KINCOPPAL–ROSE BAY
SCHOOL OF THE SACRED HEART

2009
Year 11 Preliminary
EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 1½ hours
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in every question
- Start a new booklet for each question

Total marks – 72

- Attempt Questions 1 - 6
- All questions are of equal value

Question 1 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the exact value of $\cos 75°$. **2**

(b) Given the points $A(3,1)$ and $B(-3,5)$. The point $P(x,y)$ which divides AB externally in the ratio 2 : 3. Find point P. **2**

(c) Find the gradient of the normal to the curve $y = 3x^2 - 4x + 4$ at the point $(2,5)$. **2**

(d) Find the $\lim_{x\to\infty} \dfrac{x^2-16}{2x^2+4}$ **2**

(e) Find the **acute** angle between the lines $x - 3y = 0$ and $x + 4y = 0$. **2**

(f) Find, from **first principles**, the gradient function of the curve $f(x) = x^2 - 2x + 3$. **2**

End of Question 1

Question 2 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Solve for x: $\dfrac{x^2}{2x+3}<1$ **3**

(b) A committee is to be chosen from a class of 7 boys and 13 girls.
How many distinct committees are possible if the committee is to have:

(i) 5 students. **1**

(ii) 2 boys and 3 girls. **1**

(c) Nine friends are going to dinner at a restaurant.

(i) They line up outside the restaurant. In how many different ways can they be arranged? **1**

(ii) Once inside they intend to sit around a circular table. How many distinct seating arrangements are possible? **1**

(iii) Before taking their seats they realise that Vanessa and Angie insist on sitting apart. How many seating arrangements are possible with the girls sitting apart? **1**

(d) Find the general solutions of the equation: **2**

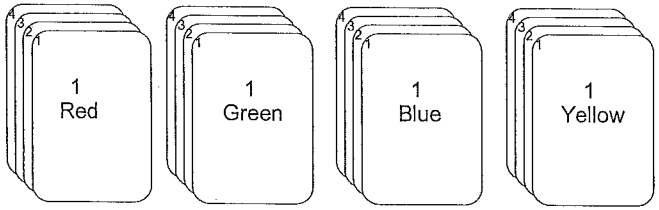
$$2\sin x-1=0$$

(e) Show $\dfrac{3^k-1}{2}+3^k=\dfrac{3^{k+1}-1}{2}$ **2**

End of Question 2

Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) A game uses a deck of 16 cards, numbered from 1 to 4 in each of four "suits" Red, Green, Blue and Yellow. **2**



All of the cards are placed in a line. How many distinct arrangements are possible in which each set of coloured cards are all together?
(ie: all blue then all red then all yellow then all green).

(b) A point $P(x,y)$ is equidistant from a fixed point $A(2,-3)$ and the line $y=3$

(i) Draw a diagram to illustrate this information. **1**

(ii) Find the locus of the point P, that follows the above conditions. **2**

(c) The roots of the equation $(x-1)(x-3)=m$ are equal. Find the value of m. **2**

(d) A parabola has equation $y^2+6x+30=0$. Find:

(i) the coordinates of the vertex. **2**

(ii) the focal length. **1**

(iii) the coordinates of the focus. **1**

(iv) the equation of the directrix. **1**

End of Question 3

Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Prove that $\tan(x+45°)+\tan(x-45°)=2\tan 2x$. **3**

(b) By expressing $\sin x-\cos x$ into the form $R\sin(\theta-\alpha)$, where $R>0$ and α is acute, solve: **3**

$$\sin x-\cos x=1 \quad \text{for} \quad 0^0\le x\le 360^0$$

(c) In the diagram below AB is a tangent to the circle. The length of AC is 9 cm and the length of CD is 7 cm. Find the length of AB. **2**

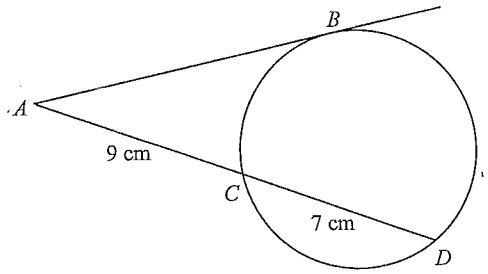


Diagram not to scale

(d) From a point P due east of the base of a tower, situated at T, the angle of elevation to the top of the tower is 42°. From another point Q, due south of the tower, the angle of elevation is 33°. The distance PQ is 450 m.

