St. Catherine's School

Year 11 Extension 1 Mathematics

Preliminary Task #3

June 2008

Time allowed: 50 minutes

INSTRUCTIONS

- There are 2 sections including a total of 10 questions of different values
- Marks for each question are indicated
- All questions should be attempted
- All necessary working should be shown
- Start each question on a new page
- Approved scientific calculators and drawing templates may be used

Section A (19 marks)

Q.1 For the parabola $(x+2)^2=-4(y-1)$, Find

(i) the focal length
(ii) the coordinates of the vertex
(iii) the coordinates of the focus
(iv) the equation of the directrix
(v) Sketch this parabola highlighting the above features.

(4m)

Q.2 Find the possible equations of the parabolas in each case, given that

Its focus is at the origin, the axis is the x axis and the focal length is 3 units

(3m)

(ii) The focus is at (2,4), the directrix is the line $y=8$ (3m)

(iii) The vertex is at (3,6) and it passes through the point (0,3) (3m)

Q.3 Find the vertex of the parabola, whose equation is given by

$x^2-2x+2y+14=0$ (3m)

Q.4 Find the equation of the locus of a point, which moves such that it is equidistant from the lines $2x+3y-1=0$ *and* $3x+2y-5=0$

(3m)

Please Turn Over for Section B

Section B (22 marks) – Start a new page

Q.5 For what values of k does the following quadratic equation

(i) $1-3x-kx^2=0$ have two distinct roots? (2m)

(ii) $kx^2-4kx-5=0$ have no real roots? (3m)

Q.6 If one root of $x^2-mx+2=0$ is double the other, find the possible values of m. (2m)

Q.7 The roots of the quadratic equation $mx^2-5x+2=0$ are reciprocals of each other, find the roots. (3m)

Q.8 If α *and* β are the roots of the equation $2x^2+3x+1=0$, find the value of:

(i) $\alpha+\beta$

(ii) $\alpha\beta$

(iii) $\alpha^2+\beta^2$

(3m)

Q.9 Find a, b and c if $2x^2+4x+5=a(x-1)^2+b(x-1)+c$ (3m)

Q.10 Solve for x:

(i) $(x^2-x)^2-8(x^2-x)+12=0$ (3m)

(ii) $4^x-12(2^x)+32=0$ (3m)

END OF PAPER

Solution

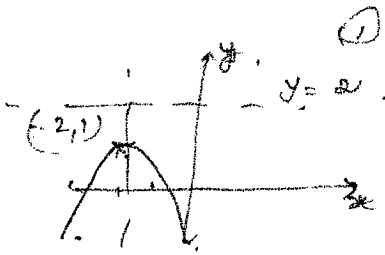
Q.1 $(x+2)^2 = -4(y-1)$

(i) focal length $= 1$ $(\frac{1}{2})$

(ii) Vertex: $(-2, 1)$ $(\frac{1}{2})$

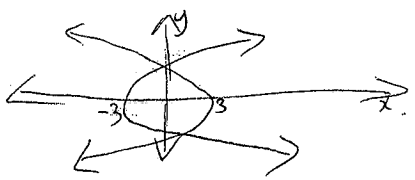
(iii) Focus: $(-2, 0)$ (1)

(iv) directrix: $y = 2$. (1)



(1)

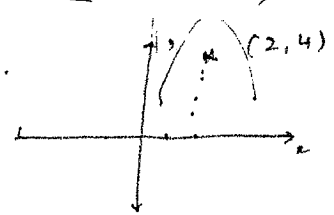
(2i) Focus: $(0,0)$ Vertex: $(\pm 3, 0)$ (1)



$y^2 = -12(x-3)$

$y^2 = 12(x+3)$

(ii)

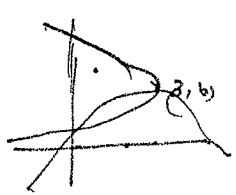


Vertex: $(2, 6)$ (1)

Focal length $= 2$ (1)

Eq: is $(x-2)^2 = -8(y-6)$ (1)

(iii)



Eq: is $(x-3)^2 = -4a(y-6)$ (1)

Sub $(0, 3)$

$9 = -4a(3-6)$

$= +12a$

$a = +\frac{3}{4}$ $(1\frac{1}{2})$

$(y-6)^2 = 4a(x-3)$ AND/OR.

