KINCOPPAL - EXT2 TRIAL HSC 2005

Total Marks – 120
Attempt Questions 1-8
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find $\int x \ln 2x \, dx$ 2

(b) Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{\sin^5 x} dx$ 3

(c) By completing the square, find $\int \frac{dx}{\sqrt{11-10x-x^2}}$ 2

(d) (i) Find A and B such that $\frac{x^2-3x+14}{(x+3)(x-1)^2} = \frac{A}{x+3} + \frac{B}{(x-1)} + \frac{3}{(x-1)^2}$ 2

(ii) Hence find $\frac{x^2-3x+14}{(x+3)(x-1)^2}$ 2

(e) Use the substitution $x = 3\sin\theta$ to evaluate $\int_0^3 \frac{x^3}{\sqrt{9-x^2}} dx$ 4

Question 2 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Let $z = 2+3i$ and $w = 4-i$.
Find in the form $x+iy$,

(i) zw 1

(ii) $\overline{\left(\frac{z}{w}\right)}$ 2

(b) Find the real numbers a and b such that $(a+bi)^2 = 16+30i$ 3

(c) Sketch the locus of z satisfying the following:

(i) $\arg(z-4) = \frac{3\pi}{4}$ 2

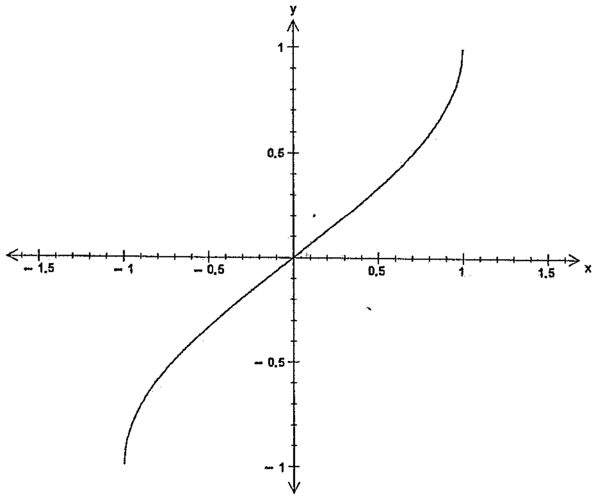
(ii) $\text{Im}\, z = |z|$ 3

(d) (i) Express $1+i$ in modulus-argument form. 2

(ii) Given that $(1+i)^n = x+iy$, where x and y are real and n is an integer, show that $x^2+y^2 = 2^n$ 2

Question 3 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) The diagram shows $y=f(x)$ which is defined in the domain $-1\le x\le 1$



Draw a neat separate sketch of each of the following graphs.
Use about one third of a page for each graph. Show all significant features.

(i) $y=(f(x))^2$ **2**

(ii) $y=|f(x)|$ **2**

(iii) $y=f(|x|)$ **2**

(iv) Draw $y=f(x)$ and $y=\sqrt{f(x)}$ on the same number plane. **2**

(v) $y=e^{f(x)}$ **2**

(vi) $y=\dfrac{1}{f(x)}$ **2**

(vii) $y=f'(x)$ **3**

Question 4 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) A polynomial is such that $P(x)=x^3-x^2+6x+4$ has roots α, β and γ.

(i) Find $\alpha^2\beta\gamma+\alpha\beta^2\gamma+\alpha\beta\gamma^2$ **2**

(ii) Evaluate $\alpha^2+\beta^2+\gamma^2$ **2**

(iii) Use your answer to (ii) to determine the number of real roots of $P(x)$. Justify your answer. **2**

(b) The equation $x^3-12x+m=0$ has a double root. Find the possible values of m. **3**

(c) Let roots α, β and γ be the roots of $x^3-2x^2+10=0$

(i) Find the polynomial equation with integer coefficients whose roots are $\alpha-2$, $\beta-2$ and $\gamma-2$ **2**

(ii) Find the polynomial equation with integer coefficients whose roots are α^2, β^2 and γ^2 **2**

(iii) Evaluate $\alpha^3+\beta^3+\gamma^3$ **2**

Question 5 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the equation of the tangent to $x^2 \sin y + 2x = 4$ at the point $(2,0)$ **3**

(b) (i) Show that $\frac{d}{dx}\ln(\sec x) = \tan x$ **1**

(ii) The length of an arc joining two points whose x-coordinates are a and b on the curve $y = f(x)$ is given by **2**

$$\text{arc length} = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx.$$

Consider the curve defined by $y = \ln(\sec x)$.

