LOCUS AND THE PARABOLA

1) Find the equation of the locus of point P that moves so that it is always 3 units from the point (2,-1).

2) Find the equation of the locus of point P that moves so that it is equidistant from the points (1,3) and (2,-2).

3) Find the equation of the locus of point P that moves so that its distance from the origin is double its distance from the point (4,3).

4) Find the equation of the locus of point P that moves so that PA is perpendicular to PB where A = (-1,-3) and B = (2,4).

5) Find the equation of the locus of point P that moves so that it is euqidistant from the point (1,2) and the line $y = -2$.

6) Find the equation of the circle with centre (-1,-1) and radius 2.

7) Find the coordinates of the centre and the radius of the circle $x^2 - 6x + y^2 - 4y + 12 = 0$

8) Find the coordinates of the centre and the radius of the circle $x^2 - 4x + y^2 + 10y + 13 = 0$

9) (a) By rotating the curve $x^2 + y^2 = r^2$ about the x-axis, show that the volume of a sphere with radius r is given by $V = \frac{4}{3}\pi r^3$.

(b) Hence or otherwise, find the exact volume of the sphere formed by rotating the circle $x^2 - 6x + y^2 + 5 = 0$ about the x-axis.

10) Find the equation of the parabola with focus (0,4) and directrix $y = -4$.

11) Find the equation of the parabola with vertex (0,0) and directrix $y = 2$.

12) Find the equation of the parabola with focus (-2,0) and directrix $x = 2$.

13) Find the equation of the parabola with axis $y = 0$, vertex (0,0) and passing through the point (1,2).

14) Find the equation of the parabola with axis $x = 0$, vertex (0,0) and focal length 9.

15) Find the coordinates of the focus and the equation of the directrix of the parabola $x^2 = 20y$

16) Find the coordinates of the focus and the equation of the directrix of the parabola $y^2 = 28x$

17) Find the coordinates of the focus and the equation of the directrix of the parabola $y^2 = -24x$

18) Find the coordinates of the focus and the equation of the directrix of the parabola $x^2 = -y$

19) A focal chord cuts the parabola $x^2 = 4y$ at the point (4,4).

(a) Find the equation of the chord.

(b) Find the point that cuts the parabola again at the other end of the chord.

20) Find the equation of the parabola with focus (2,1) and directrix $y = -3$.

21) Find the equation of the parabola with vertex (1,-3) and directrix $y = 5$.

22) Find the equation of the parabola with focus (4,-2) and directrix $x = 0$.

23) Find the equation of the parabola with vertex (3,-1) and focus (-2,-1).

24) Find the coordinates of the focus and vertex, and the equation of the directrix of the parabola $x^2 - 2x - 12y - 47= 0$

25) Find the coordinates of the focus and vertex, and the equation of the directrix of the parabola $y^2 + 8x + 24 = 0$

26) Find the coordinates of the focus and vertex, and the equation of the directrix of the parabola $y^2 - 4y - 16x - 76 = 0$

27) Find the coordinates of the focus and vertex, and the equation of the directrix of the parabola $x^2 + 20y - 60 = 0$

28) Find the equation of the tangent to the parabola $x^2 = 4y$ at the point (2,1).

29) (a) Show that the equation of the tangent to the parabola $x^2 = 8y$ at the point $\left(-1, \frac{1}{8}\right)$ is $2x + 8y + 1 = 0$.

(b) Find the coordinates of point R where this tangent meets the directrix.

30) Find the gradient of the normal to the parabola $x^2 = 10y$ at the point where $x = 3$.

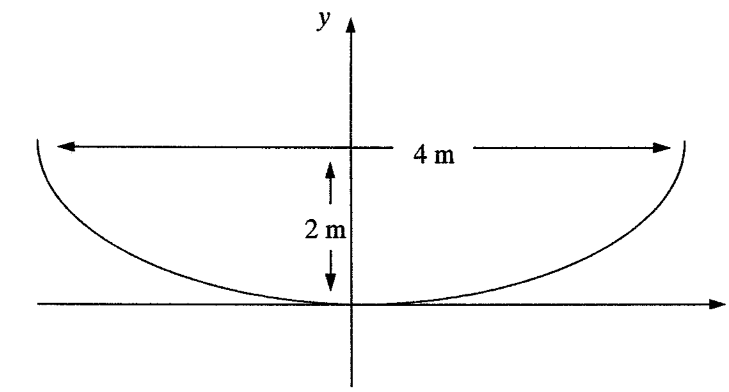
31) (a) Find the equation of the normal to the parabola $x^2 = -8y$ at the point (4,-2).

