

Exercise 1.2

1. Evaluate each of the following.

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| (a) 5^0 | (b) $36^{\frac{1}{2}}$ | (c) $49^{-\frac{1}{2}}$ |
| (d) $81^{-\frac{3}{4}}$ | (e) $(\frac{1}{2})^{-3}$ | (f) $100^{\frac{3}{2}}$ |
| (g) $(\frac{1}{4})^{-\frac{3}{2}}$ | (h) $12^{\frac{1}{2}} \cdot 3^{\frac{1}{2}}$ | (i) $32^{\frac{1}{2}} \cdot 2^{-\frac{1}{2}}$ |
| (j) $\frac{9^{\frac{1}{2}} \cdot 8^{\frac{1}{2}}}{2^{\frac{1}{2}}}$ | (k) $27^{\frac{1}{4}} \cdot 3^{\frac{1}{4}}$ | (l) $(0.0625)^{-\frac{1}{4}}$ |

2. In the following equations, find the values of x .

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|-------------------|------------------------------|------------------------------|
| (a) $2^x = 8$ | (b) $3^x = 27^{\frac{2}{3}}$ | (c) $3^x = 27^{\frac{1}{4}}$ |
| (d) $10^x = 0.01$ | (e) $(2^x)^3 = 16$ | (f) $(2^x)^2 = 2^5$ |

3. Solve the following exponential equations.

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|---------------------------------------|--------------------------------|
| (a) $2^{x-3} = 4^{x+1}$ | (b) $3^{2x} \cdot 3^{x-1} = 9$ |
| (c) $2^x \cdot 2^{x+1} = \frac{1}{2}$ | (d) $3^x \cdot 2^{2x-3} = 18$ |

4. Solve the following exponential equations.

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|--|---------------------------------------|
| (a) $2^{2x} - 9 \cdot 2^x + 8 = 0$ | (b) $3^{2x} - 10 \cdot 3^x + 9 = 0$ |
| (c) $4^x - 3 \cdot 2^{x+1} + 8 = 0$ | (d) $2^{2x+1} + 4 = 2^{x+3} + 2^x$ |
| (e) $3^{2x-3} - 4 \cdot 3^{x-2} + 1 = 0$ | (f) $16^x - 5 \cdot 2^{2x-1} + 1 = 0$ |

5. Express each of the following surds in its simplest form.

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| (a) $\sqrt{18} + \sqrt{8}$ | (b) $\sqrt{50} + \sqrt{32} - \sqrt{72}$ |
| (c) $(\sqrt{2} - \sqrt{3})^2$ | (d) $(4\sqrt{3} - 3\sqrt{2})^2$ |
| (e) $\sqrt{12} \times \sqrt{27}$ | (f) $\sqrt{32} \times \sqrt{15} \div \sqrt{24}$ |
| (g) $(2\sqrt{3} - \sqrt{5})(2\sqrt{3} + \sqrt{5})$ | (h) $(3\sqrt{7} + 5\sqrt{2})(3\sqrt{7} - 5\sqrt{2})$ |

6. Rationalise the denominator for each of the following surds.

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| (a) $\frac{10}{\sqrt{5}}$ | (b) $\frac{\sqrt{2}}{\sqrt{3}}$ | (c) $\frac{2}{2 + \sqrt{2}}$ |
| (d) $\frac{5}{\sqrt{3} + \sqrt{2}}$ | (e) $\frac{\sqrt{3}}{2\sqrt{5} - \sqrt{3}}$ | (f) $\frac{1 + \sqrt{2}}{\sqrt{3} - 1}$ |
| (g) $\frac{\sqrt{3} + 2\sqrt{2}}{\sqrt{5} - \sqrt{3}}$ | (h) $\frac{3\sqrt{3} - 4}{\sqrt{6} - 2\sqrt{3}}$ | (i) $\frac{1}{\sqrt{2} + 1} + \frac{1}{\sqrt{2} - 1}$ |
| (j) $\frac{2}{\sqrt{5} - 2} - \frac{1}{\sqrt{5} + 2}$ | | |

7. Without using tables, find the value of each of the following logarithms.

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|-------------------|--------------------|------------------------|---------------------|
| (a) $\log_3 81$ | (b) $\log_{27} 3$ | (c) $\log_4 0.5$ | (d) $\log_{100} 10$ |
| (e) $\log_8 0.25$ | (f) $\log_{0.5} 8$ | (g) $\log_5 5\sqrt{5}$ | (h) $\log_5 0.04$ |

8. Express each of the following as a single logarithm.

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|---|---|
| (a) $\log_3 8 - \log_3 6$ | (f) $\log_{10} (x+1) - \log_{10} (x^2 - 1)$ |
| (b) $4 \log_2 3 - \log_2 9$ | (g) $\frac{1}{2} \log_{10} (x-1) + 2 \log_{10} (x-2)$ |
| (c) $2 \log_a 5 + \log_a 4 - 2 \log_a 10$ | (h) $\log_{10} (p^2 - q^2) - \log_{10} (p+q) + \log_{10} p - 2 \log_{10} q$ |
| (d) $\frac{3}{2} \log_3 9 - 2 \log_3 6$ | |
| (e) $2 \log_8 (\frac{2}{3}) - \log_8 (\frac{8}{9})$ | |

