KINCOPPAL-ROSE BAY
Cor Unum

2009
Preliminary Course Task 3

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 55 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- Start a new page for each question
- All necessary working should be shown in every question
- Marks may be deducted for carelessly arranged work

Total marks – 48

- Attempt Questions 1 - 4
- All questions are of equal value

Question 1 (12 marks) Start a New Page **Marks**

(a) Given $f(x) = x^2 - 7x + 10$,

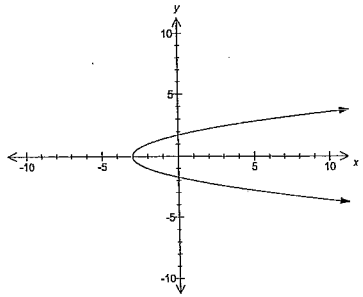
i) Find $f(4)$ **1**

ii) Find x if $f(x) = 0$ **2**

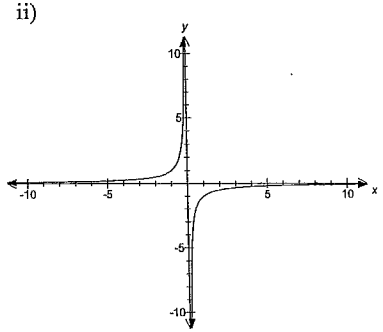
ii) Show whether the function $f(x)$ is even, odd, or neither. **2**

(b) State whether the following are functions or relations: **2**

i)



ii)



(c) Consider the function $f(x) = \begin{cases} 2x-1 & \text{if } x < 2 \\ x^2 & \text{if } x \geq 2 \end{cases}$

i) Evaluate $f(2) - 3f(-2)$ **2**

ii) Sketch the graph of this function, indicating all important points. **2**

(d) State the domain of the function $y = \sqrt{2x+3}$ **1**

End of Question 1

Question 2 (12 marks) Start a New Page **Marks**

(a) Consider the quartic function $y = x^4 + 3$. **2**

What would the new equation of the quartic $y = x^4 + 3$ be if it is to be translated on the number plane 5 units to the right and 4 units downward?

(b) Given $f(x) = \dfrac{x^2 - 16}{x + 4}$

i) Find $\lim\limits_{x \to -4} f(x)$ **2**

ii) What happens to the function $f(x)$ as $x \to 0$? **1**

(c) Sketch each graph below, labelling all intercepts, asymptotes and turning points.

i) $y = |x + 3|$ **2**

ii) $y = \dfrac{1}{x - 2}$ **2**

(d) Shade the region given by the intersection of $x^2 + y^2 < 4$ and $y \geq 2 - x$. **3**

You must clearly show all points of intersection.

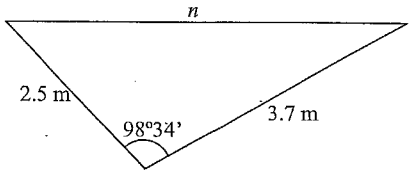
End of Question 2

Question 3 (12 marks) Start a New Page **Marks**

(a) For the triangle shown below:

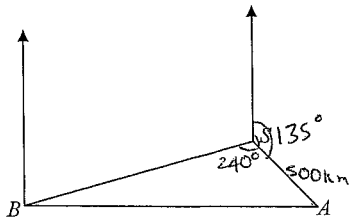
i) Evaluate n correct to one decimal place. **2**

ii) Find the area of the triangle correct to 3 significant figures. **2**



(b) Plane A leaves Sydney and flies on a bearing of 135° for 500 km. Plane B leaves Sydney at a bearing of 240° until it is due west from plane A.

i) Copy the following diagram onto your answer sheet. **1**
Fill in all missing information and show that $\angle SBA = 30°$.



ii) How far apart are the planes, correct to 1 decimal place? **2**

(c) If $\tan\theta = \dfrac{5}{12}$ and $\cos\theta < 0$, find the exact ratios of $\sin\theta$ and $\sec\theta$. **3**
(Hint: Draw a diagram)

(d) Find the exact value of $\sin^2 240°$. Show all working. **2**

End of Question 3

Question 4 (12 marks) Start a New Page **Marks**

(a) Simplify $\sqrt{4-4\sin^2\theta}$ **2**

(b) Evaluate θ if $\sec(2\theta-5)^\circ = \operatorname{cosec}(\theta+35)^\circ$ **2**

(c) i) Sketch the graph of $y = \cos x$ for $0^\circ \le \theta \le 360^\circ$, clearly indicating all intercepts. **2**

ii) On the same graph, draw the line $y = \frac{1}{2}$. **1**

iii) Hence, or otherwise, solve the equation $2\cos\theta - 1 = 0$ for $0^\circ \le \theta \le 360^\circ$ **2**

(d) Prove that $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2\sec^2\theta$. **3**

