2008

Yearly Examination

Year 9X Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A formulae sheet is provided at the back of this paper
- All necessary working should be shown in every question
- Start a new page for each question

Total marks – 100

Pages 1 - 13

100 marks

- Attempt Questions 1 – 10
- All questions are of equal value
- Start each question in a new writing booklet

Total Marks 100
Attempt Questions 1 – 10
All questions are of equal value

Answer each question starting a new page.

Question 1 (10 marks)

(a) Expand and simplify $\left(4-\sqrt{5}\right)\left(3+2\sqrt{5}\right)$ **2**

(b) Rain falling into a bucket caused the height of the water in the bucket to rise at 5 mm per minute. After the storm the water's height was 6 cm. The bucket was empty before the storm. How long did the storm last?   **2**

(c) Solve each equation to find the value of x:

   (i) $\dfrac{3}{x}=\dfrac{4}{5}$ **1**

   (ii) $3x-5=\dfrac{5+x}{2}$ **2**

   (iii) $(3x+1)(2x-5)=10$ **2**

(d) If \$6200 is invested at 6% p.a., interest **compounded quarterly**, find the balance of the account at the end of 5 years.   **1**

Question 2 (10 marks) Begin on a new page

    (a)   (i) Simplify leaving your answer in surd form: $\sqrt{128} - 2\sqrt{72} + \sqrt{200}$ 2

         (ii) Expand and simplify, leaving you answer with positive indices: $\dfrac{36h^{-4}i^{-1}}{12h^{-2}i^{-3}}$ 2

    (b)   Fully simplify: $\sqrt[4]{x^8 y^4} \div \left(x^3 y^6\right)^{\frac{1}{3}}$ 2

    (c)   Make a the subject of $3a + 4b = 12$.   1

    (d)   This back-to-back stem-and-leaf plot represents the heights of a sample of students.

Leaf			Stem	Leaf			
		9	13				
7	7	8	14	8			
	8	3	15	1	4		
		0	16	0	0	2	7
			17	2	5	7	
			18	4	6		
			19	1			

        What fraction of students are taller than 150 cm?   1
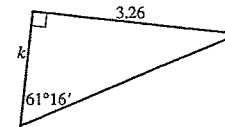
    (e)   (i) Factorise $d^2 - d - 12$ 1

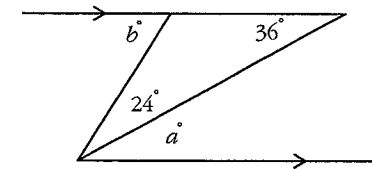   (ii) Hence, simplify: $\dfrac{d-4}{20} \div \dfrac{d^2+d-12}{5d}$.   1

Question 3 (10 marks) Begin on a new page

    (a)   Sam bought a car for $30 000 and elected to depreciate it by the rate of 15% per year. After 5 years, what was the value of the car?   2

    (b)   Find the length of the side, k, in the figure below   2

    (c)

        (i)   Find the value of a in the diagram, giving geometric reasons.   2

        (ii)   Find the value of b in the diagram, giving geometric reasons.   2

    (d)   Find the length of the third side of the triangle, <u>in exact and simplest form</u>.   1

    (e)   A standard basket ball has a diameter of 22 cm. Use the formula $V = \frac{4}{3}\pi r^3$, to find the volume of the ball, correct to the nearest cubic centimetre.   1

Question 4 (10 marks) Begin on a new page

(a) Weather records show that on the first day of the Brisbane Exhibition the weather has been fine on 65 of the past 80 years.

  Using these results, what is the probability that the weather will **not** be fine on the first day of the Exhibition in 2009?   **1**

(b) A rectangular courtyard has a total perimeter of 32 metres.   **2**
 The length is 2 metres more than the width. What are the dimensions of the courtyard?

(c) (i) Find the gradient of the straight line passing through the points (1, 2) and (-2, –13)   **1**

  (ii) Find the equation of the line passing through the points (1, 2) and (-2, -13)   **1**

(d) Prove that $\triangle ABC \equiv \triangle CDA$ **3**
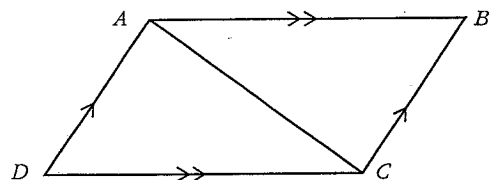
(e) A steel cattle feeding trough is in the shape of a half cylinder. It has a   **2**
  diameter of 0·8 m and is 2·2 m long. The <u>entire</u> trough (inside and outside),
  is to be painted for rust proofing. Find the area to be painted, *correct
  to one decimal place.*
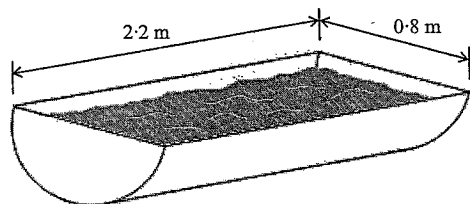
Question 5 (10 marks) Begin a new page.