9. Evaluate each of the following.

(a) $\frac{\log_a 32}{\log_a 2}$ (b) $\frac{\log_3 x}{\log_9 x}$ (c) $(\log_a 27)(\log_3 a)$

10. Given that $\log_3 2 = a$ and $\log_3 5 = b$, express each of the following in terms of a and b .
 (a) $\log_3 60$ (b) $\log_3 6.4$ (c) $\log_{10} 2$

11. Given that $\log_5 x = p$, express each of the following in terms of p .
 (a) $\log_5 5x^2$ (b) $\log_x 5$ (c) $\log_{25} x$ (d) $\log_x 0.2$

12. Find the value of x in each of the following equations.
 (a) $\log_8 x = 1.5$ (b) $\log_3 x = -0.7$ (c) $\log_3 x = 2.2$
 (d) $\log_2 x = 0.05$ (e) $\log_x 65 = -6$ (f) $\log_x 5 = 1.2$

13. Solve each of the following equations.
 (a) $5^x = 15$ (b) $2^{2x+1} = 0.01$ (c) $10^x = 0.3$
 (d) $8(2^{2x}) = 5$ (e) $3^{x-2} = 5^{2x+1}$

14. Solve each of the following equations.
 (a) $\log_2 x^4 + \log_2 4x = 12$ (b) $\log_3 x + \log_3 (x+6) = 3$
 (c) $\log_3 x = 4 \log_x 3$ (d) $2 \log_4 x + 3 \log_x 4 = 7$
 (e) $3 \log_8 x = 2 \log_x 8 + 5$ (f) $\log_5 x + \log_x 25 = 3$

15. Solve each of the following inequalities.
 (a) $2^x < 9$ (b) $(4.8)^{x+1} > 3.6$ (c) $3^{x+1} < 4^{x-1}$
 (d) $12^{x^2} > 10^{2x}$ (e) $(\frac{3}{5})^{2x+1} < 0.001$

16. Find the set of values of x for the following inequalities.
 (a) $2^{2x} - 5(2^x) + 6 > 0$ (Hint: let $2^x = y$)
 (b) $e^x - 3e^{-x} \leq 2$ (Hint: multiply both sides by e^x)

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1. (a) 1 (b) 6 (c) $\frac{1}{7}$
 (d) $\frac{1}{27}$ (e) 8 (f) 1000
 (g) 8 (h) 6 (i) 4
 (j) 6 (k) 3 (l) 2
2. (a) 3 (b) 2 (c) $\frac{3}{4}$
 (d) -2 (e) $\frac{4}{3}$ (f) $\frac{5}{2}$
3. (a) -5 (b) 1
 (c) -1 (d) 2
4. (a) 0, 3 (b) 0, 2 (c) 1, 2
 (d) -1, 2 (e) 1, 2 (f) $\pm \frac{1}{2}$
5. (a) $5\sqrt{2}$ (b) $3\sqrt{2}$ (c) $5 - 2\sqrt{6}$
 (d) $66 - 24\sqrt{6}$ (e) 18 (f) $2\sqrt{5}$
 (g) 7 (h) 13
6. (a) $2\sqrt{5}$ (b) $\frac{1}{3}\sqrt{6}$
 (c) $2 - \sqrt{2}$ (d) $5(\sqrt{3} - \sqrt{2})$
 (e) $\frac{1}{17}(2\sqrt{15} + 3)$
 (f) $\frac{1}{2}(1 + \sqrt{2})(1 + \sqrt{3})$
 (g) $\frac{1}{2}(\sqrt{3} + 2\sqrt{2})(\sqrt{5} + \sqrt{3})$
 (h) $\frac{1}{6}(4 - 3\sqrt{3})(\sqrt{6} + 2\sqrt{3})$
 (i) $2\sqrt{2}$ (j) $6 + \sqrt{5}$
7. (a) 4 (b) $\frac{1}{3}$
 (c) $-\frac{1}{2}$ (d) $\frac{1}{2}$
 (e) $-\frac{2}{3}$ (f) -3
 (g) $\frac{3}{2}$ (h) -2

8. (a) $\log_3(\frac{4}{3})$ (b) $2 \log_2 3$
 (c) 0 (d) $\log_3(\frac{3}{4})$
 (e) $-\frac{1}{3}$ (f) $-\log(x-1)$
 (g) $\log[(x-1)^{\frac{1}{2}}(x-2)^2]$ (h) $\log[\frac{p(p-q)}{q^2}]$
9. (a) 5 (b) 2 (c) 3
10. (a) $1 + 2a + b$ (b) $5a - b$ (c) $\frac{a}{a+b}$
11. (a) $1 + 2p$ (b) $\frac{1}{p}$
 (c) $\frac{1}{2}p$ (d) $-\frac{1}{p}$
12. (a) 22.627 (b) 0.463
 (c) 11.212 (d) 1.035
 (e) 0.499 (f) 3.824
13. (a) 1.683 (b) -3.822 (c) -0.523
 (d) -0.339 (e) -1.795
14. (a) 4 (b) 3 (c) $9, \frac{1}{9}$
 (d) 2, 64 (e) $\frac{1}{2}, 64$ (f) 5, 25
15. (a) $x < 3.170$ (b) $x > -0.183$
 (c) $x > 8.638$ (d) $x > 1.853$
 (e) $x > 6.261$
16. (a) $x < 1, x > 1.585$ (b) $x < 1.099$