End of Assessment Task

Q1

a) i. $f(4) = 4^2 - 7\times4 + 10$
$= 16 - 28 + 10$
$= -2$ ✓ (1)

ii. $0 = x^2 - 7x + 10$
$0 = (x-5)(x-2)$ (2)
$x = 5, 2$ ✓ ✓

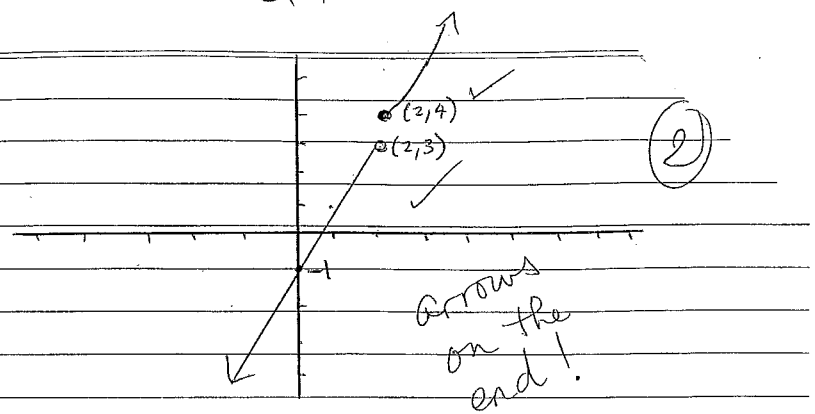
iii. $f(x) = x^2 - 7x + 10$ ✓
$f(-x) = x^2 + 7x + 10$ ✓
$f(x) \neq f(-x)$
∴ not even
$f(-x) = x^2 + 7x + 10$
$-f(x) = -x^2 + 7x - 10$
$f(-x) \neq -f(x)$ (2)
∴ not odd ✓
∴ its neither

b) i) relation ✓
ii) Function ✓ (2)

c) $2^2 - 3(2x-2-1)$ ✓
$= 4 - 3(-5)$ (2)
$= 4 + 15$
$= 19$ ✓

Q1

ii/



(2)

$y = 2x - 1$
$y = x^2$

d) D: all real $x \geq -1.5$ ✗ (0)

Q2

a) $y = x^4 + 3$

$y = x^{2+} - 1$

$y = (x-5)^4 - 1$ 2.

b) i. $\lim_{x \to 4} \frac{x^2 - 16}{x + 4}$

$\lim_{x \to 4} \frac{(x+4)(x-4)}{x+4}$

$\lim_{x \to 4} x - 4$

$4 - 4$

$= 0.$ 2.

ii. $\lim_{x \to 0} \frac{x^2 - 16}{x + 4}$

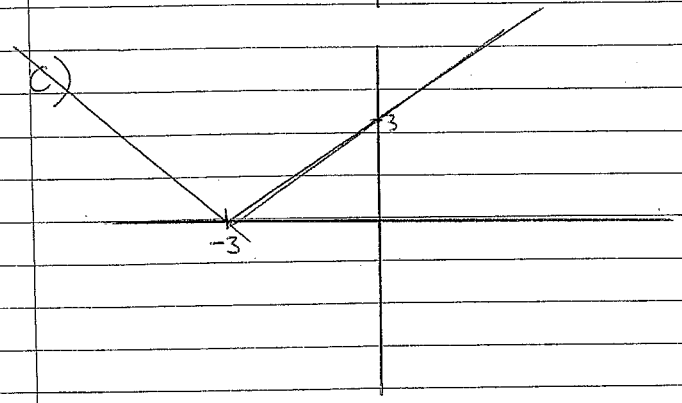
$\lim_{x \to 0} \frac{(x+4)(x-4)}{x+4}$

$\lim_{x \to 0} x - 4$

$0 - 4$ 1 2

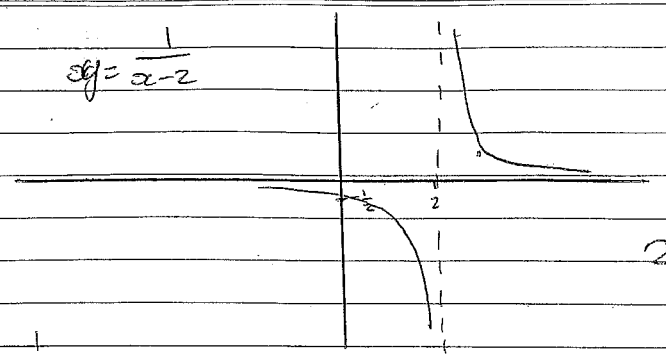
$= -4$

c)



