	MORIAH COLLEGE MATHEMATICS DEPARTMENT **Year 12 – 3 unit** Logs and Exponentials
	Name:________________ Class:____

Question 1 (10 marks)

a) State the domain of $y = \log_2(x-3)$

b) State the range of $y = e^{-3x}$

c) Write an equivalent equation to $y = x^p$ using logarithmic notation

d) Evaluate $\log_3 7$ to 3 significant figures

e) Evaluate $\log_8 \dfrac{1}{16}$ using logs laws and showing all working

f) Solve for m in terms of b if $\ln m + \ln m = b$

g) Sketch $y = e^x + 2$ showing all critical points

Question 2 (10 marks)

Differentiate the following:

a) $y = x.\log 2x$

b) $y = \dfrac{3e^{2x}}{\ln 4}$

c) $y = \sqrt{e^{3x-1}}$

d) $y = (\ln 5x)^3$

e) $y = 2^x$

f) $y = \log_3 x$

g) $y = \ln\sqrt{x+1}$

h) $y = \ln\left(\dfrac{3x+1}{2x-3}\right)$

Question 3 (5 marks)

Find the equation of the normal to the curve $y = \dfrac{e^x - 1}{e^x + 1}$ at the point $x = 0$

Question 4 (7 marks)

Evaluate the following:

a) $\int e^{4x+5}\,dx$

b) $\int_2^{e^2} 2x - \dfrac{1}{x}\,dx$

c) $\int e^x\left(e^{3x} - 3\right)dx$

d) $\int \dfrac{e^x}{e^x + 2}\,dx$

Question 5 (8 marks)

a) Find the area contained by the y-axis, the curve $y = e^{2x}$ and the line $y = e^{\frac{1}{2}}$
(leave answer in exact form)

b) A solid of revolution is formed by rotating about the x-axis the portion of the curve $y = 2\left(1 + e^{3x}\right)$ between $x = 0$ and $x = \dfrac{1}{3}$.
Find the volume of this solid. (leave answer in exact form)

Question 6 (6 marks)

Consider the function $f(x) = x - 3\ln x$, where $1 \le x \le 7$

a) There is one turning point for $f(x)$. Find its co-ordinates and determine its nature.

b) Draw a sketch of the curve $y = f(x)$ where $1 \le x \le 7$

c) What is the maximum value of the function where $1 \le x \le 7$

Question 7 (12 marks)

a) Given that $\dfrac{d^2x}{dt^2} = 6 + e^{-t}$, and that $\dfrac{dx}{dt} = -1$ at $t = 0$, and $x = 0$ at $t = 0$.

Find an expression for $x(t)$.

b) Prove that $\dfrac{d}{dx}\ln\left(\dfrac{x-1}{x+1}\right) = \dfrac{2}{(x-1)(x+1)}$

hence, evaluate $\displaystyle\int_2^5 \frac{2\,dx}{(x-1)(x+1)}$ giving your answer in simplest exact form.

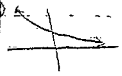
c) Find $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$ using the substitution $u = e^{\sqrt{x}}$ or otherwise.

Y12 – 3unit Total: 58

logs exponentials

#1 a) $x-3>0$

$x>3$ ✓ 1

b)  range: $y>0$ ✓ 1

c) $y=x^p$

$\log_x y = p$ ✓ 1

d) $\frac{\log 7}{\log 3} = 1.77$ ✓ 1

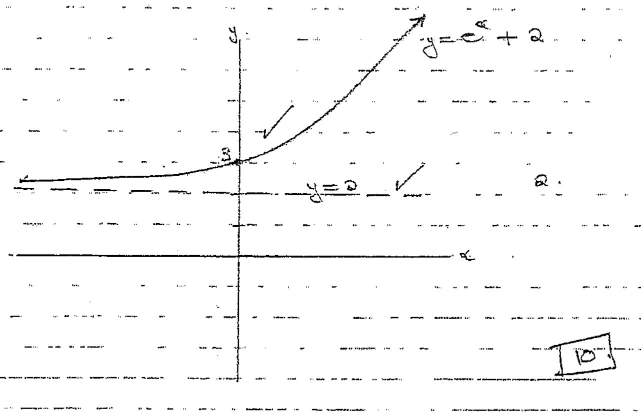
e) $\frac{\log_2 2^{-4}}{\log_2 2^3}$ ✓ $= -\frac{4}{3}$ ✓ 2

f) $2\ln m = b$

$\ln m^2 = b$ ✓ 2

$m^2 = e^b$

$m = \sqrt{e^b}$ ✓

g) 