(a) For how long would an electrician have visited, if her total charge was $111 given her initial service call charge is $48 and her hourly rate is $35 per hour?   **1**

(b) A ladder 8 m long leans against a wall 5 m high. Which of the following can be used to work out the angle that the ladder makes with the wall?   **2**
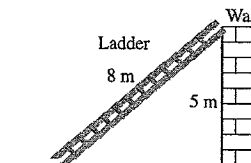
(c) Solve for *x*:
$$3^{2x-1} = 27$$ **2**

(d) If an inheritance of $20 000 needs to grow to $30 000 in 6 years, what annual interest rate would need to be used? (Assume that interest is compounded annually)   **3**

(e) A coin is flipped three times.

  (i) Draw a tree diagram to represent the possible outcomes.   **1**

  (ii) What is the probability of getting **at least two tails**?   **1**

Question 6 (10 marks) start a new page.

    (a) What is the size of one interior angle in a regular **heptagon** (7 sided shape)?   1

    (b) If an object has a volume of 20 cm^3 and it is enlarged by a factor of 3,
the volume of the image will be: 1

    (c) Find the solution to the following pair of simultaneous equations:
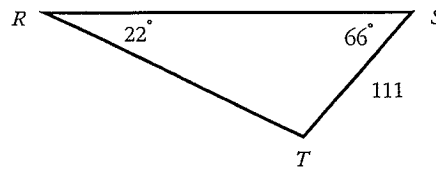
$$2x - y = -6$$
$$3x + y = -29$$

        2

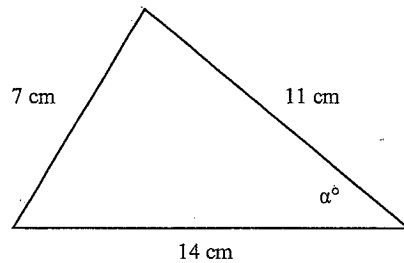   (d) Simplify: $\dfrac{x^2 + 2xy}{x^2 - xy - 6y^2}$ 2

    (e) Use the Sine Rule to find the length of side *RT* in the triangle *RST*.
Answer correct to the nearest unit.   2

    (f) Find α to the nearest minute: 2

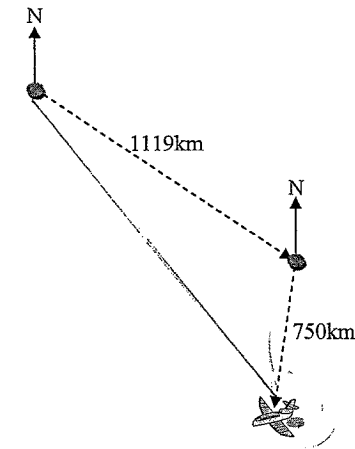
Question 7 (10 marks) Start a new page

    (a) Solve these equations simultaneously for *x* and *y*:   2

$$5x - 2y = -16$$
$$3x + 4y = -7$$

    (b) A plane flies for 1119 km on a bearing of 162°N and then for 750 km on a bearing 208°N.
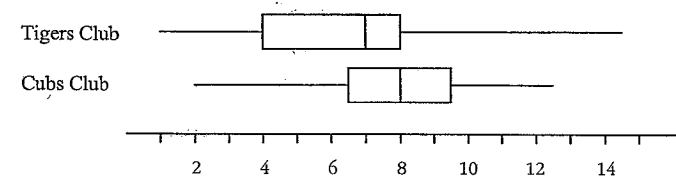

        How far is the plane from its starting point? (*To the nearest km*)   3

    (c) A piece of aluminium 144 cm by 36 cm has 36 circles of diameter 12 cm cut from it.
Find the area of the aluminium that is left over.   2

    (d) The two box-and-whisker plots show the results for the charity collections of two clubs.
The scale shows hundreds of dollars (i.e. 2 means $200).

        (i)   Compare the ranges for each club, stating which has the greatest range.   1

        (ii)   Which club achieved the better results for their collection?
Give reasons for your answer, in terms of measures of spread.   2

Question 8 (10 marks) start a new page.

(a) In a mixture of cement, sand and gravel the ratio of cement to sand is 2 : 11 and the ratio of sand to gravel is 7 : 3.
If there is 7kg of cement, how much gravel is there? **2**

(b) Expand: $\left(x+\dfrac{1}{x}\right)^2$. **1**

(c) If students spend 320 minutes a day in class, the pie graph below shows that the time spent on Maths is closest to: **1**
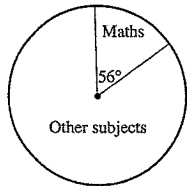

(d) Find the height *"h"*, correct to one decimal place, of the tower below if an observer, who is standing 75 m away from it, must look up 36° to see the top of the tower. **2**
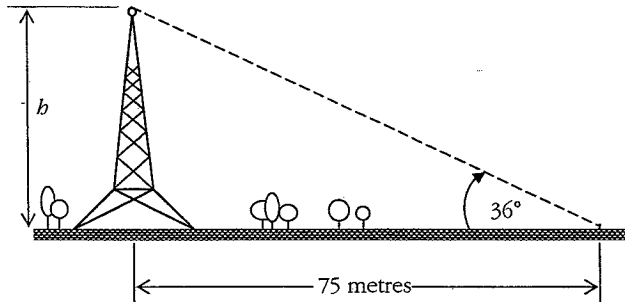
(e) Show that the line $y = 6x - 4$ passes through the point (4 , 20). **1**