Q2

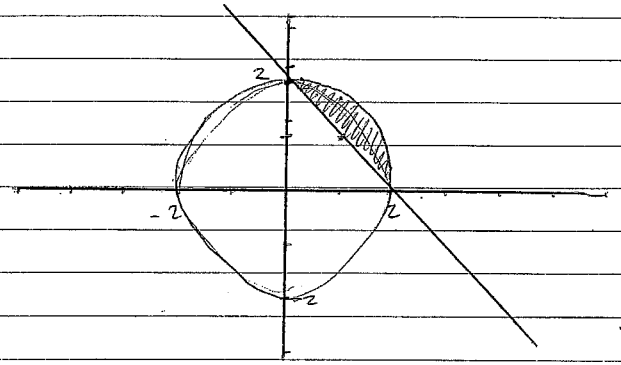
d) iii $y = \frac{1}{x-2}$



2

$y = \frac{1}{x-2}$

d)



3

$y^2 = 4 - x^2$

$y = \sqrt{4 - x^2}$

$x^2 + y^2 = 4$

$0 + 0 < 4$

$0 < 4$

$0 \geq 2 - 0$

$0 \geq 2$

Q3 (12/12) Perfect

a) i. $a^2 = b^2 + c^2 - 2bc \times \cos A$

$a^2 = 2.5^2 + 3.7^2 - 2 \times 2.5 \times 3.7 \times \cos 98°34'$

$a^2 = 19.94 + 2.75576$

$a^2 = 22.696$

$a = 4.8m$ ✓✓ (2)

ii. $\frac{1}{2}ab\sin C$

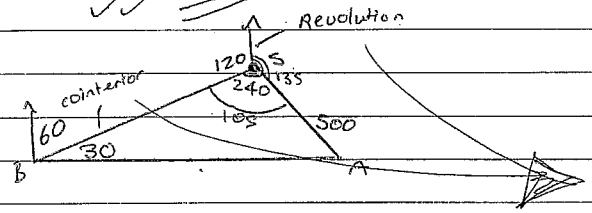
$= \frac{1}{2} \times 2.5 \times 3.7 \times \sin 98°34'$

$= 4.625 \times \sin 98°34'$

$= 4.57\ m^2$ units !! (2)

✓✓

b) i.



write up more formally next time

$\angle ASB = 240 - 135$

$= 105°$

$\angle B = 90$

$\angle SBA = 90 - 60$

$= 30°$ ✓ (1)

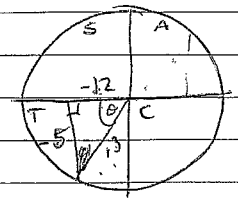
ii. $\frac{AB}{\sin 105} = \frac{500}{\sin 30}$

$AB = \frac{500 \times \sin 105}{\sin 30}$

$AB = 965.9\ km$ ✓✓ (2)

Q3

c) $\tan\theta = \frac{5}{12}$ $\cos\theta < 0$



$c^2 = 12^2 + -5^2$

$c^2 = 169$

$c = 13$

$\sin\theta = -\frac{5}{13}$ ✓

$\sec\theta = -\frac{13}{12}$ ✓✓ (3)

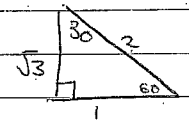
d) $\sin^2 240 = \sin^2(180 + \theta)$ (3rd quadrant)

$\theta = 60°$

$\sin 60 = \frac{\sqrt{3}}{2}$ ✓

$\sin^2 60 = \left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{3}{4}$ ✓ (2)



Q4 (10/12)

a) $\sqrt{4-4\sin^2\theta}$

$\sqrt{4(1-\sin^2\theta)}$

$\sqrt{4(\cos^2\theta)}$

$= 2\cos\theta$ ✓✓

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\sin^2\theta + \cos^2\theta = 1$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

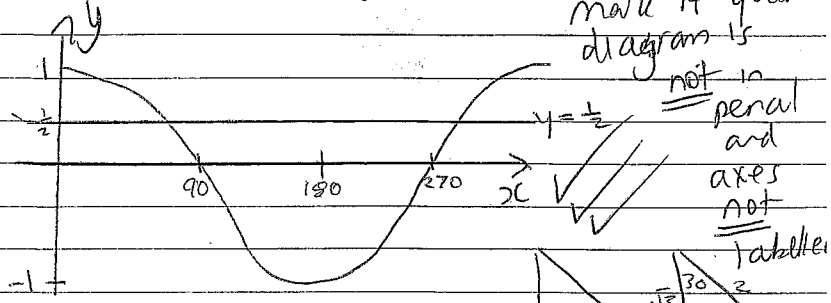
$1 + \tan^2\theta = \sec^2\theta$

b) $2\theta - 5 = \theta + 35$

$\theta = 40°$ XX

In future I will deduct mark if your diagram is not in pencil and axes not labelled

c) i/ $y = \cos x$



S A
T C

ii/ $2\cos\theta - 1 = 0$

$\cos\theta = \frac{1}{2}$

$\theta = 60°, 300°$ ✓✓

d) RHS $= \frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$

$= \frac{1-\sin\theta+1+\sin\theta}{1-\sin^2\theta}$

$= \frac{2}{1-\sin^2\theta}$

$= \frac{2}{\cos^2\theta}$ ✓✓

$= 2\sec^2\theta$ = LHS