✓ ✓ 2

$\boxed{10}$

#2 a) $y = x\ln 2x$

$y' = (\ln 2x)(1) + (x)(\frac{1}{2x})(2)$ ✓

$y' = \ln 2x + 1$ ✓ 2

2b) $y = \frac{3e^{2x}}{\ln 4}$

$y' = \frac{6e^{2x}}{\ln 4}$ ✓ 1

c) $y = e^{\frac{3x-1}{2}}$

$y' = \frac{3}{2}e^{\frac{3x-1}{2}}$

$y' = \frac{3}{2}\sqrt{e^{3x-1}}$ ✓ 1

d) $y = (\ln 5x)^3$

$y' = 3(\ln 5x)^2 \times \frac{1}{5x} \times 5$ ✓

$y' = \frac{3}{x}(\ln 5x)^2$ ✓ 2

e) $y = 2^x$

$y' = 2^x \ln$

f) $y = \log_3$

$y = \frac{\ln x}{\ln 3}$

$y' = \frac{1}{x\ln 3}$

g) $y = \ln\sqrt{}$

$y = \frac{1}{2}\ln(x$

$y' = \frac{1}{2}\frac{1}{(x+}$

10

#3. $y = \frac{e^x - 1}{e^x+1}$ $\frac{u}{v}$

$m(\tan) = y' = \frac{(e^x+1)(e^x) - (e^x-1)(e^x)}{(e^x+1)^2}$ ✓

$m(\tan) = \frac{e^x(e^x+1-e^x+1)}{(e^x+1)^2}$

$= \frac{e^x(2)}{(e^x+1)^2}$

at $x=0$

$y=0$ ✓

(6)

at $x=0$

$m(\tan) = \frac{2}{4} = \frac{1}{2}$ ✓

$m(\text{normal}) = -2$ ✓

eqn of normal: $y-0 = -2(x-0)$ ∴ $y =$

#4 a) $\int e^{4x+5}\,dx$

$= \frac{e^{4x+5}}{4} + c$ ✓ 1

b) $\int_2^{e^2} 2x - \frac{1}{x}\,dx$

$= \left[\frac{2x^2}{2} - \ln x\right]_2^{e^2}$ ✓

$= (e^4 - \ln e^2) - (4 - \ln 2)$ ✓

$= e^4 - 2 - 4 + \ln 2$

$= e^4 - 6 + \ln 2$ ✓ 3

c) $\int e^{4x} - 3e^x\,dx$ ✓

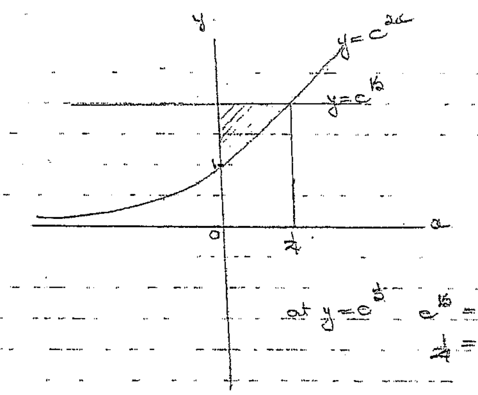
$= \frac{e^{4x}}{4} - 3e^x + c$ ✓ 2

(7)

d) $\int \frac{e^x}{e^x+2}\,dx$

$\int \frac{f'(x)}{f(x)} = \ln f(x)$

$= \ln(e^x+2) + c$ ✓ 1

#5 a) 

at $y=e^{\frac{1}{2}}$ $e^{\frac{1}{2}} = e^{2x}$

$\frac{1}{4} = x$

shaded

∴ Area = area of rectangle $- \int_0^{1/4} e^{2x}\,dx$ ✓

$= (\frac{1}{4})(e^{\frac{1}{2}}) - \left[\frac{e^{2x}}{2}\right]_0^{1/4}$ ✓

$= \frac{1}{4}\sqrt{e} - \left[\frac{\sqrt{e}}{2} - \frac{1}{2}\right]$ ✓

$= \frac{\sqrt{e}}{4} - \frac{\sqrt{e}}{2} + \frac{1}{2}$ 4

$= \frac{1}{2} - \frac{\sqrt{e}}{4}\ u^2$ ✓

#5b) $V = \pi\int_0^{\frac{1}{3}} [2(1+e^{3x})]^2\,dx$ ✓

$= \pi\int_0^{\frac{1}{3}} 4(1+2e^{3x}+e^{6x})\,dx$ ✓

(8)