(i) Draw a diagram, labelling all information given. **1**

(ii) Prove $h=\dfrac{450}{\sqrt{\cot^2 33^\circ+\cot^2 42^\circ}}$, where $h=$ height of the tower. **2**

(iii) Find the height of the tower correct to 2 decimal places. **1**

End of Question 4

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Differentiate and factorise **fully** with respect to x: $y=\dfrac{\sqrt{x^2+1}}{3x-2}$ **4**

(b) Given that $y=\sqrt{1+x^2}$, show that $\dfrac{d}{dx}\left(\sqrt{1+x^2}\right)=\dfrac{x}{y}$ **2**

(c)

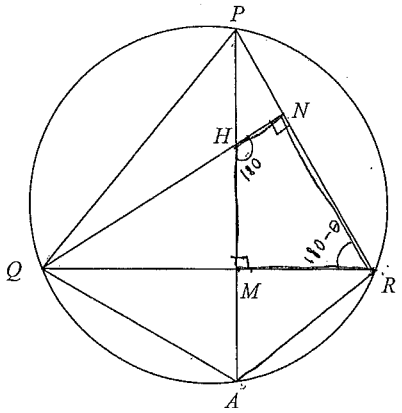


Diagram not to scale

In the diagram above, $PR\perp QN$, $QR\perp PA$ and PA and QN meet at H.

PM is produced and cuts the circle PQR at A.

(i) Prove that $HNRM$ is a cyclic quadrilateral. **1**

(ii) Explain why $\angle QHA=\angle NRM$. **1**

(iii) Prove that ΔQHA is an isosceles triangle. **2**

(iv) Hence prove $HM=MA$. **2**

End of Question 5

Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) The line $y = mx$ is a tangent to the circle whose centre is the point $(5,0)$ and whose radius is 3 units. Find the possible value(s) of m. 4

(b) A function is defined such that $f(x) = \dfrac{3x}{x^2+9}$

(i) Show that $f(x)$ is odd. 1

(ii) What is the domain of $f(x)$? 1

(iii) Draw a neat sketch of $f(x) = \dfrac{3x}{x^2+9}$, about one third of a page in size, showing all important features. 2

(c) If $0 \leq x \leq 360°$, solve for x if $7\sin^2 x + \sin x \cos x - 1 = 0$ using $\sin^2 x + \cos^2 x = 1$. 4