Sub $(0,3)$

$(-3)^2 = 4a(-3)$ $\therefore$ Eq: is $(x-3)^2 = -3(y-6)$ $(\frac{1}{2})$

$\frac{-3}{4} = a$

$\therefore (\quad)^2 = -3(x-3)$

Q.3 $x^2 - 2x + 2y + 14 = 0$

$x^2 - 2x + 1 = -2y - 14 + 1$

$(x-1)^2 = -2y - 13$ (1)

$= -2(y + \frac{13}{2})$ (1)

Vertex: $(1, -\frac{13}{2})$ (1)

Q.4 Let $P: (x_1, y_1)$ be the moving pt.

$$\left| \frac{2x_1 + 3y_1 - 1}{\sqrt{2^2+3^2}} \right| = \left| \frac{3x_1 + 2y_1 - 5}{\sqrt{3^2+2^2}} \right|$$

$2x_1 + 3y_1 - 1 = 3x_1 + 2y_1 - 5$

OR

$2x_1 + 3y_1 - 1 = -3x_1 - 2y_1 + 5$

Locus is

$2x + 3y - 1 = 3x + 2y - 5$ OR $2x + 3y - 1 = -3x - 2y + 5$

$x - y - 4 = 0$ or $5x + 5y - 6 = 0$.

One of the lines only $\frac{2}{3}$.

Q.5

$1 - 3x - kx^2 = 0$

$\Delta = (-3)^2 - 4(1)(-k)$

$= 9 + 4k.$

two distinct roots $\Rightarrow \Delta > 0$ (1)

$9 + 4k > 0$

$4k > -9$

$k > -\frac{9}{4}$ (1)

$kx^2 - 4kx - 5 = 0$

$\Delta = (-4k)^2 - 4(k)(-5)$

$= 16k^2 + 20k$ (1)

no real roots; $\Delta < 0$

$16k^2 + 20k < 0$

$4k(4k+5) < 0$ (1)

$-\frac{5}{4} < k < 0$ (1)

Q.6

$x^2 - mx + 2 = 0$

α, 2α are the roots.

$3\alpha = m$ ——— (1)

$2\alpha^2 = 2$. ——— (2) (1/2)

from (2) Sub in (1) (1/2)

$\alpha^2 = 1$.

$\alpha = \pm 1$ (1/2)

$m = \pm 3$ (1/2)

1½/2 if $\alpha = -1$ is not considered.

Q.9

$2x^2 + 4x + 5 = a(x-1)^2 + b(x-1) + c$

$= a(x^2 - 2x + 1) + bx - b + c$

comparing coefficient,

$2 = a$ ——— (1)

$4 = -2a + b$ ——— (2)

$5 = a - b + c$ ——— (3)

Sub $a = 2$ in (2)

$4 = -4 + b$

$b = 8$

Sub in (3)

$5 = 2 - 8 + c$

$c = 11$

Q.10

$(x^2 - x)^2 - 8(x^2 - x) + 12 = 0$

Let $u = x^2 - x$

$u^2 - 8u + 12 = 0$

$(u-6)(u-2) = 0$ (1M)

$u = 6$

$x^2 - x = 6$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3$; $x = -2$ (1M)

$u = 2$

$x^2 - x = 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2$; $x = -1$. (1M)

(11) $4^x - 12(2^x) + 32 = 0$

note $4^x = (2^2)^x$

$= (2^x)^2$

$u^2 - 12u + 32 = 0$ (1)

$(u-8)(u-4) = 0$

$u = 8$ $u = 4$

(1)

Q.7 $mx^2 - 5x + 2 = 0$.

$\alpha, \frac{1}{\alpha}$ are the roots.

$\alpha + \frac{1}{\alpha} = \frac{5}{m}$

$\alpha \times \frac{1}{\alpha} = \frac{2}{m}$ $\therefore \underline{m = 2}$. ——— (1M)

$\therefore \alpha + \frac{1}{\alpha} = \frac{5}{2}$. $\left(\frac{1}{2}\right)$

$2\alpha^2 + 2 = 5\alpha$

$2\alpha^2 - 5\alpha + 2 = 0$ $\left(\frac{1}{2}\right)$

$(2\alpha - 1)(\alpha - 2) = 0$ $\left(\frac{1}{2}\right)$

$\alpha = \frac{1}{2}$ & $\alpha = 2$ $\left(\frac{1}{2}\right)$

$\therefore$ The roots are 2 and $\frac{1}{2}$.

Q.8 $2x^2 + 3x + 1 = 0$.

$\alpha + \beta = -\frac{3}{2}$ $\frac{1}{2}$

$\alpha\beta = \frac{1}{2}$. $\frac{1}{2}$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ 1

$= \frac{9}{4} - 1$

$= \frac{5}{4}$ 1