Kevin

1. A and B are the points $(2,-1)$ and $(-4,3)$ respectively.
   (a) Find the distance from A to B.
   (b) Find the co-ordinates of the midpoint of the interval AB.
   (c) Find the co-ordinates of the point P which divides the interval AB externally in the ratio 5:3.
   (d) Find the gradient of the line AB.
   (e) Find, to the nearest minute, the angle of inclination of the line AB to the positive x axis.
   (f) Find, in general form, the equation of the line AB.
   (g) Find the equation of the line through A parallel to the x axis.

2. The line PQ has equation $x + 3y - 2 = 0$ and the point R has co-ordinates $(-5,4)$.
   (a) Find the co-ordinates of the point where the line PQ cuts the x axis.
   (b) Find the perpendicular distance from the point R to the line PQ.
   (c) Find, in general form, the equation of the line through R perpendicular to the line PQ.
   (d) Find the co-ordinates of the point S. S is the foot of the perpendicular from R to the line PQ.

3. Find the equation of the line inclined at 120^0 to the positive x axis and cutting off an intercept of -2 on the y axis.

4. Find, to the nearest minute, the acute angle between the lines $3x - y + 5 = 0$ and $7x + 2y - 3 = 0$.

5. Find the value of k for which the lines $2x - 5y = 9$ and $kx + 4y = 13$ are parallel to each other.

6. $M(7,2)$, $N(1,-3)$ and $P(5,-1)$ are the vertices of triangle MNP.
   (a) Find the length of the interval MP.
   (b) Show that the equation of the line MP is $3x - 2y - 17 = 0$.
   (c) Find the perpendicular distance of point N from the line MP.
   (d) Hence find the area of triangle MNP.

7. $A(-1,-3)$, $B(3,2)$, $C(-2,6)$ are three vertices of parallelogram ABCD.
   (a) Find the co-ordinates of the fourth vertex D.
   (b) Show that diagonal AC is perpendicular to diagonal BD.
   What does this imply about parallelogram ABCD ?

m.v.

Analytical Geometry Test

$A(2,-1) \quad B(-4,3)$

1)

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$AB = \sqrt{(-4-2)^2+(3--1)^2}$$
$$= \sqrt{(-6)^2+4^2}$$
$$= \sqrt{36+16}$$
$$= \sqrt{52}$$
$$= \sqrt{4} \times \sqrt{13}$$
$$= 2\sqrt{13} \text{ units}$$

$$\text{midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$
$$\text{midpoint} = \left(\frac{2+-4}{2}, \frac{-1+3}{2}\right)$$
$$= \left(\frac{-2}{2}, \frac{2}{2}\right)$$
$$= (-1, 1)$$

$$\text{point} = \left[\frac{kx_2+lx_1}{k+l}, \frac{ky_2+ly_1}{k+l}\right]$$

external division $\Rightarrow$ ratio is $-5:3$

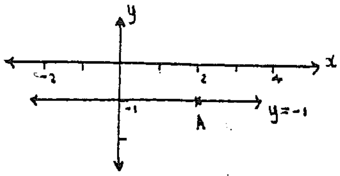
$$\text{point} = \left[\frac{-5\times-4+3\times2}{-5+3}, \frac{-5\times3+3\times-1}{-5+3}\right]$$
$$= \left[\frac{26}{-2}, \frac{-18}{-2}\right]$$
$$= (-13, 9)$$

$$m = \frac{y_2-y_1}{x_2-x_1}$$
$$m = \frac{3--1}{-4-2}$$
$$= \frac{4}{-6}$$
$$= -\frac{2}{3}$$

$$m = \tan\alpha$$
$$-\frac{2}{3} = \tan\alpha$$
$$\alpha = 180^\circ - 33^\circ 41'$$
$$\alpha = 146^\circ 19'$$

$$y - y_1 = m(x-x_1)$$
$$y - -1 = -\frac{2}{3}(x-2)$$
$$3(y+1) = -2(x-2)$$
$$3y+3 = -2x+4$$
$$2x+3y+3-4 = 0$$
$$2x+3y-1 = 0$$

9)



equation of line is $y=-1$

2)

PQ: $x+3y-2=0$, $R(-5,4)$

a) cuts x axis when $y=0$

$$x+3\times0-2=0$$
$$x=2$$

$\therefore$ point is $(2,0)$

b)

$$p.d = \left|\frac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}\right|$$
$$p.d. = \left|\frac{1\times-5+3\times4-2}{\sqrt{1^2+3^2}}\right|$$
$$= \left|\frac{-5+12-2}{\sqrt{10}}\right|$$
$$= \left|\frac{5}{\sqrt{10}}\right|$$
$$= \frac{5}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}}$$
$$= \frac{5\sqrt{10}}{10}$$
$$= \frac{\sqrt{10}}{2} \text{ units}$$

c)

$$x+3y-2=0$$
$$3y = -x+2$$
$$y = \frac{-x+2}{3}$$
$$\Rightarrow m = -\frac{1}{3}$$

for any perpendicular line, $m=3$

$$y-y_1 = m(x-x_1)$$
$$y-4 = 3(x--5)$$
$$y-4 = 3x+15$$
$$3x-y+19=0$$

d)