Find the length of the arc between $x = 0$ and $x = \frac{\pi}{4}$

(c) In a game, two players take turns at drawing, and immediately replacing, a marble from a bag containing two green and three red marbles. The game is won by player A drawing a green marble or player B drawing a green marble. A goes first. Find the probability that:

(i) A wins on her first draw. **1**

(ii) B wins on her first draw. **1**

(iii) A wins in less than four of her turns. **2**

(iv) A wins eventually. **2**

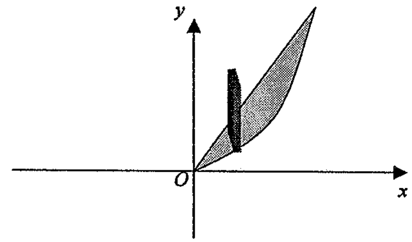
(d) A sequence is defined such that $T_1 = 5, T_2 = 7$ and $T_{n+2} = 3\times T_{n+1} - 2\times T_n$ **3**
Prove by mathematical induction that $T_n = 3 + 2^n$.

Question 6 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Use the identity $2\sin A\cos B = \sin(A+B) + \sin(A-B)$ to show that: **3**

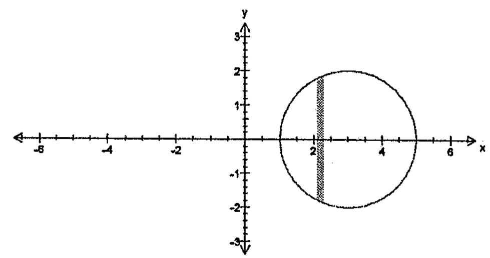
$\int_0^t \sin(\alpha x)\cos(\alpha(t-x))\,dx = \frac{t}{2}\sin(\alpha t)$, where α and t are constants

(b)



The base of a solid is the region contained by $y = x$ and $y = x^2$. Cross-sections, perpendicular to the x-axis are rectangles, with height four times the length of the base. Find the volume of the solid. **4**

(c) The graph below is of the circle $(x-3)^2 + y^2 = 4$.
The circle is to be rotated around the y-axis. Consider a strip is of width δx.



(i) Copy the diagram and draw an appropriate cylindrical shell. **1**

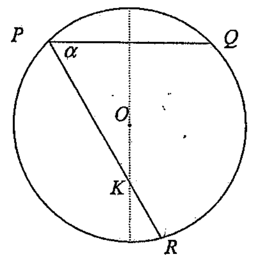
(ii) Use the method of cylindrical shells to show that the volume of the doughnut formed when the region inside the circle is rotated about the y-axis is given by **2**

$$V = 4\pi\int_1^5 x\sqrt{4-(x-3)^2}\,dx$$

(iii) Hence find the volume of the doughnut using the substitution $x - 3 = 2\sin\theta$ **5**

Question 7 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the general solution of $\tan 2x = 2\sin x \cos x$ **4**



(b) PQ is a chord of a circle. The diameter of the circle perpendicular to PQ meets another chord PR at K such that $OK = KR$ and $\angle QPR = \alpha$

(i) Prove that $OKRQ$ is a cyclic quadrilateral **3**

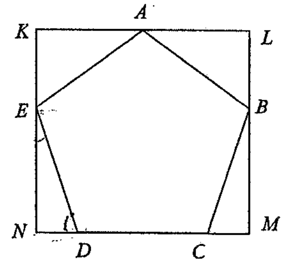
(ii) Hence deduce that KQ bisects $\angle OQR$. **3**

(c) (i) Show that $\sin\left(\sin^{-1} x - \cos^{-1} x\right) = 2x^2 - 1$ **2**

(ii) Hence solve $\sin^{-1} x - \cos^{-1} x = \sin^{-1}(1-x)$ **3**

Question 8 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) The diagram shows a regular pentagon $ABCDE$ with all sides 1 unit in length. The pentagon is inscribed in a rectangle $KLMN$.



