TRINITY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT

YEAR 12 MATHEMATICS/EXTENSION 1
2008 ASSESSMENT TASK 1

NAME: ______________________

TEACHER: ______________________

Time allowed: 30 minutes

WEIGHTING: 10% towards final result

DATE: Friday 16th November 2007.

OUTCOMES REFERRED TO: **P5, P6, P7, P8, H1, H4, H5, H6, H7, H9**

EQUIPMENT: Calculators and geometrical instruments will not be loaned. It is your responsibility to be fully prepared for this task. The only calculators are permitted are those approved by the Board of Studies.

INSTRUCTIONS:
- Attempt ALL questions
- Note that some pages are **double-sided**.
- Write your **name** and circle your **teacher** for EACH question.
- Show all necessary working in the spaces provided.
- Silent Board of Studies approved calculators are permitted.
- All questions have marks specified.

MARKS

Locus & the parabola Q1, Q2, Q3	Series & Applications Q4, Q5, Q6	**TOTAL**
/15	/15	/30

Name: ______________________

Teacher: Rogers Peterson Geddes Lammiman Kesby O'Donoghue Wymer

QUESTION 1 (5 marks) **Marked by Wymer**

Marks

(a) Find the centre and radius of the circle with equation given by **2**

$$(x+1)^2+(y+6)^2=49.$$

(b) A parabola has equation $(x+3)^2=-12(y+1)$. **3**

Find:
(i) the coordinates of its vertex;
(ii) its focal length;
(iii) the equation of its directrix.

Name: ______________________________

Teacher: Rogers Peterson Geddes Lammiman Kesby O'Donoghue Wymer

QUESTION 2 (5 marks) **Marked by O'Donoghue**

Marks

(a) Find the equation of the parabola which satisfies all the following conditions: **2**

(i) it is concave up;
(ii) the focal length is 2 units;
(iii) the directrix has equation $y = -2$;
(iv) the vertex is at (0,0).

(b) The locus of a point P moves so that PA is twice the distance of PB where A = (0,3) and B = (4,0).

(i) Show that $PA^2 = 4 \times PB^2$. **1**

(ii) Hence find the equation of the locus of the point P. **2**

Name: ______________________________

Teacher: Rogers Peterson Geddes Lammiman Kesby O'Donoghue Wymer

QUESTION 3 (5 marks) **Marked by Kesby**

Marks

(a) (i) Find the equation of the normal to the parabola $x^2 = 4y$ at the point (-8, 16). **3**

(ii) This normal in part (a)(i), cuts the parabola again at Q. Find the coordinates of Q. **2**

Name: ____________________________________

Teacher: Rogers Peterson Geddes Lammiman Kesby O'Donoghue Wymer

QUESTION 4 (5 marks) **Marked by Lammiman**

Marks

(a) The third term and the ninth term of an arithmetic series are -2 and 28 respectively. Find the:

(i) first term and the common difference, **2**

(ii) the sum of the first 9 terms. **1**

Marks

(b) Evaluate $\sum_{k=1}^{20} 2^k$. **2**

Name: ____________________

Teacher: Rogers Peterson Geddes Lammiman Kesby O'Donoghue Wymer

QUESTION 5 (5 marks) **Marked by Geddes**

Marks

(a) (i) For what values of r does the geometric series **1**

$$a + ar + ar^2 + \ldots$$

have a limiting sum?

For these values of r write down the limiting sum. **1**

(ii) An infinite geometric series has a first term of 8 and a limiting sum of 12. **3**

Calculate the common ratio.

Name: ____________________

Teacher: Rogers Peterson Geddes Lammiman Kesby O'Donoghue Wymer

QUESTION 6 (5 marks) **Marked by Peterson**

Scott borrowed \$80 000 to buy a bread shop. He agreed to repay the loan at 1% monthly reducible interest over 3 years by making 6 equal instalments of \$P at 6 monthly intervals.

An expression for the amount Scott owes immediately after he made his first repayment of \$$P$, 6 months after he took out the loan is $A_1 = 80000(1.01)^6 - P$.

Marks

(i) Show the amount Scott owes immediately after his second repayment is $A_2 = 80000(1.01)^{12} - P[(1.01)^6 + 1]$. **2**

Marks

(ii) Given that $A_3 = 80000(1.01)^{18} - P[(1.01)^{12} + (1.01)^6 + 1]$ by continuing the pattern find an expression for A_6. **1**

(iii) Hence show the value of his monthly repayments P, can be found by evaluating; **1**

$$P = \frac{80000(1.01)^{36}}{(1.01)^{30} + (1.01)^{24} + (1.01)^{18} + (1.01)^{12} + (1.01)^6 + 1}.$$

(iv) Calculate the value of Scott's repayments. **1**

END OF TASK

TRINITY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT

YEAR 12 MATHEMATICS/EXTENSION 1
2008 ASSESSMENT TASK 1

NAME: KESBY

TEACHER: SOLUTIONS

Time allowed: 30 minutes

WEIGHTING: 10% towards final result

DATE: Friday 16th November 2007.