$$x+3y-2=0 \quad \text{— ①}$$
$$3x-y+19=0 \quad \text{— ②}$$
$$\text{①}\times3 \quad 3x+9y-6=0 \quad \text{— ③}$$
$$\text{②}-\text{③} \quad -10y+25=0$$
$$10y=25$$
$$y=\frac{25}{10}$$
$$y=2\tfrac{1}{2} \quad \text{— ④}$$

sub ④ into ①

$$x+3\times2\tfrac{1}{2}-2=0$$
$$x+5\tfrac{1}{2}=0$$
$$x=-5\tfrac{1}{2}$$

$\therefore$ S is point $(-5\tfrac{1}{2}, 2\tfrac{1}{2})$

3)

$$m = \tan\alpha$$
$$m = \tan 120^\circ$$
$$= -\tan 60^\circ$$
$$= -\sqrt{3}$$

equation $\quad y = mx+b$

$$y = -\sqrt{3}x - 2$$

4)

$$3x-y+5=0$$
$$3x+5=y$$
$$\Rightarrow m_1 = 3$$

$$7x+2y-3=0$$
$$2y = -7x+3$$
$$y = \frac{-7x+3}{2}$$
$$\Rightarrow m_2 = -\frac{7}{2}$$

$$\tan\psi = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$$
$$\tan\psi = \left|\frac{3--\frac{7}{2}}{1+3\times-\frac{7}{2}}\right|$$
$$= \left|\frac{13}{2} \div -\frac{19}{2}\right|$$
$$= \left|-\frac{13}{19}\right|$$
$$= \frac{13}{19}$$
$$\psi = 34^\circ 23'$$

5)

$$2x-5y=9$$
$$2x-9=5y$$
$$y=\frac{2x-9}{5}$$
$$\Rightarrow m_1=\frac{2}{5}$$

$$kx+4y=13$$
$$4y=-kx+13$$
$$y=\frac{-kx+13}{4}$$
$$\Rightarrow m_2 = \frac{-k}{4}$$

m.Y.

for parallel lines $m_1 = m_2$

$$\frac{2}{5} = -\frac{k}{4}$$

$$k = -\frac{8}{5}$$

6)

$M(7,2)$ $\quad N(1,-3)$ $\quad P(5,-1)$

a)

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$MP = \sqrt{(7-5)^2 + (2--1)^2}$$

$$= \sqrt{2^2 + 3^2}$$

$$= \sqrt{13} \text{ units}$$

b)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m \text{ of } MP = \frac{-1-2}{5-7}$$

$$= \frac{-3}{-2}$$

$$= \frac{3}{2}$$

equation

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{3}{2}(x - 7)$$

$$2(y-2) = 3(x-7)$$

$$2y - 4 = 3x - 21$$

$$3x - 2y - 17 = 0$$

c)

$$p.d = \left| \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}} \right|$$

p.d of N from MP

$$= \left| \frac{3\times1 - 2\times-3 - 17}{\sqrt{3^2 + (-2)^2}} \right|$$

$$= \left| \frac{3 + 6 - 17}{\sqrt{13}} \right|$$

$$= \left| \frac{-8}{\sqrt{13}} \right|$$

$$= \frac{8}{\sqrt{13}} \times \frac{\sqrt{13}}{\sqrt{13}}$$

$$= \frac{8\sqrt{13}}{13} \text{ units}$$

d)

area of ΔMNP

$$= \frac{1}{2} \times MP \times \text{p.d. of } N \text{ from } MP$$

$$= \frac{1}{2} \times \sqrt{13} \times \frac{8\sqrt{13}}{13}$$

$$= \frac{1}{2} \times \frac{8 \times 13}{13}$$

$$= 4 \text{ units}^2$$

7)

$A(-1,-3)$, $\quad B(3,2)$ $\quad C(-2,6)$

a)

let D be the point (x,y)

since ABCD is a parallelogram,
midpoint of AC = midpoint of BD
(diagonals bisect each other)

$$\text{midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$\Rightarrow \left(\frac{-1+-2}{2}, \frac{-3+6}{2}\right) = \left(\frac{3+x}{2}, \frac{2+y}{2}\right)$$

$$\left(\frac{-3}{2}, \frac{3}{2}\right) = \left(\frac{3+x}{2}, \frac{2+y}{2}\right)$$

hence $\quad \frac{-3}{2} = \frac{3+x}{2} \quad$ and $\quad \frac{3}{2} = \frac{2+y}{2}$

$-3 = 3 + x \qquad 3 = 2 + y$

$x = -6 \qquad y = 1$

$\therefore$ D is the point $(-6,1)$

b)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m \text{ of } AC = \frac{6--3}{-2--1}$$

$$= \frac{9}{-1}$$

$$= -9$$

$$m \text{ of } BD = \frac{1-2}{-6-3}$$

$$= \frac{-1}{-9}$$

$$= \frac{1}{9}$$

since $\quad m$ of $AC \times m$ of BD

$$= -9 \times \frac{1}{9}$$

$$= -1$$

the diagonals AC and BD are perpendicular

$\therefore$ parallelogram ABCD is actually a rhombus.

but

$$m \text{ of } AB = \frac{2--3}{3--1}$$

$$= \frac{5}{4}$$

$$m \text{ of } BC = \frac{6-2}{-2-3}$$

$$= -\frac{4}{5}$$

and since $\quad m$ of $AB \times m$ of BC

$$= \frac{5}{4} \times -\frac{4}{5}$$

$$= -1$$

the sides AB and BC are perpendicular

$\therefore$ rhombus ABCD is actually a square.

m.v.