(i) Deduce from the diagram that $\Delta NED \equiv \Delta BMC$ **2**

(ii) Prove that $ND = \cos 72°$ **1**

(iii) Given that opposite sides of a rectangle are equal, show that $2\cos 36° = 1 + 2\cos 72°$ **2**

(iv) Hence show that $\cos 36° = \dfrac{1+\sqrt{5}}{4}$ **3**

(v) Hence calculate the exact value of $\cos 72°$ **2**

(b) (i) Using the binomial theorem write down the expansion of $(1+i)^{2m}$, where $i = \sqrt{-1}$, and m is a positive integer. **2**

(ii) Hence prove that ${}^{2m}C_0 - {}^{2m}C_2 + {}^{2m}C_4 - {}^{2m}C_6 \ldots (-1)^m\, {}^{2m}C_{2m} = 2^m \cos\dfrac{m\pi}{2}$ **3**

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EXCELLENT

1.(a). $\int x\ln 2x\,.dx$

$u = \ln 2x \quad v = \frac{x^2}{2}$

$u' = \frac{2}{x} \quad v' = x$

$I = \left[\frac{x^2\ln 2x}{2}\right] - \int x.dx$ ✓

$= \frac{x^2\ln 2x}{2} - \frac{x^2}{2}$ ✓

$= \frac{x^2}{2}[\ln 2x - 1]$

(b) $\int_{\pi/6}^{\pi/2} \frac{\cos x}{\sin^5 x}.dx$

Let $\sin x = u$. At $x = \pi/2$, $u = 1$ ✓

$du = \cos x.dx$ $x = \pi/6$, $u = 1/2$.

$I = \int_{1/2}^{1} u^{-5}.du$

$= \left[\frac{u^{-4}}{-4}\right]_{1/2}^{1}$ ✓

$= \left[-\frac{1}{4} + 4\right]$

$= \frac{15}{4}$. ✓

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(c). $\int \frac{dx}{\sqrt{11 - 10x - x^2}}$

$= \int \frac{dx}{\sqrt{-(x^2 + 10x + 25) + 36}}$ ✓

$= \int \frac{dx}{\sqrt{36 - (x+5)^2}}$

$= \sin^{-1}\left(\frac{(x+5)}{6}\right) + c$. ✓

(d). $\frac{x^2 - 3x + 14}{(x+3)(x-1)^2} = \frac{A}{x+3} + \frac{B}{(x-1)} + \frac{3}{(x-1)^2}$

$x^2 - 3x + 14 = A(x-1)^2 + B(x+3)(x-1) + 3(x+3)$

Let $x = -3$ — Let $x = 0$

$32 = 16A$ ✓ — $14 = 2 - 3B + 9$.

$A = 2$. ✓ — $3 = -3B$

$B = -1$ ✓

$\therefore \int \frac{x^2 - 3x - 14}{(x+3)(x-1)^2} = \int \frac{2}{x+3} - \frac{1}{x-1} + \frac{3}{(x-1)^2}$.

$= 2\ln(x+3) - \ln(x-1) - 3(x-1)^{-1}$

$= 2\ln(x+3) - \ln(x-1) - \frac{3}{(x-1)}$

(e). $\int_0^3 \frac{x^3}{\sqrt{9-x^2}}.dx$

Let $x = 3\sin\theta$ — At $x = 3$, $\theta = \pi/2$. ✓

$dx = 3\cos\theta.d\theta$ — $x = 0$, $\theta = 0$.

$I = \int_0^{\pi/2} \frac{27\sin^3\theta}{\sqrt{9 - 9\cos^2\theta}} . 3\cos\theta.d\theta$

$= \int_0^{\pi/2} \frac{27\sin^3\theta}{3\cancel{\cos\theta}} 3\cancel{\cos\theta}.d\theta$

$= 27\int_0^{\pi/2} \sin^3\theta.d\theta$ ✓

$= -27\int_0^{\pi/2} -\sin\theta(1 - \cos^2\theta).d\theta$

Let $u = \cos\theta$ — At $\theta = \pi/2$, $u = 0$

$du = -\sin\theta.d\theta$ ✓ — $\theta = 0$, $u = 1$.

$= -27\int_1^0 (1-u^2).du$

$= +27\int_0^1 (1-u^2).du$

$= 27\left[u - \frac{u^3}{3}\right]_0^1$

$= 27\left[1 - \frac{1}{3}\right]$ ✓

$= 18$.

