CRANBROOK
SCHOOL

Term 2, 2006

Year 12 Mathematics & Extension 1

HSC ASSESSMENT TASK

Wednesday May 31, 2006

There are 3 questions.

All questions are of equal value.

Begin a new page for each question.

All necessary working should be shown in every question.

Full marks may not be awarded if work is careless or badly arranged.

Approved silent calculators may be used.

A list of Standard Integrals has been provided on page 4

Time Allowed: 45 Minutes

Question 1 (12 Marks)   **Marked by GC**

A.   Write an equivalent equation to $y = a^3$ using logarithmic notation.   1

B.   Evaluate $\log_{\frac{1}{2}} 8$ 1

C.   Solve $8^{2x+3} = 11$, giving your answer correct to 2 decimal places   2

D.   Differentiate: 3

        i. $y = e^{2x+1}$

        ii. $y = \dfrac{e^{3x}}{x}$

E.   If $y = e^{-2x}$, find k such that $\dfrac{d^2 y}{dx^2} + 5\dfrac{dy}{dx} + ky = 0$ 3

F   Find the volume of the solid formed when the curve $y = e^{2x} + 3$ is rotated

about the $x-axis$ from $x = 1$ to $x = 3$ (Answer correct to one decimal place)

3

Question 2 (12 Marks)   **Marked by CAB**

A.   Find $\displaystyle\int \frac{x}{x^2 + 5}\,dx$ 2

B.   Find $\dfrac{dy}{dx}$ given that $y = \log_e \left(\dfrac{2x+1}{3x-7}\right)$ 2

C.   Find the equation of the normal to the curve $y = x\log_e x$ at the point (e, e) ③

D.   Solve for x the equation $\log_5(6) = \log_5(x+3) + \log_5(x-4)$.   2

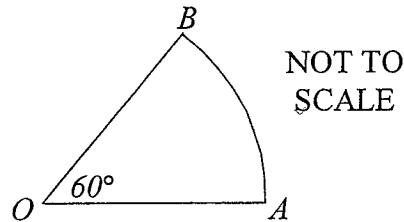E.   The diagram shows part of the hyperbola $y = \dfrac{7}{1+x}$ and the line $y = 7 - x$.   3

The hyperbola and line intersect at the points $(0, 7)$ and $(6, 1)$.

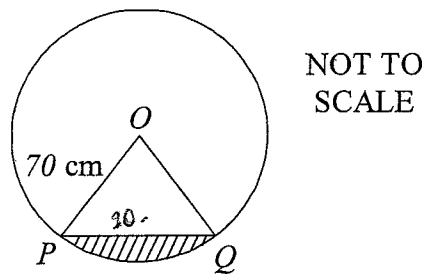Calculate the exact area of the shaded region.
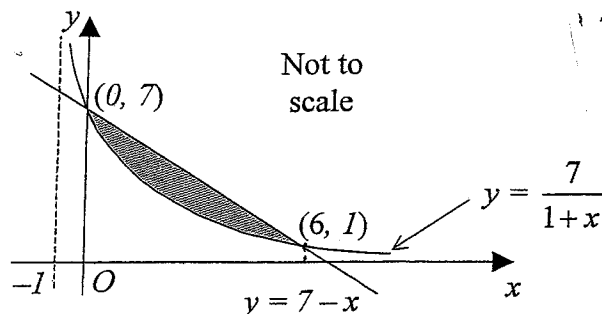
A.   The length of the arc AB of the sector OAB is 2π cm.

     i.   Convert 60° to radians, simplify your answer leaving it in terms of π.   1

     ii.   Show the length of OA is 6 cm.   2

   iii.   Calculate the exact area of the sector OAB.   1

B.   In the diagram the length of the chord PQ is 30 cm.

     The radius of the circle is 70 cm.

     i.   Show that $\angle POQ = 25°$ to the nearest degree.   2

     ii.   Convert $25°$ to radians correct to 4 decimal places.   1

     iii.   Find the shaded area correct to the nearest cm^2.   1

C.   The diagram shows a sector of a circle with radius r cm. The angle at the centre is θ radians and the perimeter of the sector is 8 cm.
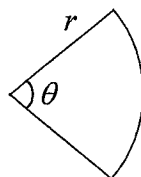
     i.   Find an expression for r in terms of θ.   2

     ii.   Show that A, the area of the sector in cm^2, is given by $A = \dfrac{32\theta}{(\theta+2)^2}$.   2