$= 4\pi\left[x + \frac{2e^{3x}}{3} + \frac{e^{6x}}{6}\right]_0^{1/3}$ ✓

$= 4\pi\left[\frac{1}{3} + \frac{2e}{3} + \frac{1}{6}e^2 - \frac{2}{3} - \frac{1}{6}\right]$

$= 4\pi\left(-\frac{1}{2} + \frac{2e}{3} + \frac{e^2}{6}\right) u^3$ ✓ 4

#6. $f(x) = x - 3\ln x \qquad 1 \le x \le 7$

$f'(x) = 1 - \frac{3}{x} = 0$

$1 = \frac{3}{x}$

$3 = x$

at $x = 3$, $y = 3 - 3\ln 3$ T.P $(3,\ 3(1-\ln 3))$

$f'(x) = 1 - 3x^{-1}$

$f''(x) = 3x^{-2} = \frac{3}{x^2} > 0 \quad \therefore \cup \text{ min}$

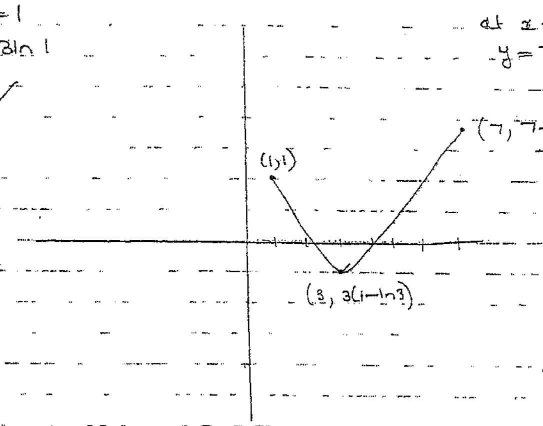
at $x = 1$

$y = 1 - 3\ln 1$

$y = 1$

at $x = 7$

$y = 7 - 3\ln 7$



max value $= 7 - 3\ln 7$

(6)

#7 a) $\frac{d^2x}{dt^2} = 6 + e^{-t}$

$\frac{dx}{dt} = \int 6 + e^{-t}\, dt = 6t + \frac{e^{-t}}{-1} + c_1$

$\frac{dx}{dt} = -1$ at $t = 0$; $-1 = -1 + c_1$

$0 = c_1$

$\frac{dx}{dt} = 6t - e^{-t}$

$x = \int 6t - e^{-t}\, dt$

$x = \frac{6t^2}{2} - \frac{e^{-t}}{-1} + c_2$

$x = 0,\ t = 0$: $0 = 1 + c_2$

$-1 = c_2$

(4)

$x = 3t^2 + \frac{1}{e^t} - 1$

b) $y = \ln\left(\frac{x-1}{x+1}\right)$

$y = \ln(x-1) - \ln(x+1)$

$\frac{dy}{dx} = \frac{1}{x-1} - \frac{1}{x+1}$

$= \frac{x+1-x+1}{(x-1)(x+1)}$

$= \frac{2}{(x-1)(x+1)}$

$\therefore y = \int \frac{2}{(x-1)(x+1)}\, dx$

(5)

$\int_2^5 \frac{2}{(x-1)(x+1)}\, dx = \left[\ln\left(\frac{x-1}{x+1}\right)\right]_2^5$

$= \ln\frac{4}{6} - \ln\frac{1}{3}$

$= \ln\left(\frac{4}{6} \div \frac{1}{3}\right) = \ln 2$

#7c) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx \qquad y = x^{\frac{1}{2}}$

$u = e^{\sqrt{x}}$

$\frac{du}{dx} = e^{\sqrt{x}} \times \frac{1}{2}x^{-\frac{1}{2}}$

$\frac{du}{dx} = \frac{1}{2}\frac{e^{\sqrt{x}}}{\sqrt{x}}$

$du = \frac{1e^{\sqrt{x}}}{2\sqrt{x}}\, dx$

$\therefore \int \frac{e^{\sqrt{x}}}{\sqrt{x}} = 2\int \frac{1}{2}\frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$

$= 2\int du$

$= 2u + c$

$= 2e^{\sqrt{x}} + c$

3.

35mins

OR using $\int f'(x)e^{f(x)} = e^{f(x)} + c$

$2\int \frac{1}{2}\frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx = 2e^{\sqrt{x}} + c$