2.(a). $z = 2+3i$, $w = 4-i$

(i) $zw = (2+3i)(4-i)$

$= 11 + 10i$ ✓

(ii) $\overline{\left(\frac{z}{w}\right)} = \overline{\left(\frac{2+3i}{4-i} \times \frac{4+i}{4+i}\right)}$

$= \overline{\left(\frac{5+14i}{17}\right)}$ ✓

$= \frac{5-14i}{17}$ ✓

(b). $(a+bi)^2 = 16 + 30i$

$a^2 - b^2 + 2iab = 16 + 30i$

$a^2 - b^2 = 16$ — $2ab = 30$.

$b = \frac{15}{a}$.

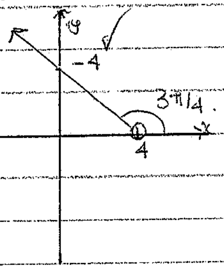
$a^2 - \frac{225}{a^2} = 16$.

$a^4 - 16a^2 - 225 = 0$.

$a^2 = 25$, $a^2 = -9$. ✓

$\therefore a = \pm 5$, $b = \pm 3$. $\therefore z = \pm(5+3i)$ ✓

(c)(i) $\arg(z-4) = 3\pi/4$.



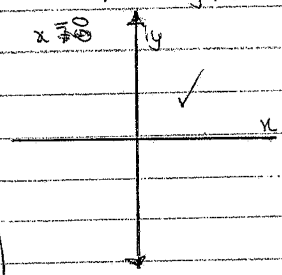
(ii) $\operatorname{Im}(z) = |z|$ $\therefore y \geq 0$.

$y = \sqrt{x^2+y^2}$

$y^2 = x^2+y^2$

$x^2 = 0$ $\therefore y \geq 0$.

$x = 0$



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(d) (i) $z = 1+i$

$= \sqrt{2}\operatorname{cis}\pi/4$

(ii) $(1+i)^n = x+iy$

$(\sqrt{2}\operatorname{cis}\pi/4)^n = (x+iy)$

$(\sqrt{2}\operatorname{cis}\pi/4)^{2n} = (x+iy)^{2n}$

$= (\sqrt{2})^{2n}\left(\cos\frac{2n\pi}{4} + i\sin\frac{2n\pi}{4}\right) = (x^2+2ixy-y^2)$

$2^n(0+i) = (x^2+2ixy-y^2)$

$2^n i = x^2+2ixy-y^2$

$(1+i)^n = (x+iy)$

$(1+i)^{-n} = (x+iy)^{-1}$

$\sqrt{2}\left(\cos\frac{-2n\pi}{4} + i\sin\frac{2n\pi}{4}\right) = \frac{1}{(x^2+2ixy-y^2)}$

$= (\sqrt{2})^{-n}(0+i) = \frac{1}{x^2+2ixy-y^2}$

$= 2^{-n}(i)$

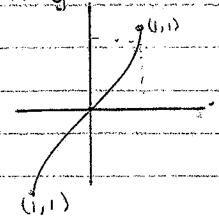
ii $(1+i)^n = x+iy$

$\left(\left|(\sqrt{2})^n\operatorname{cis}\frac{n\pi}{4}\right|\right)^2 = (|x+iy|)^2$

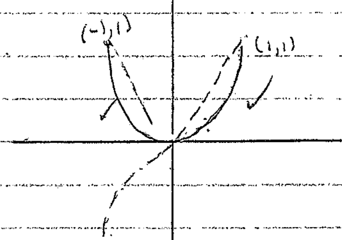
$((\sqrt{2})^n)^2 = (\sqrt{x^2+y^2})^2$

$2^n = x^2+y^2$.

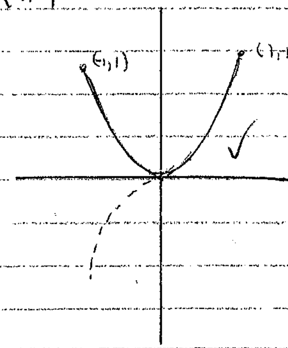
3.(a). $y = f(x)$.



