Student Name / Number ____________________

SOUTH SYDNEY HIGH SCHOOL

Year 12 June Assessment Task

2001
MATHEMATICS

Instructions :

Time Allowed: *1 hours*

- Attempt ALL questions.
- ALL questions are of equal value.
- All necessary working should be shown.
- Marks may be deducted for poorly arranged or missing working.
- Use a SEPARATE Writing Booklet for each question.
- Write your Name on every page.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n \neq -1;\ x \neq 0,\ \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a \neq 0$$

$$\int \frac{1}{\sqrt{(a^2-x^2)}}\,dx = \sin^{-1}\frac{x}{a},\quad a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{(x^2-a^2)}}\,dx = \ln\left\{x+\sqrt{(x^2-a^2)}\right\},\quad |x| > |a|$$

$$\int \frac{1}{\sqrt{(x^2+a^2)}}\,dx = \ln\left\{x+\sqrt{(x^2+a^2)}\right\}$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Question 1 (10 marks) **Start a NEW page.** **Marks**

(a) Given that $\log_a b = 3.75$ and $\log_a c = 1.25$ find the value of **4**

(i) $\log_a\left(\frac{b}{c}\right)$ (ii) $\log_a \sqrt{ab}$

(b) Calculate correct to 3 decimal places $165e^{-2.4}$. **1**

(c) Differentiate the following **5**

(i) $4e^{2x}$ (ii) $x^3 \ln x$

(iii) $\frac{\ln 4x}{4x}$

Continue next page ....

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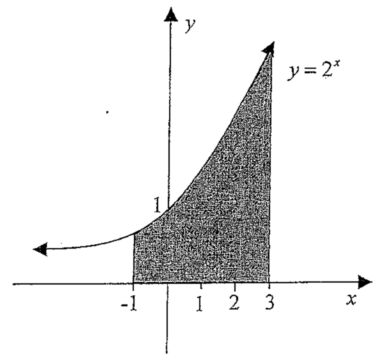
Question 2(10 marks) **Start a NEW page.** **Marks**

(a) Consider the function $y = 2^x$

x	-1	0	1	2	3
y					

(i) Copy and complete the table. **1**

(ii) Using Simpson's rule with these five function values, find an estimate for the area shaded in the graph below **3**



(b) Find (i) $\int e^{3x}dx$ (ii) $\int \frac{dx}{3x+5}$ (iii) $\int x^2 + \frac{2}{x}\,dx$ **4**

(c) Find the equation of the tangent to the curve $y = e^{2x}$ at the point where $x = 1$. **2**

Continue next page ....

Question 3(10 marks) **Start a NEW page.** **Marks**

(a) Solve for x 2

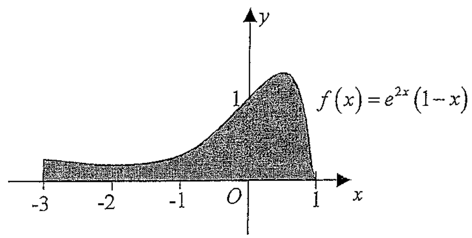
$2\log_5 3 = \log_5 x - \log_5 6$.

(b) Consider the function $f(x) = e^{2x}(1-x)$ where $-3 \le x \le 1$.

(i) Copy and complete the table of values. 1
Give values correct to two decimal places.

x	-3	-2	-1	0	1
$f(x)$	0.01	0.05			

(ii) Using the trapezoidal rule with five function values, approximate the area under the curve below. 3



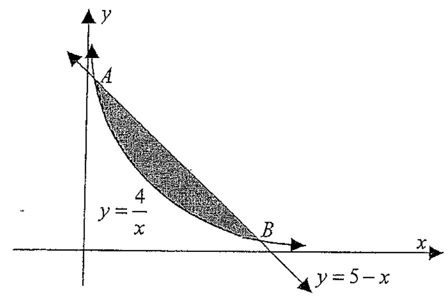
(c) Evaluate: (a) $\int_0^1 e^{2x} + 1\, dx$ (b) $\int_3^5 \frac{2x}{x^2-1}\, dx$ 4

Continue next page ...

