1 check concavity
(d)	as $x \to \infty$ $xe^{x} \to \infty$	✓	
	$x \to -\infty$ $xe^{x} \to 0$ from beneath	✓	
	$\because$ $f(x) = xe^{x}$ try $x=-100$ $= -100\,e^{-100}$ $= \frac{-100}{e^{100}}$ v.v. small + (−) ve		
(e)	See over		

Qn	Solutions	Marks	Comments+Criteria
4(e) cont'			1 SP's, POI 1½ intercepts/shape ½ endpts of domain
(f)	see above	✓	correct region