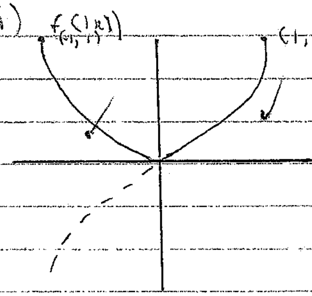
(i) $y = (f(x))^2$



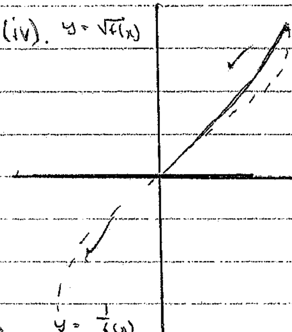
(ii)



(iii) $f(|x|)$



(iv). $y = \sqrt{f(x)}$



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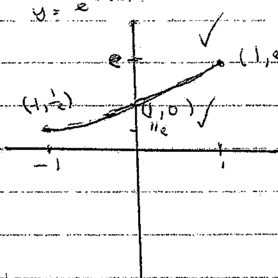
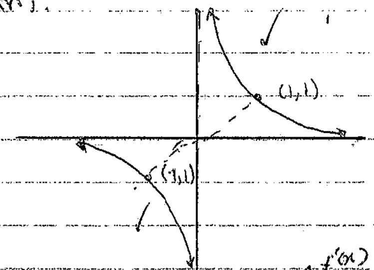
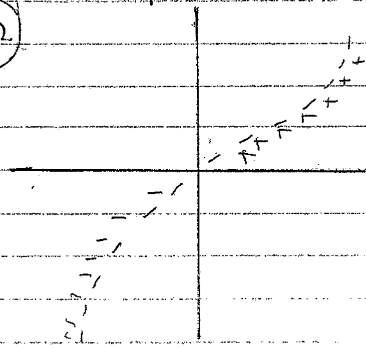
(v). $y = e^{f(x)}$.



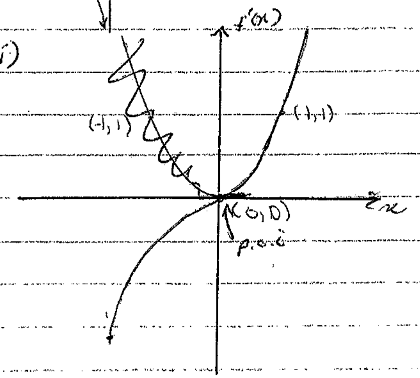
(vi). $y = \frac{1}{f(x)}$



(vi)



? (vii)



4. (a)(i) $P(x) = x^3 - x^2 + 6x + 4.$

$\alpha^2 \beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^3$

$= \alpha\beta\gamma(\alpha + \beta + \gamma)$

$= (-4)(1)$

$= -4.$

(ii) $\alpha^2 + \beta^2 + \gamma^2$

$= (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\beta)$

$= (+1)^2 - 2(6)$

$= -11$

(iii) Since cubic, 1 real root, since the sum of the ~~square~~ roots of is negative. & sum of roots is 1 2 imaginary roots.

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(b). $x^3 - 12x + m = 0.$

$P'(x) = 3x^2 - 12 = 0.$

$3x^2 = 12$

$x^2 = 4.$

$x = \pm 2.$

$P(2) = 8 - 24 + m = 0.$ $\qquad$ $P(-2) = -8 + 24 + m = 0.$

$m = 16$ $\qquad$ $16 + m = 0.$

$m = -16.$

(c). $P(x) = x^3 - 2x^2 + 10 = 0.$

(i) $\alpha - 2$, ie $y = x - 2$

$y + 2 = x$

$P(y+2) = (y+2)^3 - 2(y+2)^2 + 10.$

$= y^3 + 6y^2 + 12y + 8 - 2y^2 - 8y - 4 + 10.$

$= y^3 + 4y^2 + 4y + 14.$

(ii) α^2 ie $y = x^2$, $\sqrt{y} = x.$

$P(\sqrt{y}) = y\sqrt{y} - 2y + 10 = 0.$

$y\sqrt{y} = 2y - 10.$

$y^3 = (2y - 10)^2$

$y^3 = 4y^2 - 40y + 100$

$y^3 - 4y^2 + 40y - 100.$

(iii) $\alpha^3 + \beta^3 + \gamma^3$

since α is a root, then $\alpha^3 = 2\alpha^2 + 10$

$\therefore \alpha^3 + \beta^3 + \gamma^3 = 2(\alpha^2 + \beta^2 + \gamma^2) - 30.$

