Year 9 Mathematics

Common Task

Term 3, 2006

Name: ....

Class Teacher (circle) ***RBL*** ***GP*** ***MWA***

Topics:
Surds
Recurring Decimals
Properties of Geometric Figures
Circle Geometry
Coordinate Geometry

Time: **50 minutes**

Instructions:
Answer all questions.

Calculators may be used.

Marks may not be awarded for untidy or careless work.

Show all necessary working.

Diagrams are NOT drawn to scale

Marks

1. Simplify the following surds:

(a) $2\sqrt{3}+5\sqrt{3}-4\sqrt{2}$ **1**

(b) $\sqrt{20}+\sqrt{45}-\sqrt{80}$ **1**

(c) $\left(3-2\sqrt{5}\right)^2$ **2**

2. Rationalise the denominator in the following:

(a) $\dfrac{\sqrt{3}}{\sqrt{5}}$ **1**

(b) $\dfrac{6+\sqrt{2}}{4-\sqrt{2}}$ **2**

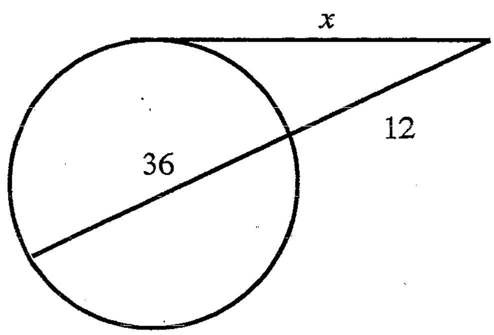
Marks

3. Convert $0.\dot{3}\dot{8}$ to a fraction in its simplest form. 2

4. Show that $0.\dot{9} = 1$. 2

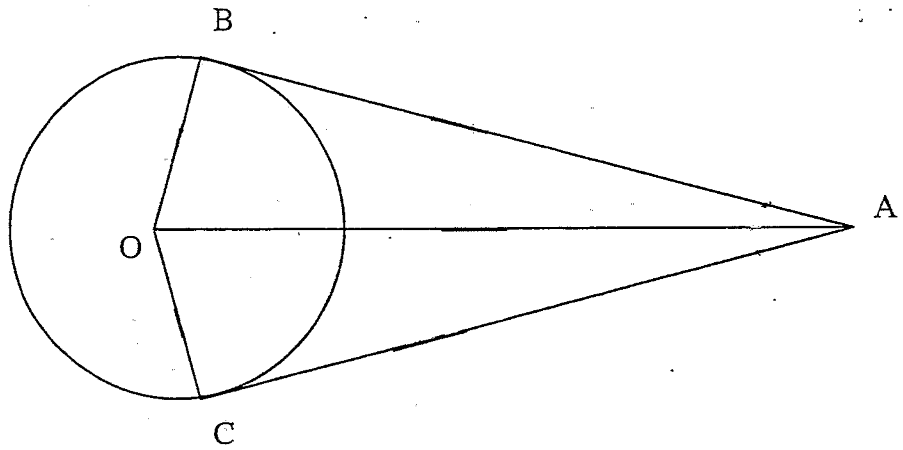
2

5. Find the value of x in the following, giving reasons. 2



Marks

6. AB and AC are tangents to a circle with centre O.

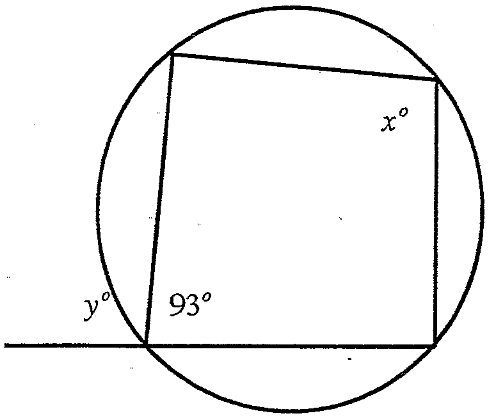


(a) Using your circle geometry rules, prove that $\triangle ABO \equiv \triangle ACO$. 2

