KINCOPPAL–ROSE BAY
SCHOOL OF THE SACRED HEART

2010
HSC Course Task 3

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 55 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- Start a new writing booklet for each question
- All necessary working should be shown in every question
- Marks may be deducted for carelessly arranged work

Total marks – 44

- Attempt Questions 1 - 3

Question 1 (12 marks) Start a New Page **Marks**

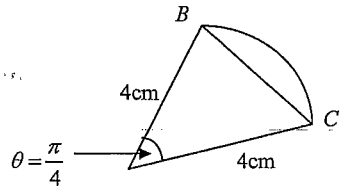
(a) Evaluate $\lim_{x\to 0}\frac{3\sin x}{7x}$ **1**

(b) Change 1.324 radians into degrees to the nearest minute. **2**

(c) A circular disc has area $A=16\pi\ \text{cm}^2$. A sector, with area $2\pi\ \text{cm}^2$, is cut out of the disc.

(i) Show that the sector has radius = 4 cm and angle $\theta=\frac{\pi}{4}$ radians. **2**

(ii) Show that the exact area of the minor segment cut off by the chord BC is $(2\pi-4\sqrt{2})\ \text{cm}^2$ **2**



(d) Consider the function $y=\frac{1}{2}\sin 2x$.

(i) State the period of the graph. **1**

For parts (ii) and (iii) use the extra sheet provided.

(ii) Sketch the graph for the domain $0\le x\le 2\pi$. **2**

(iii) How many solutions does the equation $\frac{1}{2}\sin 2x-x^2=0$ have in the domain $0\le x\le 2\pi$
Justify your answer. **1**

(iv) **Use your graph** to find any solution(s), correct to 1 decimal place. **1**

End of Question 1

Question 2 (14 marks) Start a New Page **Marks**

(a) Differentiate

(i) $-\sin(3-x^2)$ **2**

(ii) $\ln(\cos 5x)$ **2**

(iii) $e^{2x}\tan x$ **2**

(b) Find the exact value of:

$$\int_0^{\frac{\pi}{3}} (\sin 3x - x)dx$$ **3**

(c) (i) Use a trigonometric identity to show that $\int \tan^2 x\,dx = \tan x - x + c$ **2**

(ii) The curve $y = \tan x$, between $x = 0$ *and* $x = \frac{\pi}{4}$, is rotated about the $x-$axis.
Using the result in part (i), find the volume of the resulting solid of revolution. **3**

End of Question 2

Question 3 (18 marks) Start a New Page **Marks**

(a) Find the 75^{th} term of the series $0.3+0.7+1.1+1.5+.....$ **2**

(b) Find the first positive term of the series

$$-125-121-117-113.....$$ **2**

(c) How many terms must be added in the series

$$5+11+17+23+.....$$

to make the sum 208? **3**

(d) The 8^{th} term of an arithmetic series is 44 and the sum of the first 21 terms is 1365. Find the sum of 37 terms. **4**

(e) If $17+x+153+.....$ is a geometric series, find x. **2**

(f) Find the first value of n for which the terms of the series

$$\frac{1}{7}+1+7+.....$$

exceed 2000. **2**

(g) Evaluate the series $\frac{3}{2}+4\frac{1}{2}+13\frac{1}{2}+.....+364\frac{1}{2}$ **3**

End of Task

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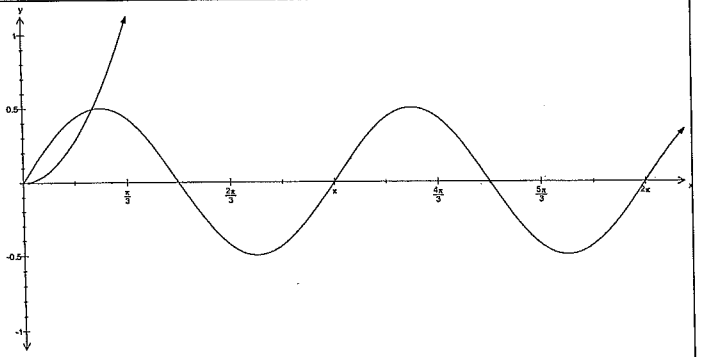
Question 1 12 Marks	Criteria	Marks
1(a)	$\frac{3}{7}$	1
1(b)	$1.324\times\frac{180}{\pi}$ $=75°52'$	1 1
1(c)(i)	$\pi r^2=16\pi \quad \therefore r=4$ $\frac{1}{2}(4)^2\theta=2\pi \quad \therefore \theta=\frac{\pi}{4}$	1 1
1(c)(ii)	$Area=\frac{1}{2}(4)^2\left[\frac{\pi}{4}-\sin\frac{\pi}{4}\right]$ $=2\pi-8\sin\frac{\pi}{4}$ $=2\pi-8\times\frac{1}{\sqrt{2}}=2\pi-4\sqrt{2}$	1 1
1(d)(i)	Period $=\pi$	1
1(d)(ii)		2
(iii)	Two solution as the graphs intersect at only one point in the given domain.	1
(iv)	$x=0 \quad or \quad x=0.6-0.8$	1