(f) The cumulative frequency graph shows the number of seeds in each special variety of pawpaw called 'papaya' investigated in research by the agricultural department.
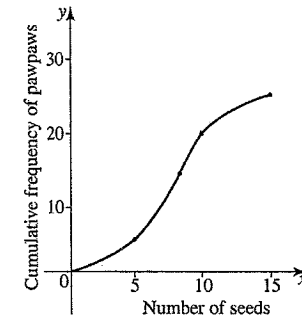
Estimate the number of pawpaws with 10 seeds or fewer? **1**

(g) Solve $2(3-x) \le 4-x$ and graph the answer on a number line. **2**
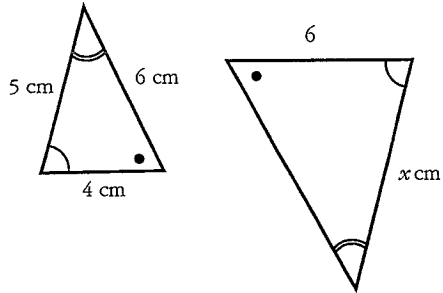
Question 9 (10 marks) Start a new page

(a) Rationalise the denominator: $\dfrac{5}{3\sqrt{10}}$ 2

(b) Expand and simplify $2ab(3b-2a)$ 1

(c) These two triangles are similar with corresponding angles shown.   2
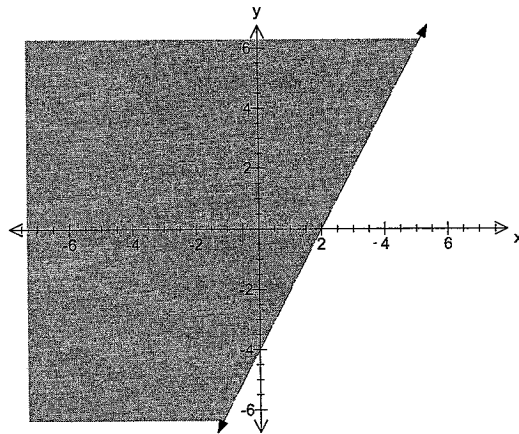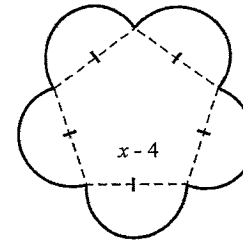
Find the value of x.

(d) Find the equation of the shaded region: 2

(e) Find the perimeter of the shape in the figure below in terms of x 2

(f) If $\sin A = \dfrac{5\cdot6}{7\cdot2}$, find $\angle A$ correct to the nearest minute if $\angle A$ is **obtuse**.   1
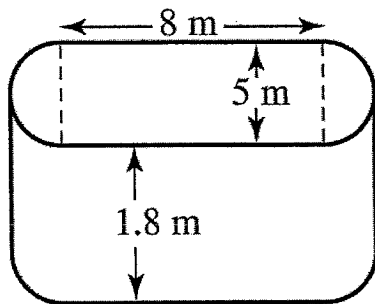
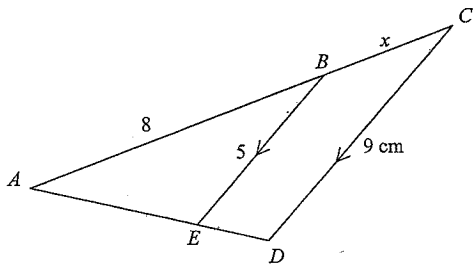**Go on to the back of this page for Question 10**

Question 10 (10 marks) Start a new page

(a) Find the volume of water required to fill the swimming pool depicted in the figure below is: **3**

(b) In the diagram BE is drawn parallel to CD.
$AB = 8$ cm, $BE = 5$ cm and $CD = 9$ cm.

(i) Prove that the two triangles ABE and ACD are similar. **3**

(ii) Hence find the length of interval BC (labelled x) **2**

(c) What simple interest rate would I need to be offered if I wanted my term deposit of $5000 to become $8000 in 7 years. Round to 2 decimal places. **2**

END OF EXAM

9X 22/11/08

9X Solutions

2008
Yearly Examination

Year 9X Mathematics

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Total marks – 100

Pages 1 - 13

100 marks

- Attempt Questions 1 – 10
- All questions are of equal value
- Start each question in a new writing booklet

Total Marks 100
Attempt Questions 1 – 10
All questions are of equal value

Answer each question starting a new page.

Question 1 (10 marks)

(a) Expand and simplify $(4-\sqrt{5})(3+2\sqrt{5})$ **2**

$12 + 8\sqrt{5} - 3\sqrt{5} - 2\times 5$ ①
$2 + 5\sqrt{5}$ ①

(b) Rain falling into a bucket caused the height of the water in the bucket to rise at 5 mm per minute. After the storm the water's height was 6 cm. The bucket was empty before the storm. How long did the storm last?   *12 mins*   **2**

(c) Solve each equation to find the value of x:

(i) $\dfrac{3}{x}=\dfrac{4}{5}$ $x = \dfrac{15}{4}$ or 3.75 **1**

(ii) $3x-5=\dfrac{5+x}{2}$ $6x-10 = 5+x$ ①
$5x = 15$
$x = 3$ ①   **2**

(iii) $(3x+1)(2x-5)=10$ $x=3$ ① and ~~$-5/6$~~ ①   **2**

(d) If $6200 is invested at 6% p.a., interest **compounded quarterly**, find the balance of the account at the end of 5 years.   **1**