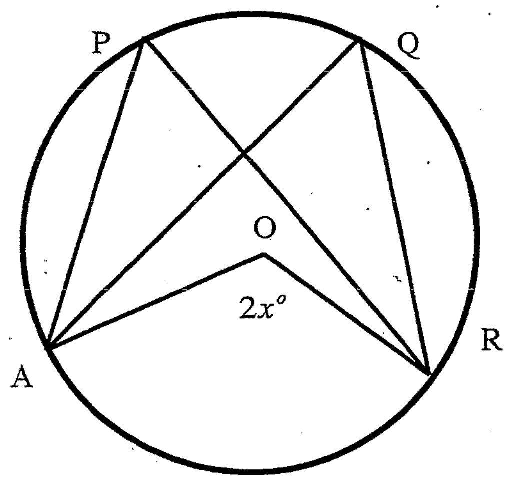
(b) Hence prove that OA bisects angle BAC. 1

Marks

7. Find the value of x and y in the diagram below. Give reasons for your answer. **2**



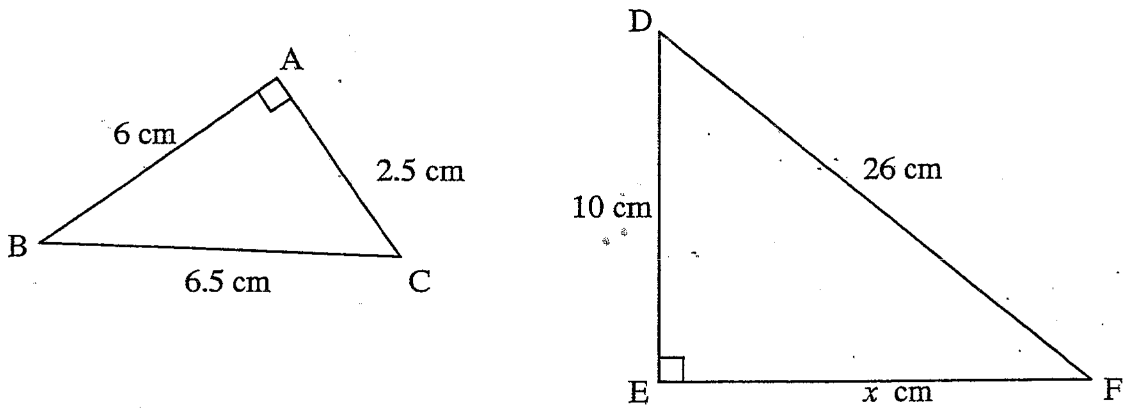
8. In the following diagram, O is the centre of a circle and $\angle AOR = 2x^o$. Find the values of $\angle APR$ and $\angle AQR$ in terms of x. Give reasons for your answers. **2**



Marks

9. Show that the external angle sum of an octagon is 360^o. **2**

10.



(a) Prove that $\triangle$ABC is similar to $\triangle$DEF. **2**

(b) Hence find the value of x. **1**

Marks

11. Given an interval between the points $A(3, 4)$ and $B(6, -8)$, find showing **all** working:

(a) The midpoint of the interval AB. **1**

(b) The gradient of the interval AB. **2**

(c) The length of the interval AB. **2**

(d) Show that the equation of a line passing through A and B is $y = -4x + 16$ **2**

Marks

12. Find the equation of a line perpendicular to the line $y = -4x + 16$ and passing through the point $P\,(1,\,1)$. Write your answer in general form. **3**

13. Complete the following sentences:

(a) $y = 0$ is the equation of ____ **1**

(b) $y = 4 - \dfrac{x}{4}$ has a y-intercept of **1**

(c) $x = 4$ is a line parallel to which axis? **1**

(d) $y = \dfrac{3x}{5} + 4$ has a gradient of **1**

End of Examination

Marks

1. Simplify the following surds:

(a) $2\sqrt{3}+5\sqrt{3}-4\sqrt{2}$ **1**

$= 7\sqrt{3} - 4\sqrt{2}$

(b) $\sqrt{20}+\sqrt{45}-\sqrt{80}$ **1**

$= 2\sqrt{5} + 3\sqrt{5} - 4\sqrt{5}$

$= \sqrt{5}$

(c) $(3-2\sqrt{5})^2$ **2**

$= (3)^2 - 2\times 3\times 2\sqrt{5} + (-2\sqrt{5})^2$

$= 9 - 12\sqrt{5} + 20$

$= 29 - 12\sqrt{5}$

2. Rationalise the denominator in the following:

(a) $\frac{\sqrt{3}}{\sqrt{5}}$ **1**

$= \frac{\sqrt{3}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{\sqrt{15}}{5}$