End of Test

KINCOPPAL–ROSE BAY
SCHOOL OF THE SACRED HEART

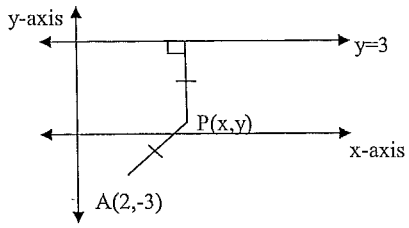
2009
YEAR 11 PRELIMINARY EXAMINATION

Mathematics Extension 1
SOLUTIONS
And
Marking Criteria

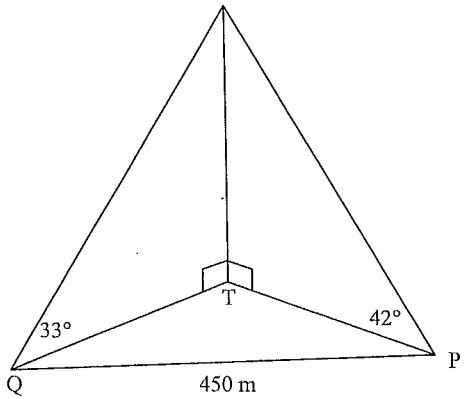
Question 1	Criteria	Marks

Question 2	Criteria	Marks
1(a)	$\cos(30°+45°)=\cos 30°\cos 45°-\sin 30°\sin 45°$	1
	$=\frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}}-\frac{1}{2}\times\frac{1}{\sqrt{2}}$	
	$=\frac{\sqrt{3}-1}{2\sqrt{2}}=\frac{\sqrt{6}-\sqrt{2}}{4}$	1
1(b)	$x=\frac{(2)(-3)+(-3)(3)}{2+(-3)} \quad \therefore x=15$	
	$y=\frac{(2)(5)+(-3)(1)}{2+(-3)} \quad \therefore y=-7$	1
	$P(x,y)=(15,-7)$	1
1(c)	$m_1=\frac{dy}{dx}=6x-4$	1
	$m_1=6(2)-4=8$	
	$m_2=\frac{-1}{8}$	1
1(d)	$\lim_{x\to\infty}\frac{\frac{x^2-16}{x^2}}{\frac{2x^2+4}{x^2}}=\lim_{x\to\infty}\frac{1-\frac{16}{x^2}}{2+\frac{4}{x^2}}=\frac{1}{2}$	1
		1
1(e)	$y=\frac{1}{3}x, m_1=\frac{1}{3}$	
	$y=-\frac{1}{4}x, m_2=-\frac{1}{4}$	1
	$\tan\theta=\left\lvert\frac{\frac{1}{3}-(\frac{-1}{4})}{1+(\frac{1}{3})(\frac{-1}{4})}\right\rvert$	
	$\tan\theta=\frac{7}{11}$	
	$\theta=32°28'$	1
1(f)	$f(x)=x^2-2x+3$	
	$f(x+h)=(x+h)^2-2(x+h)+3$	
	$f'(x)=\lim_{h\to\infty}\frac{x^2+2xh+h^2-2x-2h+3-x^2+2x-3}{h}$	1
	$f'(x)=\lim_{h\to\infty}\frac{2xh+h^2-2h}{h}$	
	$f'(x)=2x-2$	1

Question 2	Criteria	Marks
2(a)	$\frac{x^2}{2x+3}<1 \quad \therefore\ x\neq\frac{-3}{2}$ ✓ $x^2=2x+3$ $x^2-2x-3=0$ $(x-3)(x+1)=0$ $\therefore x=3 \ \ or\ -1$ ✓ ☑ ☒ ☑ ☒ $x=-1.5 \qquad x=-1 \qquad x=3$ $\therefore\ x<-1.5 \quad or \quad -1<x<3$ ✓	1 1 1
2(b)(i)	${}^{20}C_5=15504\ ways$ ✓	1
2(b)(ii)	${}^{7}C_2\times{}^{13}C_3=6006\ ways$ ✓	1
2(c)(i)	${}^{9}P_9$ *or* $9!=362\,880\ ways$ ✓	1
2(c)(ii)	${}^{8}P_8$ *or* $(9-1)!=40320\ ways$ ✓	1
2(c)(iii)	*Sitting together* $7!\times2=10080\ ways$ $\therefore$ *Sitting apart* $= 8!-7!\times2=30240\ ways$ ✓	1
2(d)	$2\sin x-1=0$ $\sin x=\frac{1}{2} \qquad \therefore\ x=30°$ ✓ $\therefore\ x=180n+(-1)^n\times30°$ ✓	 1 1
2(e)	$LHS=\frac{3^k-1}{2}+3^k$ $=\frac{3^k-1+2\times3^k}{2}$ $=\frac{3^k(1+2)-1}{2}$ $=\frac{3^k(3)-1}{2}$ ✓ $=\frac{3^{k+1}-1}{2}$ ✓	 1 1

3(a)	Arrangements with colours together $= 4!\times4!$ $=576$	1
3(b)(i)		1
3(b)(ii)	$PA^2=PB^2$ $(x-2)^2+\left[y-(-3)^2\right]=(x-x)^2+(y-3)^2$ $(x-2)^2+(y+3)^2=(y-3)^2$ $x^2-4x+4+y^2+6y+9=y^2-6y+9$ $x^2-4x+12y+4=0$	 1 1
3(c)	$x^2-4x+3=m$ $x^2-4x+(3-m)=0$ For equal roots, $b^2-4ac=0$ $(-4)^2-4(1)(3-m)=0$ $16-12+4m=0$ $4m=-4$ $m=-1$	 1 1
3(d)(i)	$y^2+6x+30=0$ $y^2=-6(x+5)$ $(y-k)^2=-4a(x-h)$ $k=0, a=1\frac{1}{2}, h=-5$ Vertex $(h,k)=(-5,0)$	 1
3(d)(ii)	Focal Length $a=1\frac{1}{2}$	 1
3(d)(iii)	$(h-a,k)=(-6\frac{1}{2},0)$	1
3(d)(iv)	Equation of the Directrix $x=h+a$ $x=-3\frac{1}{2}$	 1