$= 2(4) + 30.$

5. (a). $x^2 \sin y + 2x = 4.$

$f'(x) = 2x$ $\quad u = x^2 \quad v = \sin y$

$u' = 2x \quad v' = \cos y \cdot \frac{dy}{dx}$

$f'(x) = x^2 \cos y \cdot \frac{dy}{dx} + 2x \sin y + 2 = 0.$

$\frac{dy}{dx} = \frac{-2 - \sin y(2x)}{x^2 \cos y}$

At $(2, 0)$, $m = \frac{-2 - \sin(0) \times 2}{4\cos(0)}$

$= \frac{-2}{4}.$

$m = -1/2.$

$y - 0 = -1/2(x - 2)$

$2y = -x + 2$

$x + 2y - 2 = 0.$

(b) (i) $\frac{d}{dx} \ln(\sec x)$

$= \frac{\sec x \tan x}{\sec x}$

$= \tan x$

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(ii) $\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$

$y = \ln \sec x$

$\frac{dy}{dx} = \tan x$

$I = \int_0^{\pi/4} \sqrt{1 + \tan^2 x} \, dx$

$= \int_0^{\pi/4} \sec x \, dx.$

~~$= \int_0^{\pi/4} (\cos x)^{-1} dx$~~

$= \left[\ln|\sec x + \tan x|\right]_0^{\pi/4}$

$= \ln(\sqrt{2} + 1) - \ln(1)$

$= \ln(\sqrt{2} + 1)$

(c). (i) $2/5.$

(ii) $3/5 \times 2/5.$

$= \frac{6}{25}.$

(iii) $2/5 + 3/5 \times 3/5 \times 2/5 + 3/5 \times 3/5 \times 3/5 \times 3/5 \times 2/5.$

$= \frac{2}{5} + \frac{18}{125} + \frac{162}{3125}.$

$= \frac{1862}{3125}.$

(iv). GP. ~~$\frac{a(1-r^n)}{(1-r)}$ $a = 2/5$, $r = 9/25$.~~

$P(A \text{ wins}) = \frac{a}{1-r}$ $\quad a = {}^{4}/_{5},\ r = {}^{9}/_{25}.$

$= \dfrac{\frac{4}{5}}{1 - {}^{9}/_{25}}$

$= {}^{5}/_{4}.$

6. (a). $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$\int_0^t \sin(ax)\cos(a(t-x)).dx$

$ax = A,\quad a(t-x) = B.$

$I = \int_0^t \sin(ax + at - ax) + \sin(ax - at + ax).dx$

$= \int_0^t \sin(at).dx + \int_0^t \sin(2ax - at).dx$

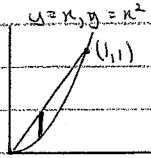
$= [x(\sin(at))]_0^t - [\frac{\cos(2ax - at)}{2a}]_0^t$

$= [t(\sin(at))] - \frac{\cos(at) + \cos(-at)}{2a}$

$\cos(a) = \cos(-a)$

$= t\sin(at)$

(b). $y = x, y = x^2$



$y = y_1 - y_2$
$= x - x^2$

$\therefore V = \lim_{\delta x \to 0} A(x).dx$

$= \lim_{\delta x \to 0} \sum A(x).dx.$

$= \int_0^1 4y^2.dx.$

$= \int_0^1 4(x - x^2)^2.dx.$

$= \int_0^1 4(x^2 - 2x^3 + x^4).dx$

$= 4(\frac{x^3}{3} - \frac{2x^4}{4} + \frac{x^5}{5}]_0^1.$

${}^{4}/_{30}.$

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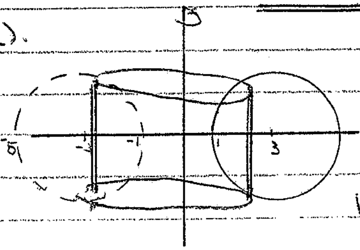
(c).



(ii) $V = 2\pi \lim_{\delta x \to 0} \sum A r h.dx.$

$(x-3)^2 + y^2 = 4.$ $\quad = 2\pi\int_1^5 x2y.dx$

$y^2 = 4 - (x-3)^2$ $\quad 2\pi\int_1^5 x2\sqrt{4-(x-3)^2}.dx$

$y = \sqrt{4 - (x-3)^2}$ $\quad = 4\pi\int_1^5 x\sqrt{4-(x-3)^2}.dx$

(iii). ~~$V = 4\pi\int_1^5 x\sqrt{4-(x-3)^2}\ dx.$~~

~~Let $x - 3 = 2\sin\theta$ $\quad dx = 2\cos\theta.d\theta$~~

~~$4\pi\int_{-\pi/2} 2\sin$ $\quad$ At $x = 5$, $\theta = \pi/2$ $\quad x = 1$, $\theta = -\pi/2$~~

kin coppe!.