OUTCOMES REFERRED TO: **P5, P6, P7, P8, H1, H4, H5, H6, H7, H9**

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MARKS

Locus & the parabola Q1, Q2, Q3	Series & Applications Q4, Q5, Q6	**TOTAL**
/15	/15	/30

Name: ______________________

Teacher: Rogers Peterson Geddes Lammiman Kesby O'Donoghue Wymer

QUESTION 1 (5 marks) **Marked by Wymer**

Marks

(a) Find the centre and radius of the circle with equation given by **2**

$$(x+1)^2+(y+6)^2=49.$$

Centre: $(-1,-6)$ ①

radius = 7 units ①

(b) A parabola has equation $(x+3)^2=-12(y+1)$. **3**

Find:

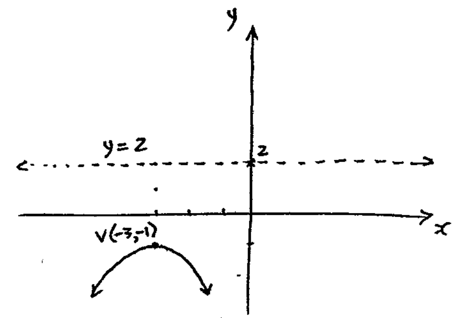
(i) the coordinates of its vertex;

(ii) its focal length;

(iii) the equation of its directrix.

$$(x+3)^2=-12(y+1)$$

$$\left.\begin{aligned}(x-(-3))^2&=-4\times3\times(y-(-1))\\(x-h)^2&=-4a(y-k)\end{aligned}\right\}$$



(i) vertex: $V(-3,-1)$ ①

(ii) focal length: $a=3$ ①

(iii) directrix: $y=2$ ①

Name: ____________________

Teacher: Rogers Peterson Geddes Lammiman Kesby O'Donoghue Wymer

QUESTION 2 (5 marks) **Marked by O'Donoghue**

Marks

(a) Find the equation of the parabola which satisfies all the following conditions: **2**

(i) it is concave up;
(ii) the focal length is 2 units;
(iii) the directrix has equation $y=-2$;
(iv) the vertex is at (0,0).

$x^2 = 4ay$ ①

$x^2 = 4\times 2\times y$

$x^2 = 8y$ ①

(b) The locus of a point P moves so that PA is twice the distance of PB where A = (0,3) and B = (4,0).

(i) Show that $PA^2 = 4\times PB^2$. **1**

$PA = 2\times PB$

$PA^2 = (2\times PB)^2$ ①

$PA^2 = 4\times PB^2$

(ii) Hence find the equation of the locus of the point P. **2**

$\overset{x_2\ y_2}{P(x,y)}$ $\overset{x_1\ y_1}{A(0,3)}$ $\overset{x_2\ y_2}{P(x,y)}$ $\overset{x_1\ y_1}{B(4,0)}$

$PA^2 = 4\times PB^2$

$(x-0)^2 + (y-3)^2 = 4\times\left((x-4)^2 + (y-0)^2\right)$

$x^2 + y^2 - 6y + 9 = 4\times\left(x^2 - 8x + 16 + y^2\right)$ ①

$\cancel{x^2} + \cancel{y^2} - \cancel{6}y + 9 = \cancel{4x^2} - \cancel{32x} + 64 + \cancel{4y^2}$

$0 = 3x^2 - 32x + 3y^2 + 6y + 55$

$\therefore$ Equation is $3x^2 - 32x + 3y^2 + 6y + 55 = 0$. ①

Name: ____________________

Teacher: Rogers Peterson Geddes Lammiman Kesby O'Donoghue Wymer

QUESTION 3 (5 marks) **Marked by Kesby**

Marks

(a) (i) Find the equation of the normal to the parabola $x^2 = 4y$ at the point (-8, 16). **3**

$y = \frac{x^2}{4}$

$y = \frac{1}{4}x^2$

$\frac{dy}{dx} = \frac{1}{4}\times 2x = \frac{1}{2}x$. ①

At $x=-8$, $\frac{dy}{dx} = \frac{1}{2}(-8) = -4$ $\therefore m_T = -4$

$\therefore m_N = \frac{1}{4}$. ①

$\overset{x_1\ y_1}{(-8,16)}$ $m_N = \frac{1}{4}$

Using pt. gradient form,

$y - y_1 = m_N(x - x_1)$

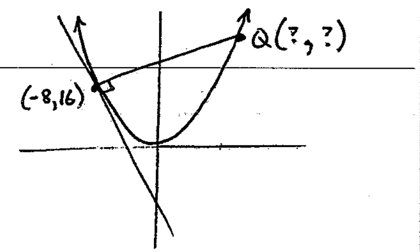
$y - 16 = \frac{1}{4}(x+8)$

$4y - 64 = x + 8$

$0 = x - 4y + 72$

$\therefore$ equation is $\boxed{x - 4y + 72 = 0}$ ①

(ii) This normal in part (a)(i), cuts the parabola again at Q. Find the coordinates of Q. **2**



$y = \frac{1}{4}x^2$ — ①

$x - 4y + 72 = 0$ — ②

Subst. ① into ②:

$x - 4\left(\frac{1}{4}x^2\right) + 72 = 0$

$x - x^2 + 72 = 0$

$x^2 - x - 72 = 0$

$(x-9)(x+8) = 0$

$x = 9, -8$ ①

When $x=9$ in equation ①: $y = \frac{1}{4}\times(9)^2 = \frac{81}{4} = 20\frac{1}{4}$.

