Student Number: ____________

Class Teacher (*circle*): MC CG DL GP

KAMBALA

YEAR 12 MATHEMATICS

HSC Assessment Task 1

December 2010

Time allowed: 50 minutes

- There are three questions, each worth 12 marks.
- The mark for each part is indicated next to that part.
- Start each question on a new page.
- Approved calculators may be used.

Question 1 (12 marks) Start a new page.

(a) Differentiate the following with respect to x.

(i) $f(x)=\dfrac{1}{\sqrt{x}}$ 2

(ii) $y=(2x+5)^3$ 2

(iii) $y=\dfrac{x-1}{3x+1}$ check! 2

(b) Find the equation of the tangent to the curve $y=2x^3+1$ at the point where $x=1$. 3

(c) Evaluate $\sum_{k=1}^{4} 2^k$. 1

(d) The third term and the ninth term of an arithmetic series are -2 and 28 respectively. 2

Find the first term and the common difference.

Question 2 (12 marks) Start a new page.

(a) The curve $y = x^2 + mx$ has a stationary point when $x = -1$. Find the value of m. **2**

(b) If $2+x$, $6+x$ and $13+x$ are the first three terms of a geometric series, find the value of x. **3**

(c) Consider the curve $f(x) = x^3 - 3x^2 + 1$.

(i) Find any stationary points and determine whether the stationary points are maximum or minimum turning points. **3**

(ii) Find any points of inflexion. **2**

(iii) Sketch the graph of the curve showing all of the above features. You do not need to find any x-intercepts. **1**

check

(iv) For which values of x is the curve increasing and concave down? **1**

check

Question 3 (12 marks) Start a new page.

(a) Find the sum to infinity of the geometric series $1 + (\sqrt{3}-1) + (\sqrt{3}-1)^2 + \ldots$ **2**

Give your answer as a surd with a rational denominator.

(b) Amanda decides to run to improve her fitness level.

On the first day she runs 1200m. On each day after that, she runs 300m more than the previous day. That is, she runs 1500m on the second day, 1800m on the third day and so on.

(i) Write down a formula for the distance T_n she runs on the n^{th} day. **1**

(ii) How far does she run on the 8^{th} day? **1**

(iii) What is the total distance she runs in the first 8 days? **1**

(iv) After how many days will the total distance she has run equal 39 kilometres? **2**

(c) The sum of the radii of two circles is 100 centimetres. Let one of the circles have a radius of x centimetres.

(i) Show that the sum of the areas of the two circles is given by $A = 2\pi\left(x^2 - 100x + 5000\right)$ square centimetres. **2**

(ii) Hence find the value of x that gives the least area and find this area. **3**

End of Assessment Task

12/12

Question One

a) i. $f(x) = \frac{1}{\sqrt{x}}$

$y = x^{-\frac{1}{2}}$

$y' = -\frac{1}{2}x^{-\frac{3}{2}}$

$= -\frac{1}{2x^{3/2}}$ ✓✓

ii. $y = (2x+5)^3$

$y' = 3(2x+5)^2 \cdot 2$

$= 6(2x+5)^2$ ✓✓

iii. ~~$y = \frac{(x-1)}{(3x+1)}$~~ ~~$u = x-1$, $u' = 1$~~ ~~$v = (3x+1)^{-1}$, $v' = -3(3x+1)^{-2}$~~