$A = \$8350.50$

Question 2 (10 marks) Begin on a new page

(a) (i) Simplify leaving your answer in surd form: $\sqrt{128} - 2\sqrt{72} + \sqrt{200}$ **2**

$8\sqrt{2} - 12\sqrt{2} + 10\sqrt{2} = 6\sqrt{2}$

(ii) Expand and simplify, leaving you answer with positive indices: $\dfrac{36h^{-4}i^{-1}}{12h^{-2}i^{-3}}$ **2**

$3h^{-2}i^{2} = \dfrac{3i^{2}}{h^{2}}$

(b) Fully simplify: $\sqrt[4]{x^{8}y^{4}} \div (x^{3}y^{6})^{-\frac{1}{3}}$ $x^{2}y^{1} \div x^{-1}y^{-2} = x^{3}y^{3}$ **2**

(c) Make a the subject of $3a + 4b = 12$. $3a = 12 - 4b$, $a = 4 - \dfrac{4}{3}b$ **1**

(d) This back-to-back stem-and-leaf plot represents the heights of a sample of students.

Leaf	Stem	Leaf
	9 | 13 |	
7 7	8 | 14 |	8
8	3 | 15 |	1 4
	0 | 16 |	0 0 2 7
	17 |	2 5 7
	18 |	4 6
	19 |	1

What fraction of students are taller than 150 cm? $\dfrac{15}{20} = \dfrac{3}{4}$ **1**

(e) (i) Factorise $d^{2} - d - 12$ $(d-4)(d+3)$ **1**

(ii) Hence, simplify: $\dfrac{d-4}{20} \div \dfrac{d^{2}+d-12}{5d}$. **1**

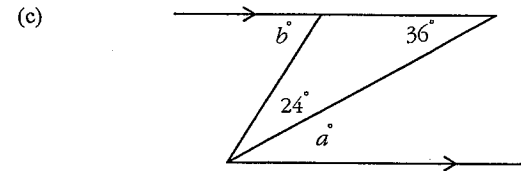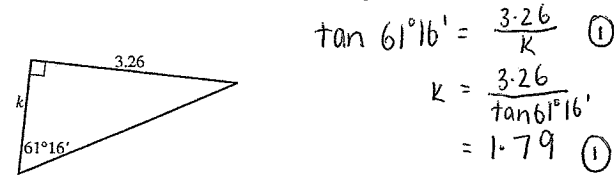$\dfrac{5d}{420(d+3)} = \dfrac{d}{4d+12}$

or

$\dfrac{(d-4)}{20} \div \dfrac{(d+4)(d-3)}{5d}$

$= \dfrac{(d-4)}{20} \times \dfrac{5d}{(d+4)(d-3)}$

$= \dfrac{5d(d-4)}{20(d+4)(d-3)} = \dfrac{d(d-4)}{4(d+4)(d-3)}$

Question 3 (10 marks) Begin on a new page

(a) Sam bought a car for $30 000 and elected to depreciate it by the rate of 15% per year. After 5 years, what was the value of the car? $30000(0.85)^{5} = \$13311.16$ **2**

(b) Find the length of the side, k, in the figure below **2**

$\tan 61°16' = \dfrac{3.26}{k}$ ①

$k = \dfrac{3.26}{\tan 61°16'}$ ①

$= 1.79$ ①

(c)

(i) Find the value of a in the diagram, giving geometric reasons. $36°$ ① Alternate ① **2**

(ii) Find the value of b in the diagram, giving geometric reasons. $60°$ ① Alternate ① **2**

(d) Find the length of the third side of the triangle, in exact and simplest form. **1**
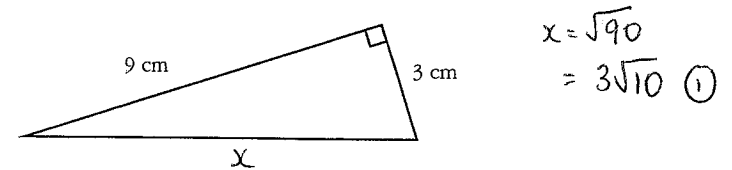
$x = \sqrt{90}$

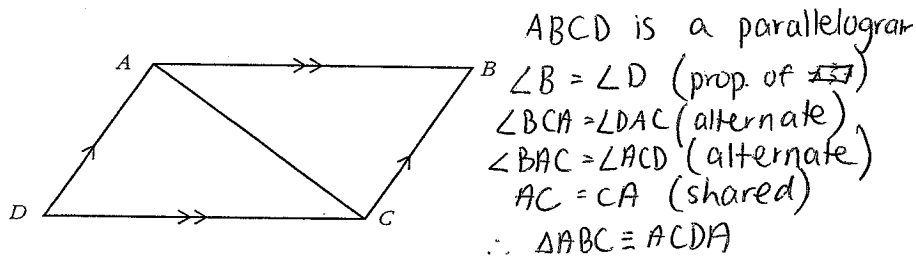$= 3\sqrt{10}$ ①

9 cm   3 cm x

(e) A standard basket ball has a diameter of 22 cm. Use the formula $V = \frac{4}{3}\pi r^{3}$, to find the volume of the ball, correct to the nearest cubic centimetre. **1**

$V = 5575 cm^{3}$

Question 4 (10 marks) Begin on a new page

(a) Weather records show that on the first day of the Brisbane Exhibition the weather has been fine on 65 of the past 80 years.