Question 4 (10 marks) **Start a NEW page.** **Marks**

(a) The gradient of a curve at any point on it is $\frac{2}{2x+1}$ and the curve passes through the point $(1, \log_e 3)$. Find the equation of the curve. 3

(b)

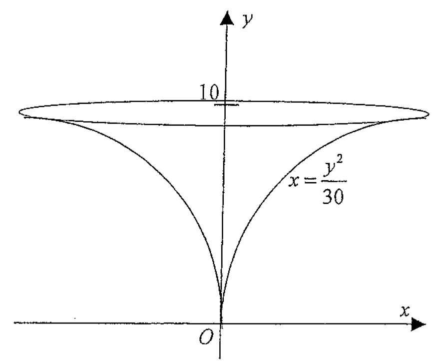


The graph shows $y = \frac{4}{x}$ and $y = 5 - x$ intersecting at A and B.

(i) Find the x coordinates of the points A and B. 2

(ii) Find the area of the shaded region between $y = \frac{4}{x}$ and $y = 5 - x$. 2

(c)



A glass has a shape obtained by rotating part of the parabola $x = \frac{y^2}{30}$ about the y-axis as shown. The glass is 10 cm deep. Find the volume of liquid which the glass will hold. 3

End of assessment task

June Assessment

Question 1

40/40 Excellent

a) i. $\log_a\left(\frac{b}{c}\right)$
$= \log_a b - \log_a c$
$= 3.75 - 1.25$
$= 2.5$ ✓ 2

(ii) $\log_a \sqrt{ab}$
$= \log_a (ab)^{\frac{1}{2}} = \frac{1}{2}\log_a(ab)$
$= \frac{1}{2}(\log_a a + \log_a b)$
$= \frac{1}{2}(1 + 3.75)$
$= 2.375$ ✓ 2

) $165e^{-2.4} = 14.96846...$ $= 14.968$ (to 3 decimal places) ✓ 1

) i. $\frac{d}{dx} 4e^{2x}$
$= 4e^{2x} \times 2$
$= 8e^{2x}$ ✓ 1

ii. $\frac{d}{dx} x^3 \ln x$
$= vu' + uv'$
$= \ln x \times 3x^2 + x^3 \cdot \frac{1}{x}$
$= 3x^2 \ln x + x^2$
$= x^2(3\ln x + 1)$ ✓ 2

(iii) $\frac{d}{dx} \frac{\ln 4x}{4x}$ $(= \frac{u}{v})$
$= \frac{vu' - uv'}{v^2}$
$= \frac{4x \times \frac{4}{4x} - \ln 4x \times 4}{(4x)^2}$
$= \frac{4 - 4\ln 4x}{16x^2}$
$= \frac{4(1 - \ln 4x)}{16x^2}$
$= \frac{(1 - \ln 4x)}{4x^2}$ ✓ 2

10

i. $y = 2^x$

x	-1	0	1	2	3
y	0.5	1	2	4	8

✓ 1

i. $A \doteqdot \frac{h}{3}[y_0 + y_n + 4(y_1 + y_3 ...) + 2(y_2 + y_4 ...)]$
$= \frac{1}{3}[0.5 + 8 + 4(1 + 4) + 2(2)]$
$= \frac{1}{3} \times 32.5$
$= 10.8\dot{3}$ units^2 ✓ 3

i. $\frac{1}{3}\int 3e^{3x} dx$
$= \frac{1}{3}e^{3x} + c$ ✓ 1

ii) $\int (3x+5)^{-1} dx$
$= \frac{1}{3}\ln(3x+5) + c$ ✓ 1

i. $\int x^2 + \frac{2}{x} dx$
$= \frac{x^3}{3} + 2\ln x + c$ ✓ 2

10

. $y = e^{2x}$
$\frac{dy}{dx} = 2e^{2x}$
but $x = 1$
$\therefore \frac{dy}{dx} = 2e^2$ ✓

at $x = 1$, $y = e^{2\times1} = e^2$

$\therefore$ eqn of tangent $\Rightarrow$
$y - e^2 = 2e^2(x - 1)$
$y - e^2 = 2e^2 x - 2e^2$
$0 = 2e^2 x - y - e^2$ ✓ 2