(b) This normal meets the parabola again at point N. Find the coordinates of N.

32) (a) Show that the equation of the parabola with focus (0,4) and directrix $y = 0$ is given by $x^2 = 8(y - 2)$.

(b) Find the exact volume of the paraboloid formed if the parabola $x^2 = 8(y - 2)$ is rotated about the y-axis from $y = 2$ to $y = 4$.

33)



A satellite dish in the shape of a parabola has a diameter of 4 metres and a depth of 2 metres. Find the focal length of the dish.

ANSWERS

1) $x^2 - 4x + y^2 + 2y - 4 = 0$
2) $x - 5y + 1 = 0$
3) $3x^2 - 32x + 3y^2 - 24y + 100 = 0$
4) $x^2 - x + y^2 - y - 14 = 0$
5) $x^2 - 2x - 8y + 1 = 0$
6) $x^2 + 2x + y^2 + 2y - 2 = 0$
7) Centre (3,2), radius 1
8) Centre (2,-5), radius 4
9) (a)

$$V = \pi \int_a^b y^2 dx$$

$$= \pi \int_{-r}^{r} (r^2 - x^2) dx$$

$$= \pi \left[r^2 x - \frac{x^3}{3} \right]_{-r}^{r}$$

$$= \pi \left[(r^3 - \frac{r^3}{3}) - (-r^3 + \frac{r^3}{3}) \right]$$

$$= \frac{4}{3} \pi r^3$$

(b) $\frac{32\pi}{3}$ units^3

10) $x^2 = 16y$
11) $x^2 = -8y$
12) $y^2 = -8x$
13) $y^2 = 4x$
14) $x^2 = \pm 36y$
15) Focus (0,5), directrix $y = -5$
16) Focus (7,0), directrix $x = -7$
17) Focus (-6,0), directrix $x = 6$
18) Focus $\left(0, -\frac{1}{4}\right)$), directrix $y = \frac{1}{4}$
19) (a) $3x - 4y + 4 = 0$ (b) $\left(-1, \frac{1}{4}\right)$
20) $x^2 - 4x - 8y - 4 = 0$
21) $x^2 - 2x + 32y + 97 = 0$
22) $y^2 + 4y - 8x + 20 = 0$
23) $y^2 + 2y + 20x - 59 = 0$
24) Focus (1,-1), vertex (1,-4), directrix $y = -7$
25) Focus (-5,0), vertex (-3,0), directrix $x = -1$
26) Focus (-1,2), vertex (-5,2), directrix $x = -9$

27) Focus (0,-2), vertex (0,3), directrix $y = 8$

28) $x - y - 1 = 0$

29) (a) $y = \dfrac{x^2}{8}$

$y^1 = \dfrac{2x}{8} = \dfrac{x}{4}$

When $x = -1$, $y^1 = -\dfrac{1}{4}$

$y - y_1 = m(x - x_1)$

$y - \dfrac{1}{8} = -\dfrac{1}{4}(x + 1)$

$8y - 1 = -2(x + 1)$

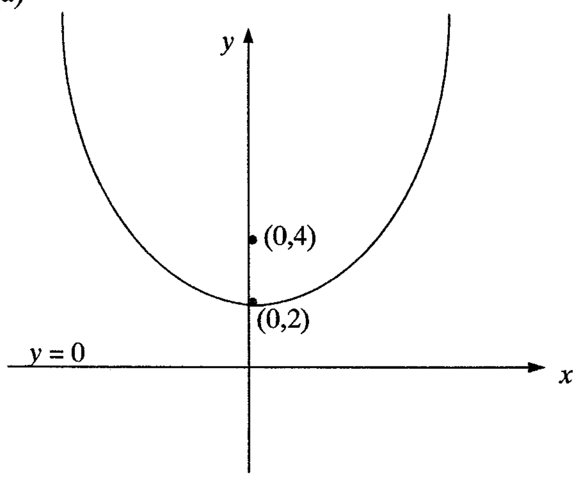
$2x + 8y + 1 = 0$

(b) $R = \left(7\dfrac{1}{2}, -2\right)$

30) $-\dfrac{5}{3}$

31) (a) $x - y - 6 = 0$ (b) (-12,-18)

32) (a)



Vertex (h,k) = (0,2) and $a = 2$

$(x - h)^2 = 4a(y - k)$

$(x - 0)^2 = 4(2)(y - 2)$

$x^2 = 8(y - 2)$

(b) 16π units^3

33) 0.5 metre