Using these results, what is the probability that the weather will **not** be fine on the first day of the Exhibition in 2009? $\frac{3}{16}$ or 0.1875 — **1**

(b) A rectangular courtyard has a total perimeter of 32 metres. L= 9m , W= 7m — **2**
The length is 2 metres more than the width. What are the dimensions of the courtyard?

(c) (i) Find the gradient of the straight line passing through the points (1, 2) and (-2, −13) **1**
     $m = 5$
     (ii) Find the equation of the line passing through the points (1, 2) and (-2, -13) **1**
     $y = 5x - 3$

(d) Prove that $\triangle ABC \equiv \triangle CDA$ — **3**

ABCD is a parallelogram
∠B = ∠D (prop. of //gm)
∠BCA = ∠DAC (alternate)
∠BAC = ∠ACD (alternate)
AC = CA (shared)
∴ △ABC ≡ △CDA

(e) A steel cattle feeding trough is in the shape of a half cylinder. It has a diameter of 0·8 m and is 2·2 m long. The <u>entire</u> trough (inside and outside), is to be painted for rust proofing. Find the area to be painted, *correct to one decimal place.* — **2**
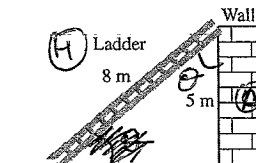
2·2 m   0·8 m
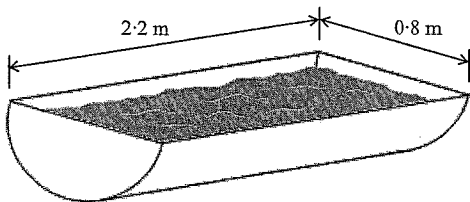
$$\text{Inside} = \frac{1}{2} \times \left[\pi r^2 + 2\pi r \times h + \pi r^2 \right]$$
$$= \frac{1}{2} \times \left[0.5 + 5.5 + 0.5 \right]$$
$$= 3.25$$

Whole trough = 2 × 3.25
$$= 6.5 \, m^2$$

**Question 5** (10 marks) Begin a new page.

1.8hr = 1hr 48mins

1. (a) For how long would an electrician have visited, if her total charge was $111 given her initial service call charge is $48 and her hourly rate is $35 per hour? **1**

2. (b) A ladder 8 m long leans against a wall 5 m high. Which of the following can be used to Find work out the angle that the ladder makes with the wall? To the nearest degree.

Wall
Ladder
8 m
5 m

$\sin \theta = \frac{o}{4}$
$\cos \theta = \frac{A}{H}$
$f = \theta = \frac{8}{8}$
$\cos \theta = \frac{5}{8}$
$\theta = 38.68°$
$\theta = 40°$
$\theta = 51.3°$
or $\theta = 51\frac{2}{7}°$

(c) Solve for x:
$$3^{2x-1} = 27$$

(d) If an inheritance of $20 000 needs to grow to $30 000 in 6 years, what annual interest rate would need to be used? (Assume that interest is compounded annually) **3**

(e) A coin is flipped three times.

   (i) Draw a tree diagram to represent the possible outcomes. **1**

   (ii) What is the probability of getting **at least two tails**? **1**

(c) $3^{2x-1} = 3^3$
$2x - 1 = 3$
$2x = 4$
$x = 2$

(e) (i)

H ⟨ H ⟨ H / T , T ⟨ H / T ⟩ T ⟨ H ⟨ H / T , T ⟨ H / T ⟩

✗
✗
✗
✓
✗
✓
✓
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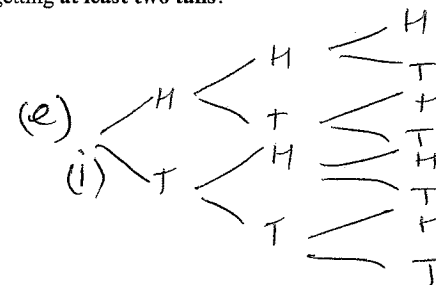
(ii) $\frac{1}{2}$

(d) $20000(1+r)^6 = 30000$
$(1+r)^6 = 1.5$
$1+r = \sqrt[6]{1.5}$
$r = \sqrt[6]{1.5} - 1$
$= 0.0699...$
$r = 6.99\%$

Question 6 (10 marks) start a new page.