$V = 4\pi\int_1^5 x\sqrt{4-(x-3)^2}.dx$

Let $x - 3 = 2\sin\theta$
$x = 2\sin\theta + 3.$
$dx = 2\cos\theta.d\theta$

At $x = 5$, $\theta = \pi/2$
$x = 1$, $\theta = -\pi/2$.

$V = 4\pi\int_{-\pi/2}^{\pi/2} (2\sin\theta + 3)\sqrt{4 - 4\sin^2\theta}\,(2\cos\theta\ d\theta)$

$= 4\pi\int_{-\pi/2}^{\pi/2} (2\sin\theta + 3)\times 2\cos^2\theta\,(2\cos\theta.d\theta$

$= 16\pi\int_0^{\pi/2} \cos^3\theta(2\sin\theta) + 3\cos^3\theta.d\theta$

$= 32\pi\int_0^{\pi/2} \cos^3\theta\sin\theta + 48\pi\int_0^{\pi/2} \cos^3\theta.d\theta$

Let $u = \cos\theta$, $du = -\sin\theta.d\theta$, At $\theta = \pi/2$, $u = 0$, $\theta = 0$, $u = 1$.

$= -32\pi\int_1^0 u^3.du.$ $\quad + 48\pi\int_0^{\pi/2} \cos\theta(1 - \sin^2\theta).d\theta$

Let $\phi = \sin\theta$, $d\phi = \cos\theta.d\theta$, At $\theta = \pi/2$, $\phi = 1$, $\theta = 0$, $\phi = 0$

$= 32\pi\int_0^1 u^3.du.$ $\quad + 48\pi\int_0^1 1 - \phi^2.d\phi$

$= 32\pi[\frac{u^4}{4}]_0^1$ $\quad + 48\pi[\phi - \frac{\phi^3}{3}]_0^1$

$= 8\pi$ $\quad + 48\pi[1 - {}^{1}/_{3}]$

$+ 48\pi[{}^{2}/_{3}]$

$+ 32\pi.$

$\therefore V = 40\pi$

7. (a). $\tan 2x = 2\sin x\cos x.$ $\quad x \neq n\pi/2.$

$\frac{\sin 2x}{\cos 2x} = \sin 2x$

$\sin 2x = \cos 2x\sin 2x$

$\cos 2x\sin 2x - \sin 2x = 0.$

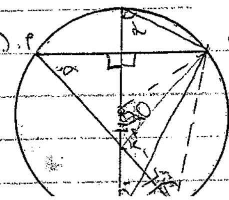
$\sin 2x(\cos 2x - 1) = 0.$

$\sin 2x = 0.$ $\quad \cos 2x = 1.$

$x = \frac{n\pi}{2}$ $\quad x \neq n\pi/2.$ $\quad \cos 2x = 2n\pi$

$\therefore x = n\pi$

(b).



$\therefore R\hat{O}Q = 2\alpha$ (double angle at centre)

$OR = OQ$ (radius)

$\therefore O\hat{Q}R = \pi/2 - \alpha$

But $P\hat{K}O = \pi/2 - \alpha$ (Angle Sum)

$\therefore$ External opposite concylic angles =.

$\therefore$ OKRQ is a cyclic quad.

$KO = KR$

$\therefore K\hat{O}R = K\hat{R}O$ (base Angle of Isos $\triangle$'s)

But $O\hat{R}K = O\hat{Q}K$ (~~base~~ angles on the same arc KR)

$K\hat{O}R = K\hat{Q}R$ (" " " ")

$\therefore O\hat{Q}K = K\hat{Q}R$

$\therefore KQ$ bisects $O\hat{Q}R$

(c)(i). $\sin(\sin^{-1} x - \cos^{-1} x) = 2x - 1$

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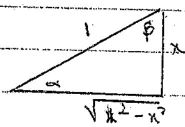
Let $\sin^{-1} x = \alpha$. $\cos^{-1} x = \beta$

$\sin\alpha = x$. $\cos\beta = x$.