(b) $\frac{6+\sqrt{2}}{4-\sqrt{2}}$ **2**

$= \frac{6+\sqrt{2}}{4-\sqrt{2}} \times \frac{4+\sqrt{2}}{4+\sqrt{2}}$

$= \frac{(6+\sqrt{2})(4+\sqrt{2})}{4^2-(\sqrt{2})^2}$

$= \frac{24+6\sqrt{2}+4\sqrt{2}+2}{16-2}$

$= \frac{26+10\sqrt{2}}{14}$

$= \frac{13+5\sqrt{2}}{7}$

Marks

3. Convert $0.\dot{3}\dot{8}$ to a fraction in its simplest form. **2**

Let $n = 0.383838\ldots$

$100n = 38.383838\ldots$

$99n = 38$

$n = \frac{38}{99}$

$\therefore 0.\dot{3}\dot{8} = \frac{38}{99}$

4. Show that $0.\dot{9} = 1$. **2**

Let $n = 0.99999\ldots$

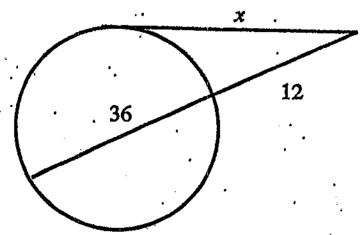
$10n = 9.9999\ldots$

$9n = 9$

$n = 1$

$\therefore 0.\dot{9} = 1$

5. Find the value of x in the following, giving reasons. **2**



$x^2 = 12 \times (36+12)$

$= 12 \times 48$

$= 576$

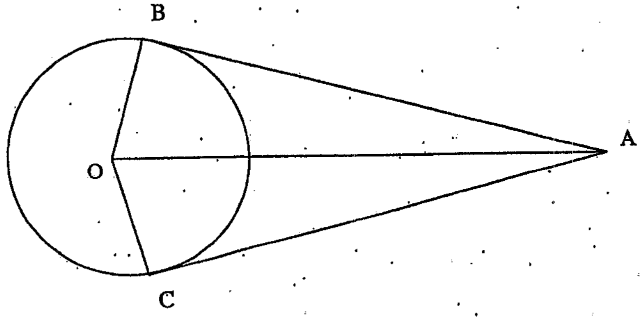
$x = \sqrt{576}$

$= 24$

- The square of the length of the tangent to a circle from an external point equals the product of the intercepts of any secant from the point.

Marks

6. AB and AC are tangents to a circle with centre O.



(a) Using your circle geometry rules, prove that $\triangle ABO \equiv \triangle ACO$. 2

OB = OC (radii of circle)

∠OBA = ∠OCA = 90° (tangent to a circle is perpendicular to the radius drawn to the point of contact)

OA is a common side and is the hypotenuse of △ACO and of △ABO

∴ △ABO ≡ △ACO (RHS)

(b) Hence prove that OA bisects angle BAC. 1

∠BAO = ∠CAO (Corresponding angles in congruent △s)

∠BAC = ∠BAO + ∠CAO

= ∠BAO + ∠BAO

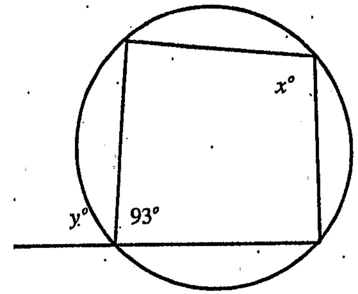
= 2∠BAO

∴ ∠BAO = ½ ∠BAC = ∠CAO

∴ OA bisects ∠BAC

Marks

7. Find the value of x and y in the diagram below. Give reasons for your answer. 2



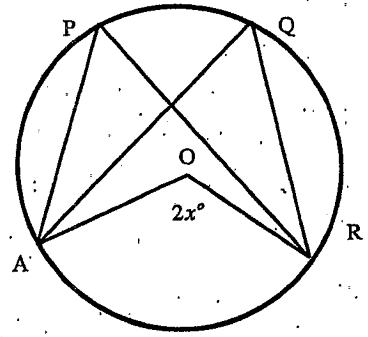
x = 180° − 93° – Opposite angles of a cyclic quadrilateral

= 87° are supplementary

y = x = 87° – Exterior angle of a cyclic quadrilateral is equal to the interior opposite angle

OR y = 180 − 93 = 87° – Straight line has 180°

8. In the following diagram, O is the centre of a circle and $\angle AOR = 2x°$. Find the values of $\angle APR$ and $\angle AQR$ in terms of x. Give reasons for your answers. 2



∠APR = x° – Angle at the centre of a circle is two times the angle at the circumference standing on same arc.