Name: ____________________

Teacher: Rogers Peterson Geddes Lammiman Kesby O'Donoghue Wymer

QUESTION 4 (5 marks) **Marked by Lammiman**

Marks

(a) The third term and the ninth term of an arithmetic series are –2 and 28 respectively. Find the:

(i) first term and the common difference, **2**

$T_3 = -2 = a + 2d$ — 1

$T_9 = 28 = a + 8d$ — 2

2 − 1 : $30 = 6d$

$\underline{d = 5}$ 1

Subst. $d = 5$ into 1 :

$a + 2(5) = -2$

$a + 10 = -2$

$\underline{a = -12}$ 1

(ii) the sum of the first 9 terms. **1**

$T_9 = L = 28$

$a = -12$

$n = 9$

$S_n = \frac{n}{2}(a + L)$

$S_9 = \frac{9}{2}(-12 + 28)$

$S_9 = 72.$ 1

Marks

(b) Evaluate $\sum_{k=1}^{20} 2^k$. **2**

$$\sum_{k=1}^{20} 2^k = \underbrace{2^1 + 2^2 + 2^3 + \dots + 2^{20}}_{\text{G.S.}\ a = 2,\ r = 2,\ n = 20} \quad 1$$

$$= \frac{a(r^n - 1)}{r - 1}$$

$$= \frac{2(2^{20} - 1)}{2 - 1}$$

$$= 2^{21} - 2$$

$$= 2\ 097\ 150. \quad 1$$

Name: ____________________

Teacher: Rogers Peterson Geddes Lammiman Kesby O'Donoghue Wymer

QUESTION 5 (5 marks) **Marked by Geddes**

Marks

(a) (i) For what values of r does the geometric series 1

$a + ar + ar^2 + ...$

have a limiting sum?

$|r| < 1$ OR $-1 < r < 1$. ①

For these values of r write down the limiting sum. 1

$S_\infty = \frac{a}{1-r}$. ①

(ii) An infinite geometric series has a first term of 8 and a limiting sum of 12. 3
Calculate the common ratio.

$a = 8$
$S_\infty = 12$
$r = ?$

$S_\infty = \frac{a}{1-r}$

$12 = \frac{8}{1-r}$ ①

$12(1-r) = 8$

$12 - 12r = 8$ ①

$-12r = -4$

$r = \frac{-4}{-12}$

$\underline{r = \frac{1}{3}}$. ①

Name: ____________________

Teacher: Rogers Peterson Geddes Lammiman Kesby O'Donoghue Wymer

QUESTION 6 (5 marks) **Marked by Peterson**

Scott borrowed \$80 000 to buy a bread shop. He agreed to repay the loan at 1% monthly reducible interest over 3 years by making 6 equal instalments of \$P at 6 monthly intervals.

An expression for the amount Scott owes immediately after he made his first repayment of \$$P$, 6 months after he took out the loan is $A_1 = 80000(1.01)^6 - P$.

Marks

(i) Show the amount Scott owes immediately after his second repayment is $A_2 = 80000(1.01)^{12} - P[(1.01)^6 + 1]$. 2

$A_2 = A_1(1.01)^6 - P$

$= [80\,000(1.01)^6 - P](1.01)^6 - P$ ①

$= 80\,000(1.01)^{12} - P(1.01)^6 - P$ ①

$= 80\,000(1.01)^{12} - P[(1.01)^6 + 1]$

Marks

(ii) Given that $A_3 = 80000(1.01)^{18} - P[(1.01)^{12} + (1.01)^6 + 1]$ by continuing the pattern find an expression for A_6. **1**

$$A_6 = 80\,000(1.01)^{36} - P\left[(1.01)^{30} + (1.01)^{24} + \dots + (1.01)^6 + 1\right] \quad ①$$

(iii) Hence show the value of his monthly repayments P, can be found by evaluating; **1**

$$P = \frac{80000(1.01)^{36}}{(1.01)^{30} + (1.01)^{24} + (1.01)^{18} + (1.01)^{12} + (1.01)^6 + 1}.$$

After 3 years $A_6 = 0$, ① with substitution

ie. $0 = 80\,000(1.01)^{36} - P\left[(1.01)^{30} + (1.01)^{24} + \dots + (1.01)^6 + 1\right]$

$\therefore P\left[(1.01)^{30} + (1.01)^{24} + \dots + (1.01)^6 + 1\right] = 80\,000(1.01)^{36}$

$$\therefore \quad P = \frac{80\,000(1.01)^{36}}{(1.01)^{30} + (1.01)^{24} + \dots + (1.01)^6 + 1}.$$

(iv) Calculate the value of Scott's repayments. **1**

$P = \$16\,346.79$ (nearest cent) ①

END OF TASK