$\sin(\alpha - \beta) = 2x - 1$

$\sin\alpha\cos\beta - \sin\beta\cos\alpha = 2x - 1$

$x \cdot x - \frac{\sqrt{1-x^2} \cdot \sqrt{1-x^2}}{1} = 2x - 1$

$x^2 - (1 - x^2) = 2x - 1$

$2x^2 - 1 = 2x - 1$

$2x^2 - 2x = 0$.

$2x(x - 1) = 0$.

$x = 0 \qquad x = 1$

$\sin^{-1} x - \cos^{-1} x = \sin^{-1}(1 - x)$

Take sin of both sides.

$\sin(\sin^{-1} x - \cos^{-1} x) = 1 - x$.

$\therefore$ Similarly $: 2x^2 - 1 = 1 - x$.

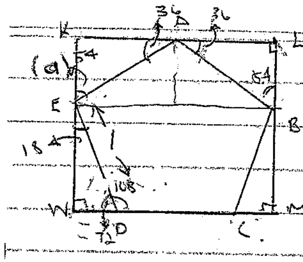
$2x^2 + x - 2 = 0$.

$x = \frac{-1 \pm \sqrt{1 + 16}}{4}$.

$= \frac{-1 \pm \sqrt{17}}{4}$.

But $-1 \le x \le 1$. $\therefore x \ne \frac{-1-\sqrt{17}}{4}$.

$\therefore x = -1 + \sqrt{17}$



(a) (i) In $\triangle$'s NED & BMC

$E\hat{N}D = B\hat{M}C = \pi/2$ (Rectangle Property)

$ED = CB = 1$ (given)

$D\hat{C}B = 3\pi/5$. (pentagon angle sum

$\therefore B\hat{C}M = 2\pi/5$. similarly $E\hat{D}N = 2\pi/5$

$\therefore E\hat{D}N = B\hat{C}M$

$\therefore \triangle NED \equiv \triangle BMC$ (AAS)

(ii). $E\hat{D}C = \frac{3\pi}{5}$ (pentagon Angle sum).

$\therefore E\hat{D}N = \frac{2\pi}{5}$.

$\cos\frac{2\pi}{5} = \frac{ND}{ED}$

But $ED = 1$ (given)

$\therefore \cos 72° = ND$

(iii) $NM = ND + DC + CM$

$= 2\cos 72 + 1$

But $NM = KL$

$KL = 2 \times KA$

But $KA = \cos 36$

$KL = 2\cos 36°$

$\therefore 2\cos 72° + 1 = 2\cos 36°$

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(iv) $2\cos 72° + 1 = 2\cos 36°$

$2\cos(2 \times 36°) + 1 = 2\cos 36°$

$2(2\cos^2 36 - 1) + 1 = 2\cos 36°$

$4\cos^2 36 + ~~8~~ = 2\cos 36°$

$4\cos^2 36 - 2\cos 36 + ~~1~~ = 0$.

$\cos 36 = \frac{2 \pm \sqrt{4 + 16}}{8}$

$= \frac{1 \pm \sqrt{5}}{4}$

(v). $\cos 36 = \frac{1 + \sqrt{5}}{4}$.

$\therefore \cos 2A = 2\cos^2 A - \sin^2 A$

$= 2\cos^2 A - 1$

$\cos 72 = 2\left(\frac{1 + \sqrt{5}}{4}\right)^2 - 1$

$= \frac{1 + 2\sqrt{5} + 5}{8} - 1 = \frac{\sqrt{5}}{4} - \frac{1}{4}$.

$= \frac{\sqrt{5} - 1}{4}$.

(b). $(1+i)^{2m} = \binom{2m}{0} + \binom{2m}{1} i - \binom{2m}{2} - i\binom{2m}{3} + \binom{2m}{4} + \ldots + (i)^k \binom{2m}{k}$

$LHS = (1+i)^{2m} = \left(\sqrt{2}\, \text{cis}\, \frac{\pi}{4}\right)^{2m}$ $\qquad$ RHS = "

$= (\sqrt{2})^{2m}\, \text{cis}\, \frac{2m\pi}{4}$. (By DeMoivre's)

$= 2^m\, \text{cis}\, \frac{2m\pi}{4}$.

Compare $Re(z)$: $\binom{2m}{0} - \binom{2m}{2} + \binom{2m}{4} - \binom{2m}{6} + \ldots (-1)^m \binom{2m}{2m}$

$= 2^m \cos\left(\frac{m\pi}{4}\right)$