Question 2 14 Marks	Criteria	Marks
2(a)(i)	$-\cos(3-x^2)\times(-2x)$ $=2x\cos(3-x^2)$	1 1
2(a)(ii)	$\frac{1}{\cos 5x}\times(-5\sin 5x)$ $=\frac{-5\sin 5x}{\cos 5x}$ $OR \quad (=-5\tan 5x)$	1 1
2(a)(iii)	$2e^{2x}\tan x+e^{2x}\sec^2 x$ $=e^{2x}(2\tan x+\sec^2 x)$	1 1
2(b)	$\left[\frac{-\cos 3x}{3}-\frac{x^2}{2}\right]_0^{\frac{\pi}{3}}$ $=(\frac{-\cos\pi}{3}-\frac{\pi^2}{18})-(\frac{-\cos 0}{3}-0)$ $=\frac{1}{3}-\frac{\pi^2}{2}+\frac{1}{3}+0$ $=\frac{2}{3}-\frac{\pi^2}{18}$	1 1 1
2(c)(i)	$\tan^2 x=-1+\sec^2 x$ $\int\tan^2 x dx=\int(-1+\sec^2 x)dx$ $=-x+\tan x+c$	1 1
2(c)(ii)	$\pi\int_0^{\frac{\pi}{4}}\tan^2 x dx=\pi\left[\tan x-x\right]_0^{\frac{\pi}{4}}$ $=\pi\left[\tan\frac{\pi}{4}-\frac{\pi}{4}\right]-\pi\left[\tan 0-0\right]$ $=\pi(1-\frac{\pi}{4}) \quad units^3$	1 1 1

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Question 3 18 Marks	Criteria	Marks
3(a)	$d=0.4, a=0.3$ $T_{75}=0.3+74\times 0.4$ $=29.9$	1 1
3(b)	$d=4, a=-125$ $T_n=-125+(n-1)\times 4>0$ $4n-129>0$ $n>32\frac{1}{4} \quad \therefore n=33$ $T_{33}=3$	1 1
3(c)	$d=6, a=5$ $S_n=208=\frac{n}{2}[10+(n-1)\times 6]$ $416=n[6n+4]$ $6n^2+4n-416=0$ $3n^2+2n-208=0$ $(3n+26)(n-8)=0 \therefore n=-\frac{26}{8}(reject)$ $n=8$	1 1 1
3(d)	$S_{21}=\frac{21}{2}[2a+20d]=1365$ $42a+420d=2730, \quad T_8=a+7d=44$ $42a+294d=1848$ $\therefore 126d=882$ $\therefore d=7$ $a+7\times 7=44$ $\therefore a=-5 \quad and \quad d=7$ $S_{37}=\frac{37}{2}[2(-5)+36\times 7]=4477$	1 1 1 1
3(e)	$\frac{x}{17}=\frac{153}{x}$ $x^2=2601$ $x=\pm 51$	1 1

Question 3 Continued..	Criteria	Marks
3(f)	$\frac{1}{7}(7)^{n-1}>2000$ $7^{n-1}>14000$ $(n-1)\ln 7>\ln 14000$ $n=\frac{\ln 14000}{\ln 7}+1$ $n>5.906...$ $n=6$	1 1
3(g)	$a=\frac{3}{2}, \quad r=3, \quad T_n=364\frac{1}{2}$ $\frac{3}{2}\times 3^{n-1}=364\frac{1}{2}$ $3^{n-1}=243$ $(n-1)\ln 3=\ln 243$ $n=\frac{\ln 243}{\ln 3}+1$ $\therefore n=6$ $\therefore S_6=\frac{\frac{3}{2}[3^6-1]}{3-1}=546$	 1 1 1