Question 3

Halan Ektas

a) $2\log_5 3 = \log_5 x - \log_5 6$

$\log_5 3^2 = \log_5\left(\frac{x}{6}\right)$

$\log_5 9 = \log_5\left(\frac{x}{6}\right)$

$\therefore \frac{x}{6} = 9$

$x = 6 \times 9$

$= 54$ ✓ 2

$f(x) = e^{2x}(1-x)$

b) i.

x	-3	-2	-1	0	1
$f(x)$	0.01	0.05	0.27	1	0

✓ 1

ii. $A \doteqdot \frac{h}{2}[y_0 + y_n + 2(y_1 + y_2 \ldots)]$

$\doteqdot \frac{1}{2}[0.01 + 0 + 2(0.05 + 0.27 + 1)]$

$= \frac{1}{2} \times 2.65$ ✓ 3

$= 1.325 \text{ units}^2 = 1.33\,u^2$ (2dp)

c) i. $\int_0^1 e^{2x} + 1\, dx$

$= \left[\frac{e^{2x}}{2} + x\right]_0^1$

$= \left[\left(\frac{e^2}{2} + 1\right) - \left(\frac{e^0}{2} + 0\right)\right]$

$= \frac{e^2}{2} + 1 - \frac{1}{2} - 0$

$= \frac{e^2}{2} + \frac{1}{2}$

$= \frac{1}{2}[e^2 + 1]$ ✓ 2

ii. $\int_3^5 \frac{2x}{x^2-1}\, dx$

$= \left[\ln(x^2-1)\right]_3^5$

$= \ln(25-1) - \ln(9-1)$

$= \ln 24 - \ln 8$

$= \ln\left(\frac{24}{8}\right)$

$= \ln 3$ ✓ 2

(10)

Question 4

Halan Ektas.

a) $\frac{dy}{dx} = \frac{2}{2x+1}$

$\therefore y = \int \frac{2}{2x+1}\, dx$

$= \ln(2x+1) + C$

but passes through $(1, \log_e 3)$

$\therefore \ln 3 = \ln(2+1) + C$

~~$\ln 3$~~ $=$ ~~$\ln 3$~~ $+ C$

$C = 0$ ✓

$\therefore$ eqn of curve

$\Rightarrow y = \ln(2x+1)$ ✓ 3

b) i. $\left(y = \frac{4}{x},\ y = 5 - x\right)$

$\therefore \frac{4}{x} = 5 - x$

$4 = 5x - x^2$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$\therefore x = 4$

& $x = 1$

x co-ordinates of

$A = 1,\quad B = 4$ ✓ 2

ii. $A = \int_1^4 5 - x - \left(\frac{4}{x}\right) dx$

$= \left[5x - \frac{x^2}{2} - 4\ln x\right]_1^4$

$= (20 - 8 - 4\ln 4) - (5 - \frac{1}{2} - 4\ln 1)$

$= 12 - 4\ln 4 - 4\frac{1}{2} + 0$

$= 7\frac{1}{2} - 4\ln 4 \text{ units}^2$ ✓

(OR $A = \frac{15}{2} - \ln 256 \text{ units}^2$) 2

c). $V_y = \pi \int_a^b x^2\, dy$

$= \pi \int_0^{10} \left(\frac{y^2}{30}\right)^2 dy$

$= \pi \int_0^{10} \frac{y^4}{900}\, dy$

$= \pi \left[\frac{y^5}{5 \times 900}\right]_0^{10}$

$= \pi \left[\frac{y^5}{4500}\right]_0^{10}$

$= \pi \left(\frac{100000}{4500} - \frac{0}{4500}\right)$

$= 22\frac{2}{9}\pi$

$= \frac{200\pi}{9} u^3$ ✓ 3

(10)