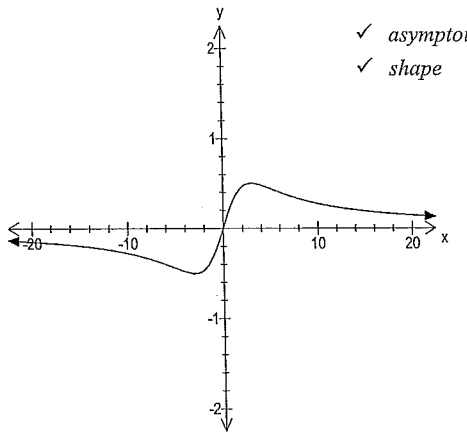
Question 4	Criteria	Marks
4(a)	$LHS = \dfrac{\tan x+\tan 45}{1-\tan x\tan 45}+\dfrac{\tan x-\tan 45}{1+\tan x\tan 45}$ ✓	1
	$=\dfrac{\tan x+1}{1-\tan x}+\dfrac{\tan x-1}{1+\tan x}$	
	$=\dfrac{(\tan x+1)^2+(\tan x-1)(1-\tan x)}{(1-\tan x)(1+\tan x)}$	
	$=\dfrac{\tan^2 x+2\tan x+1+\tan x-\tan^2 x-1+\tan x}{1-\tan^2 x}$	
	$=\dfrac{4\tan x}{1-\tan^2 x}$ ✓	1
	$=\dfrac{2(2\tan x)}{1-\tan^2 x}$	
	$=2\tan 2x$ ✓	1
4(b)	$\sin x-\cos x$	
	$R=\sqrt{1^2+(1)^2}$ $\quad\therefore R=\sqrt{2}$	
	$\tan\alpha=\dfrac{1}{1}$ $\quad\therefore\ \alpha=45°$	
	$\therefore\sqrt{2}\sin(x-45°)=1$ ✓	1
	$\sin(x-45°)=\dfrac{1}{\sqrt{2}}$	
	$x-45°=45°$ *or* $135°$ ✓	1
	$x=90°$ *or* $180°$ ✓	1
4(c)	$AB^2=AC^2\times AD^2$ ✓	1
	$=9\times(9+7)$	
	$=9\times16$	
	$=144$	
	$\therefore AB=\pm\sqrt{144}$	
	$=\pm12$	
	since AB is a length then $AB=12\ cm$ ✓	1

Question 4	Criteria	Marks
4(d)(i)		1
4(d)(ii)	*height of tower* $=h$	
	$\therefore\ \tan 42=\dfrac{h}{x}$ *and* $\tan 33=\dfrac{h}{y}$	
	$PQ^2=PT^2+QT^2$	
	$450^2=\dfrac{h^2}{\tan^2 42}+\dfrac{h^2}{\tan^2 33}$ ✓	1
	$450^2=h^2\left(\dfrac{1}{\tan^2 42}+\dfrac{1}{\tan^2 33}\right)$	
	$450^2=h^2\left(\cot^2 42+\cot^2 33\right)$	
	$\therefore h=\dfrac{450}{\sqrt{\cot^2 33^0+\cot^2 42^0}}$	
	since $h>0$ *as h is a length* ✓	1
4(d)(iii)	$h=\dfrac{450}{\sqrt{\cot^2 33^0+\cot^2 42^0}}$	
	$h=237.02\ m$ ✓	1