(a) What is the size of one interior angle in a regular **heptagon** (7 sided shape)? $128.6°$ 1

(b) If an object has a volume of 20 cm³ and it is enlarged by a factor of 3, the volume of the image will be: $540cm^3$ 1

✱ (c) Find the solution to the following pair of simultaneous equations: 2

$$2x - y = -6$$
$$3x + y = -29$$

$x = -7$ ①
$y = -8$ ①

✦ (d) Simplify: $\dfrac{x^2 + 2xy}{x^2 - xy - 6y^2}$ 2

$$= \frac{x(x+2y)}{(x+2y)(x-3y)} \text{①} = \frac{x}{(x-3y)} \text{①}$$

✦ (e) Use the Sine Rule to find the length of side RT in the triangle RST.
Answer correct to the nearest unit.   2

$270.6° \doteqdot 271 \text{ unik ②}$

✦ (f) Find α to the nearest minute: 2

7 cm   11 cm

α°

14 cm
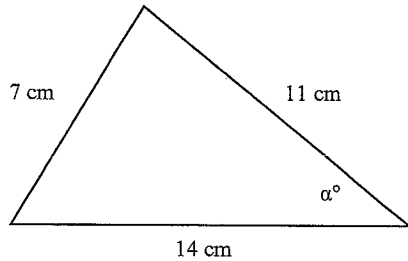
$29° 32'$ ②

Question 6 (10 marks) start a new page.

(a) What is the size of one interior angle in a regular **heptagon** (7 sided shape)? $128.6°$ 1

(b) If an object has a volume of 20 cm³ and it is enlarged by a factor of 3, the volume of the image will be: $540cm^3$ 1

(c) For the parabola with equation $y = x^2 + 3x - 18$, find
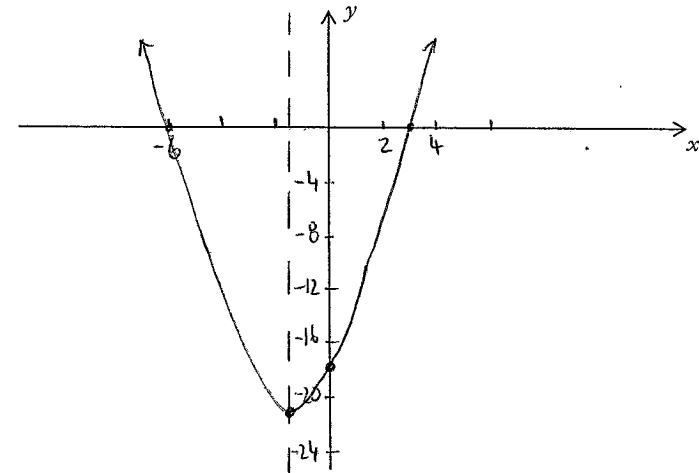
  (i) x-intercepts, -6 and 3 ① ①   2

  (ii) y-intercept, -18 1

  (iii)   equation of the axis of symmetry, $x = -1.5$ 1

  (iv)   coordinates of the vertex $(-1.5, -20.25)$ 1

  (v)   Draw a neat and clear sketch of the parabola on the axes, showing the information from parts (a) to (d).   3
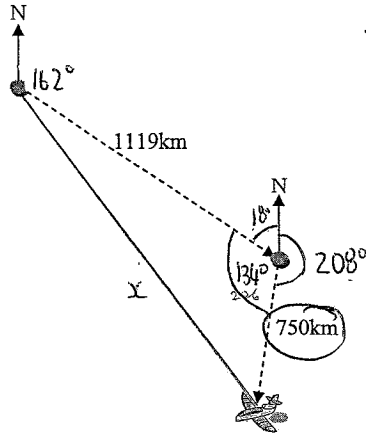
Question 7 (10 marks) Start a new page

(a) Solve these equations simultaneously for x and y: 2

$$5x - 2y = -16 \qquad x = -3$$
$$3x + 4y = -7 \qquad y = 0.5$$

(b) A plane flies for 1119 km on a bearing of 162°N and then for 775 km on a bearing 208°N.

with 775:

$x = 1748.61$

$\approx 1749 \text{km}$

N

162°

1119km

N

18°

134°

208°

2°

750km

x

with 750:

$x^2 = 1119^2 + 750^2 - 2x$

$1119 \times 750 \times \cos 134°$

$x = \sqrt{2980645}$

$= 1726.45$

$\approx 1726 \text{km}$

(or) 1749km

How far is the plane from its starting point? (*To the nearest km*) 3

(c) A piece of aluminium 144 cm by 36 cm has 36 circles of diameter 12 cm cut from it.
Find the area of the aluminium that is left over. 2

Alum: 5184cm² Circles = 4071.5cm² Left over = 1112.5cm²

(d) The two box-and-whisker plots show the results for the charity collections of two clubs.
The scale shows hundreds of dollars (i.e. 2 means $200).

Tigers Club

Cubs Club

2 4 6 8 10 12 14
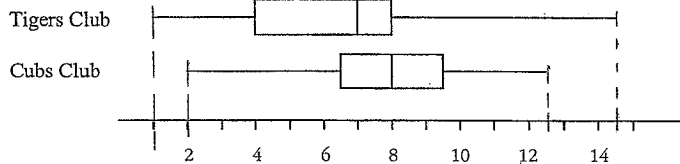
(i) Compare the ranges for each club, stating which has the greatest range. 1

T = 13.5 C = 10.5 Tiger Club

(ii) Which club achieved the better results for their collection?
Give reasons for your answer, in terms of measures of spread. 2

Cubs — higher median, the top 50% did better than 75% of the Tigers.