∠AQR = x° – Same reason as above

OR Angles at the circumference of a circle on the same arc are equal.

Marks

9. Show that the external angle sum of an octagon is $360°$. 2

At each vertex of the octagon, the sum of the interior and exterior angles is $180°$ (a straight line).

The sum of interior angles is given by $(n-2)\times180° = (8-2)\times180$
$= 1080°$

Sum of Exterior Angles = Sum of all interior and exterior angles − sum of interior angles

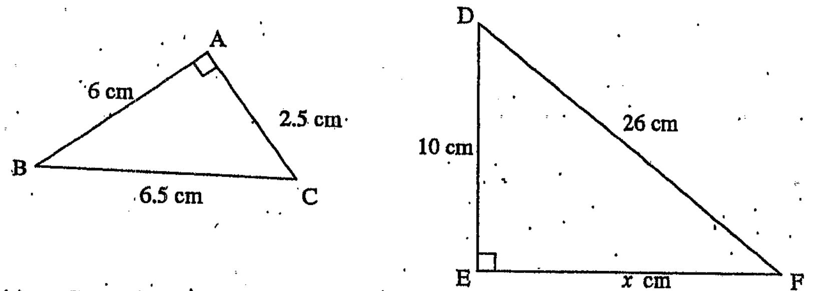
$= 8\times180° - 1080°$

$= 1440° - 1080°$

$= 360°$

$\therefore$ External angle sum of an octagon is $360°$.

10.



(a) Prove that $\triangle ABC$ is similar to $\triangle DEF$. 2

For similarity, the ratio of corresponding sides must be equal.

Hypotenuse − $\frac{BC}{DF} = \frac{6.5}{26} = \frac{1}{4} = 0.25$ − hypotenuse and one other side

Other Side − $\frac{AC}{DE} = \frac{2.5}{10} = \frac{1}{4} = 0.25$

$\therefore \frac{BC}{DF} = \frac{AC}{DE}$ $\therefore \triangle ABC ||| \triangle DEF$ (RHS)

(b) Hence find the value of x. 1

$\frac{AC}{DE} = \frac{AB}{EF}$

$\frac{2.5}{10} = \frac{6}{x}$

$2.5x = 60$

$x = 24$ cm

Marks

11. Given an interval between the points $A(3, 4)$ and $B(6, -8)$, find showing all working:

(a) The midpoint of the interval AB. 1

$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{3+6}{2}, \frac{4+-8}{2}\right)$

$= \left(\frac{9}{2}, \frac{-4}{2}\right)$

$= \left(4\frac{1}{2}, -2\right)$

(b) The gradient of the interval AB. 2

$m = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{-8-4}{6-3}$

$= \frac{-12}{3}$

$= -4$

(c) The length of the interval AB. 2

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(6-3)^2+(-8-4)^2}$

$= \sqrt{3^2+(-12)^2}$

$= \sqrt{9+144}$

$= \sqrt{153}$

$= 12.37$ (2 d.p)

(d) Show that the equation of a line passing through A and B is $y = -4x + 16$

$y - y_1 = m(x - x_1)$

$y - 4 = -4(x-3)$

$y - 4 = -4x + 12$

$y = -4x + 16$

Marks

12. Find the equation of a line perpendicular to the line $y = -4x + 16$ and passing through the point P (1, 1). Write your answer in general form. 3

For perpendicular lines : $m_1 = \frac{-1}{m_2}$

∴ Gradient of perpendicular line is $\frac{-1}{-4} = \frac{1}{4}$

Equation of Line: $y - y_1 = m(x - x_1)$

$y - 1 = \frac{1}{4}(x - 1)$

$4y - 4 = x - 1$

$x - 4y + 3 = 0$

∴ Equation of line perpendicular to $y = -4x + 16$ passing through $P(1, 1)$ is

$x - 4y + 3 = 0$

13. Complete the following sentences:

(a) $y = 0$ is the equation of the x-axis. 1

(b) $y = 4 - \frac{x}{4}$ has a y-intercept of 4 or (0, 4). 1

(c) $x = 4$ is a line parallel to which axis? the y-axis 1

(d) $y = \frac{3x}{5} + 4$ has a gradient of $\frac{3}{5}$. 1

End of Examination