5(a)	$u=(x^2+1)^{\frac{1}{2}}, u'=x(x^2+1)^{\frac{-1}{2}}$ $v=3x-2, v'=3$ $\frac{u'v-uv'}{v^2}=\frac{x(x^2+1)^{\frac{-1}{2}}.(3x-2)-3.(x^2+1)^{\frac{1}{2}}}{(3x-2)^2}$ $=\frac{x(x^2+1)^{\frac{-1}{2}}.(3x-2)-3.(x^2+1)^{\frac{1}{2}}\times\frac{(x^2+1)^{\frac{1}{2}}}{(x^2+1)^{\frac{1}{2}}}}{(3x-2)^2}$ $=\frac{x(3x-2)-3(x^2+1)}{(3x-2)^2(x^2+1)^{\frac{1}{2}}}$ $=\frac{3x^2-2x-3x^2-3}{(3x-2)^2\sqrt{x^2+1}}$ $=\frac{-(2x+3)}{(3x-2)^2\sqrt{x^2+1}}$	1 1 1 1
5(b)	$y=(1+x^2)^{\frac{1}{2}}$ $\frac{dy}{dx}=\frac{1}{2}(1+x^2)^{\frac{-1}{2}}\times 2x=x(1+x^2)^{\frac{-1}{2}}$ $\frac{dy}{dx}=\frac{x}{(1+x^2)^{\frac{1}{2}}}=\frac{x}{\sqrt{1+x^2}}=\frac{x}{y}$	1 1
5(c)(i)	$\angle HNR=\angle HMR=90°$ (opposite angles are supplementary). $\therefore$ HNRM is a cyclic quadrilateral.	1
5(c)(ii)	$\angle QHA=\angle NRM$ (external angle at a vertex of a cyclic quadrilateral is equal to the interior opposite angle).	1
5(c)(iii)	$\angle QHM=\angle NRM$ (part (ii)). $\angle PRQ=\angle PAQ$ (angles in the same segment are equal). $\angle QHA=\angle HAQ$ (base angles are equal). $\therefore \Delta QHA$ is isosceles	1 1 1
5(c)(iv)	$\angle AQM=\angle MQH$ ($\Delta AQM=\Delta MQH$ congruent triangles and equiangular) $\therefore QM$ bisects AH $\therefore HM=MA$	1 1

Question 6	Criteria	Marks
6(a)	$y=mx \quad \therefore mx-y=0$ $\therefore a=m,\ b=-1,\ c=0$ *at centre* $(5,0)$ $d=\frac{\lvert ax+by+c\rvert}{\sqrt{a^2+b^2}}$ $d=\frac{\lvert m(5)-(1)(0)+0\rvert}{\sqrt{m^2+(-1)^2}}$ ✓ $d=\frac{\lvert 5m\rvert}{\sqrt{m^2+1}}$ since radius is $3cm$ $\therefore \frac{\lvert 5m\rvert}{\sqrt{m^2+1}}=3$ ✓ $\therefore\ 5m=3\sqrt{m^2+1}$ *or* $-5m=3\sqrt{m^2+1}$ ✓ $25m^2=9(m^2+1)$ *or* $25m^2=9(m^2+1)$ $16m^2=9$ $\therefore\ m=\pm\frac{3}{4}$ ✓	1 1 1 1

Question 6	Criteria	Marks
6(b)(i)	$f(x)=\frac{3x}{x^2+9}$ $f(-x)=\frac{3(-x)}{(-x)^2+9}=\frac{-3x}{x^2+9}$ $-f(x)=\frac{-3x}{x^2+9}$ since $f(-x)=-f(x)$ $\therefore$ *odd function* ✓	1
6(b)(ii)	*Domain*: $x\in\mathbb{R}$ ✓	1

6(b)(iii)	 ✓ *asymptote* $y=0$ ✓ *shape*	**2**
6(c)	$7\sin^2 x + \sin x\cos x - 1 = 0$ since $\sin^2 x + \cos^2 x = 1$ $\therefore 7\sin^2 x + \sin x\cos x - (\sin^2 x + \cos^2 x) = 0$ ✓ $6\sin^2 x + \sin x\cos x - \cos^2 x = 0$ $(3\sin x - \cos x)(2\sin x + \cos x) = 0$ ✓ $\therefore 3\sin x + \cos x = 0$ *or* $2\sin x - \cos x = 0$ $3\sin x = -\cos x$ $\quad$ $2\sin x = \cos x$ $\tan x = \frac{-1}{3}$ $\quad$ $\tan x = \frac{1}{2}$ ✓ $x = 161°34'$, $\quad$ $x = 26°34'$, $341°55'$ $\quad$ $206°34'$ ✓	**1** **1** **1** **1**