C : S : G   6 : 5   S : G
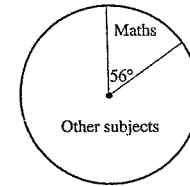
2 : 11   7 : 3   (1) Working

C : S : G
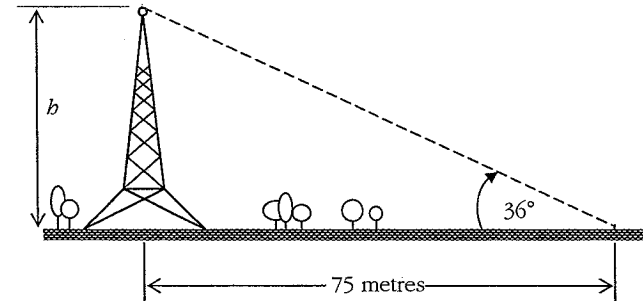
14 : 77 : 33

Question 8 (10 marks) start a new page.

(a) In a mixture of cement, sand and gravel the ratio of cement to sand is 2 : 11 and the ratio of sand to gravel is 7 : 3.
If there is 7kg of cement, how much gravel is there? 2

16.5 kg (1)

(b) Expand: $\left(x + \dfrac{1}{x}\right)^2$. $= x^2 + 2 + \dfrac{1}{x^2}$ (1) 1

(c) If students spend 320 minutes a day in class, the pie graph below shows that the time spent on Maths is closest to: 1

Maths

56°

Other subjects

(d) Find the height "h", correct to one decimal place, of the tower below if an observer, who is standing 75 m away from it, must look up 36° to see the top of the tower. 2

h

36°

75 metres

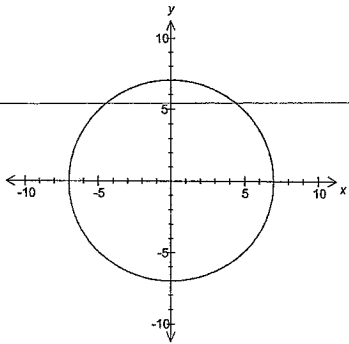(e) Show that the line $y = 6x - 4$ passes through the point (4 , 20). 1

Question 8 (10 marks) start a new page.

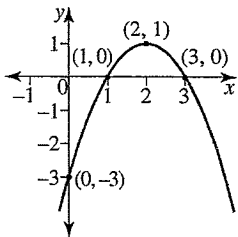(a) Write the equations of the following functions:
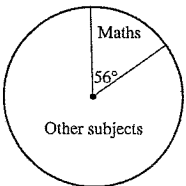
(i)   1

$$x^2 + y^2 = 49$$

(ii)   2

$$y = -(x-2)^2 + 1$$

(b) If students spend 320 minutes a day in class, the pie graph below shows that the time spent on Maths is closest to: 1
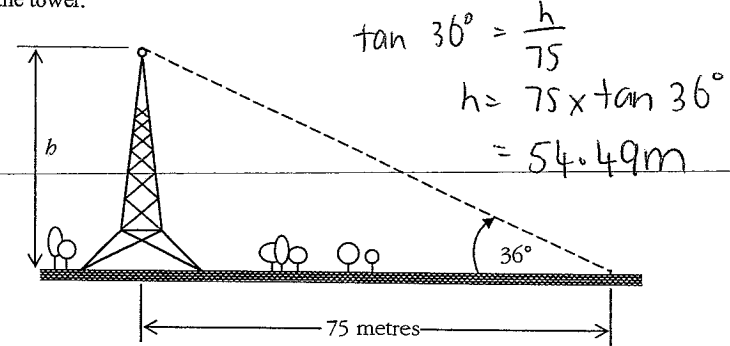

$$\frac{56°}{360°} \times 320\,mins$$

$$= 49.7\dot{}$$

$$\approx 50\,minutes$$

(c) Find the height *"h"*, correct to one decimal place, of the tower below if an observer, who is standing 75 m away from it, must look up 36° to see the top of the tower.   2
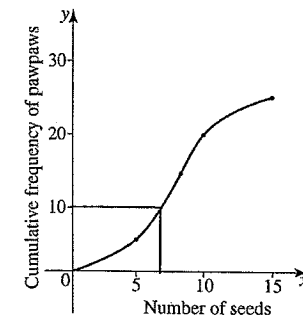

$$\tan 36° = \frac{h}{75}$$

$$h = 75 \times \tan 36°$$

$$= 54.49m$$

75 metres

(d) Show that the line $y = 6x - 4$ passes through the point (4 , 20).   1

$$20 = 6 \times 4 - 4$$
$$20 = 24 - 4 \checkmark$$

(e) The cumulative frequency graph shows the number of seeds in each special variety of pawpaw called 'papaya' investigated in research by the agricultural department.