~~$y' = \frac{-3(x-1)}{(3x+1)^2} + \frac{1}{(3x+1)}$~~

~~$= \frac{-3(x-1)+3x+1}{(3x+1)^2}$~~

~~$= \frac{-3x+3+3x+1}{(3x+1)^2}$~~

~~$= \frac{4}{(3x+1)^2}$~~

$y = \frac{x-1}{3x+1}$

$u = x-1$, $u' = 1$; $v = 3x+1$, $v' = 3$

$y' = \frac{uv' - v'u}{v^2}$

$= \frac{1(3x+1) - 3(x-1)}{(3x+1)^2}$

$= \frac{3x+1-3x+3}{(3x+1)^2}$

$= \frac{4}{(3x+1)^2}$ ✓✓

~~$y' = \frac{vu' + uv'}{v^2}$~~

~~$= \frac{1(3x+1)+3(x-1)}{(3x+1)^2}$~~

~~$= \frac{3x+1+3x-3}{(3x+1)^2}$~~

~~$= \frac{-2}{(3x+1)^2}$~~

6/6

b) Since $x=1$, $y = 2(1)^3+1$ Pt $(1,3)$

$= 3$

$y = 2x^3+1$

$y' = 6x^2$ → ∴ At $x=1$, $m=6$ ✓

$y-3 = 6(x-1)$ ✓

$y-3 = 6x-6$

$\boxed{y = 6x-3}$ ✓

3/3

c) $\sum_{k=1}^{4} 2^k = 2^1 + 2^2 + 2^3 + 2^4$

$= 2+4+8+16$

$= 30$ ✓

d) $T_3 = -2$ $T_9 = 28$

$-2 = a+2d$ $28 = a+8d$

$-2-2d = 28-8d$

$6d = 30$

$\boxed{d = 5}$ ✓✓

$\boxed{\therefore a = -12}$

3/3

11

Question Two

a) $y = x^2 + mx$ → stat pt when $x=-1$

$y' = 2x+m$

$0 = 2x+m$

$0 = 2(-1)+m$

$\boxed{2 = m}$ ✓ 2

b) $T_1 = 2+x$ $T_2 = 6+x$ $T_3 = 13+x$

$\frac{T_2}{T_1} = \frac{T_3}{T_2}$

$\frac{6+x}{2+x} = \frac{13+x}{6+x}$

$(6+x)^2 = (13+x)(2+x)$

$36+12x+\cancel{x^2} = 26+15x+\cancel{x^2}$

$10 = 3x$

$\boxed{x = \frac{10}{3}}$ ✓ 3

c) $f(x) = x^3-3x^2+1$

i. $f'(x) = 3x^2-6x$

$0 = 3x^2-6x$

$0 = 3x(x-2)$

∴ $x=0$, $y=1$, $\boxed{(0,1)}$ ✓ $x=2$, $y=-3$, $\boxed{(2,-3)}$ ✓

x	-0.1	0	0.1
y'	7.83	0	-0.57
	+	—	—

∴ Max pt ✓

x	1.9	2	2.1
y'	-0.57	0	$\frac{63}{100}$
	—		+

∴ Min pt. ✓

3

ii. $f''(x) = 6x-6$

$0 = 6x-6$

$\boxed{x=1, y=-1}$ ✓

x	-0.1	1	0.1
y''	$-\frac{33}{5}$	0	$-\frac{27}{5}$
	—		—

∴ Horizontal turning pt

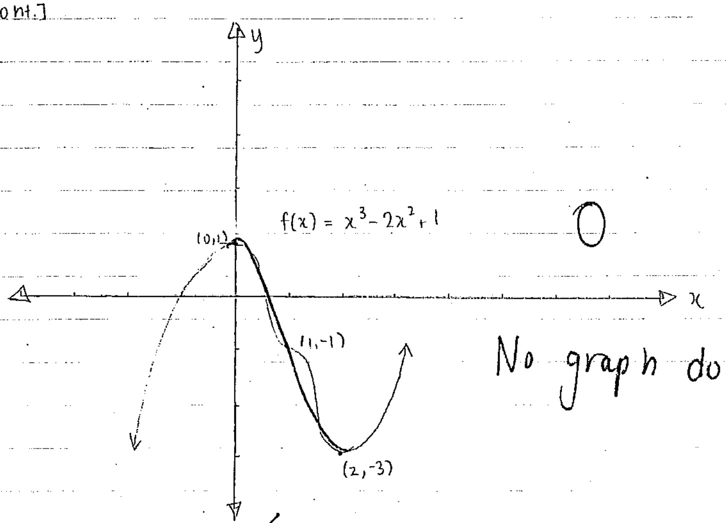
Just a pt of inflexion ✓ Careful!

2

Question Two [cont.]

c) [cont.]

iii.



0

No graph does not do this

iv. $x < 0$

1

12

Question Three

a) $a = 1$ $r = (\sqrt{3} - 1)$ $\boxed{-1 < r < 1 \quad \checkmark r = 0.7\ldots}$

$S_\infty = \frac{a}{1-r}$

$= \frac{1}{1-(\sqrt{3}-1)}$

$= \frac{1}{1-\sqrt{3}+1}$

$= \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$= \frac{2+\sqrt{3}}{4-3}$

$= 2+\sqrt{3}$

2.

b) $T_1 = 1200$ $d = 300$

i. $T_n = a + (n-1)d$

$T_n = 1200 + 300(n-1)$

$T_n = 1200 + 300n - 300$

$T_n = 900 + 300n$

1

ii. $T_8 = 900 + 300(8)$

$= 3300\,m$

1

iii. $S_n = \frac{n}{2}(a + l)$

$S_8 = \frac{8}{2}(1200 + 3300)$

$= 4(4500)$

$= 18\,000\,m$

1

iv. $S_n = \frac{n}{2}(\overset{2400}{\cancel{1200}} + 300(n-1))$

$39\,000 = \frac{n}{2}(2400 + 300n - 300)$

$78\,000 = n(2100 + 300n)$

$78\,000 = 2100n + 300n^2$

$780 = 21n + 3n^2$

$260 = 7n + n^2$

$n^2 + 7n - 260 = 0$

$(n+20)(n-13) = 0$

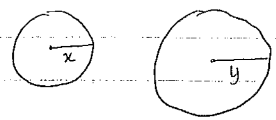
$n = -20$ $\boxed{n = 13} \rightarrow n > 0$ (integer)

$\therefore$ After 13 days

2

Question Three [cont.]

c)



Let the radius of the other circle be y.

i. $x + y = 100$

$y = 100 - x$

$A_{total} = \pi x^2 + \pi y^2$

$= \pi x^2 + \pi(100 - x)^2$

$= \pi x^2 + \pi(10\,000 - 200x + x^2)$

$= \pi x^2 + 10\,000\pi - 200\pi x + \pi x^2$

$= 2\pi x^2 + 10\,000\pi - 200\pi x$

$\boxed{A = 2\pi(x^2 - 100x + 5000)\ \text{cm}^2}$

2

ii. $A = 2\pi x^2 + 10000\pi - 200\pi x$

$A' = 4\pi x - 200\pi$

Let $A' = 0$

$0 = 4\pi x - 200\pi$

$0 = x - 50$

$\boxed{x = 50}$

$A'' = 4\pi$

> 0

$\therefore$ it has a minimum

$\therefore$ When $x = 50$, it gives the least area.

3

Since $x = 50$, $y = 50$ ($y = 100 - x$)

$A = \pi(50)^2 + \pi(50)^2$

$= 2(2500\pi)$

$= 5000\pi\ \text{cm}^2$

exact area

$= 15\,707.96\ \text{cm}^2$ (to 2dp)