Estimate the number of pawpaws with 10 seeds or fewer?   1

20 paw paws (± 2)

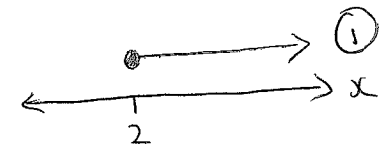(f) Solve $2(3-x) \le 4-x$ and graph the answer on a number line.   2

$$6 - 2x \le 4 - x$$
$$6 - x \le 4$$
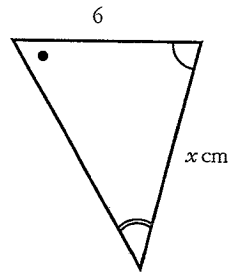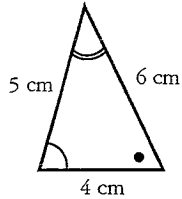$$-x \le -2$$
$$x \ge 2 \quad ①$$

Question 9 (10 marks) Start a new page

(a) Rationalise the denominator: $\dfrac{5}{3\sqrt{10}} \times \dfrac{\sqrt{10}}{\sqrt{10}} = \dfrac{5\sqrt{10}}{3\sqrt{100}} = \dfrac{5\sqrt{10}}{30} = \dfrac{\sqrt{10}}{6}$ ① ① **2**

(b) Expand and simplify $2ab(3b-2a)$ $6ab^2 - 4a^2b$ **1**

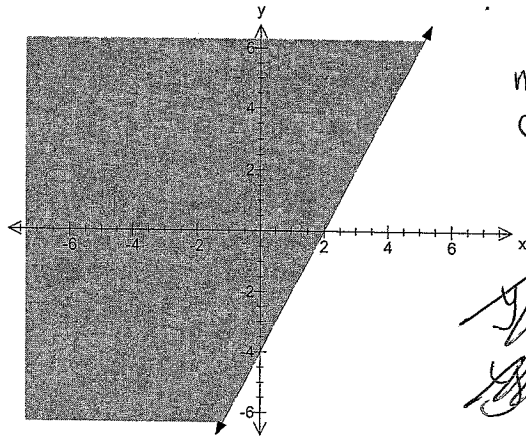(c) These two triangles are similar with corresponding angles shown. **2**

Find the value of x.

$s.f = 1.5$

5 cm   6 cm   4 cm   6 x cm

$x = 7.5cm$

(d) Find the equation of the shaded region: **2**

$m = 2$
$c = -4$

$y > 2x - 4$

$\dfrac{\text{tert}}{\sqrt{(0,0)}}$

$(1,0)$

(e) Find the perimeter of the shape in the figure below in terms of x **2**

$P = 5x \left(\cancel{} \times \pi \times (x-4) \right)$ 0.5

$= \cancel{}$

$= \cancel{}$

$= \dfrac{5}{2} \pi (x-4)$

or

$= \dfrac{5}{2}\pi x - 10x$

(f) If $\sin A = \dfrac{5\cdot6}{7\cdot2}$, find $\angle A$ correct to the nearest minute if $\angle A$ is **obtuse**. **1**
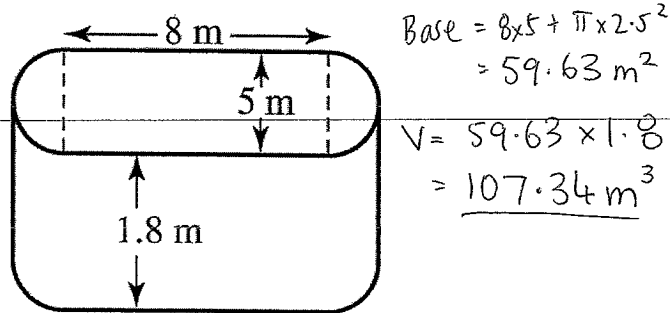
$128°57'$

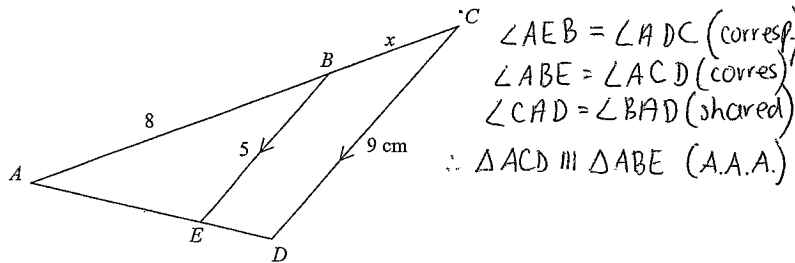**Go on to the back of this page for Question 10**

Question 10 (10 marks) Start a new page

(a) Find the volume of water required to fill the swimming pool depicted in the   **3**
figure below is:

$$Base = 8 \times 5 + \pi \times 2.5^2$$
$$= 59.63 \, m^2$$

$$V = 59.63 \times 1.8$$
$$= 107.34 \, m^3$$

(b) In the diagram BE is drawn parallel to CD.
$AB = 8$ cm, $BE = 5$ cm and $CD = 9$ cm.

$\angle AEB = \angle ADC$ (corresp.)
$\angle ABE = \angle ACD$ (corres)
$\angle CAD = \angle BAD$ (shared)
$\therefore \triangle ACD \, ||| \, \triangle ABE$ (A.A.A.)

(i)   Prove that the two triangles ABE and ACD are similar.   **3**

(ii) Hence find the length of interval BC (labelled x)   **2**

$S.f. = \dfrac{9}{5} = 1.8$ $AC = 8 \times 1.8$ $x = 6.4 \, cm$
                                                $= 14.4$

(c) What simple interest rate would I need to be offered if I wanted my term
deposit of $5000 to grow to $8000 in 7 years. Round to 2 decimal places.   **2**

$3000 = 5000 \times i \times 7$
$i = 0.0857 \times 100$
$= 8.57\%$ **END OF EXAM**