Moriah College

2015

TRIAL
HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics

- **General Instructions**
- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations

Total Marks – 100

Section I Pages 2 – 5

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 5 – 14

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0,\ \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\ x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\ a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\ a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\ a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x,\ x > 0$

Section I

10 marks

Attempt Questions 1 – 10.

Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 10.

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1. $\left(\dfrac{2a}{3b}\right)^{-5} = ?$

(A) $\dfrac{2a^5}{3b^5}$

(B) $\dfrac{3b^5}{2a^5}$

(C) $\dfrac{243b^5}{32a^5}$

(D) $\dfrac{1}{243b^5}$

2. Let α and β be the solutions of $2x^2 - 5x - 9 = 0$. Find the value of $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$.

(A) $-\dfrac{9}{2}$

(B) $-\dfrac{9}{5}$

(C) $-\dfrac{5}{9}$

(D) $\dfrac{5}{2}$

3. Find $\lim\limits_{x \to \infty} \dfrac{3\sqrt{x}}{x-2}$.

(A) $\sqrt{x}$

(B) 3

(C) $\dfrac{3}{x}$

(D) 0

4. The period and amplitude of $y = 3\cos 2x$ is:

(A) Amplitude = 2, Period = $\dfrac{2\pi}{3}$

(B) Amplitude = 3, Period = π

(C) Amplitude = π , Period = 3

(D) Amplitude = $\dfrac{2\pi}{3}$, Period = 2

5. What is the value of $8e^{-2}$ correct to 3 significant figures?

(A) 1.08

(B) 1.082

(C) 1.083

(D) 1.10

6. When simplified fully $\cos^2\left(\dfrac{\pi}{2} - \theta\right)\cot\theta$ is:

(A) $\cos^2\theta \cot\theta$

(B) $\sin\theta \cos\theta$

(C) $\dfrac{\sin^3\theta}{\cos\theta}$

(D) $\sin^2\theta \cot\theta$

7. Find the $\int_2^7 \frac{5}{x}\,dx$.

(A) $5(\ln 7 - \ln 2)$

(B) $\frac{1}{5}(\ln 7 - \ln 2)$

(C) $\frac{5}{49} - \frac{5}{4}$

(D) 0

8. The equation of the normal to the curve $x^2 = 4y$ at the point where $x = 2$ is:

(A) $y = 1$

(B) $x - y - 1 = 0$

(C) $y = -1$

(D) $y + x - 3 = 0$

9. Find the value of $\log_5 200 - 3\log_5 2$.

(A) 1.4

(B) 2.0

(C) 3.2

(D) 2.5

10. A particle is moving in a straight line. It's distance (x metres) from a fixed point O is given by $x = 2\cos 2t$, where t is the time in seconds.

At which times is the particle at rest?

(A) $t = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \ldots\ldots$

(B) $t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \ldots\ldots\ldots$

(C) $t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \ldots\ldots\ldots$

(D) $t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \ldots\ldots\ldots$

End of Section I

Section II

90 marks

Attempt Questions 11 – 16.

Allow about 2 hours and 45 minutes for this section.

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 writing booklet.

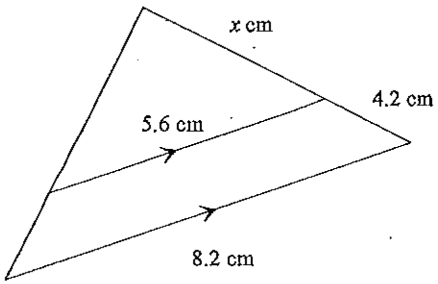
(a) Solve $x^2 - 2x - 7 = 0$, expressing your answer in simplest surd form. 2

(b) Find $\int \frac{3x}{x^2+1}\,dx$. 1

(c) Simplify fully : 2

$$\frac{2}{\sqrt{7}+3} - \frac{3\sqrt{7}}{\sqrt{7}-3}$$

(d) Find the value of x (correct to the nearest mm). 2



(e) Find the coordinates of the vertex and focus of the parabola $x^2 - 5y + 5 = 0$. 2

Question 11 continues on page 6

(f) Find the sum of the 10th to the 30th terms of the arithmetic series $5 + 9 + 13 + \ldots$.. 2

(g) Evaluate $\int_0^{\ln 6} e^x\,dx$. 2

(h) Shade the following regions bounded by the curves $y < \sqrt{4-(x-2)^2}$ and $y > \frac{x^2}{2}$. 2

End of Question 11

Question 12 (15 marks) Use the Question 12 writing booklet.

(a) Differentiate:

(i) $y = \sin^2(4x)$. 1

(ii) $y = x^3 e^{3x}$. 1

(iii) $y = \dfrac{e^x}{(x+3)^2}$. (Full simplification of your answer is not required.) 2

(iv) $y = \dfrac{\cos x}{x^2}$ 2

(b) Solve $\sqrt{3}\cos x = \sin x$ for $0 \le \theta \le 2\pi$. 2

(c) Use Simpson's Rule with four equal subintervals to find an approximation for 2

$$\int_0^1 \tan x \, dx.$$

(d) Find the values of A, B and C if $3x^2 + x + 1 \equiv A(x-1)(x+2) + B(x+1) + C$. 2

(e) A curve has the equation $y = x\cos x$.

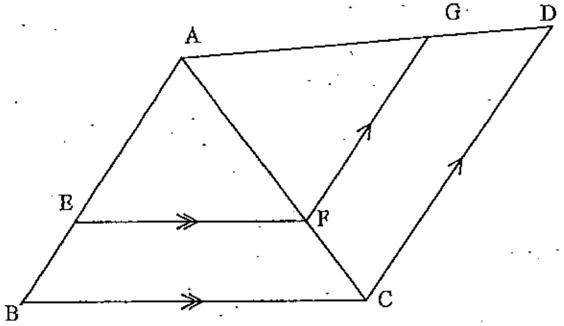
(i) Show that $P\left(\dfrac{\pi}{2}, 0\right)$ is the first point to the right of the origin where the curve crosses the x axis. 1

(ii) Find the equation of the tangent at point P. 2

End of Question 12

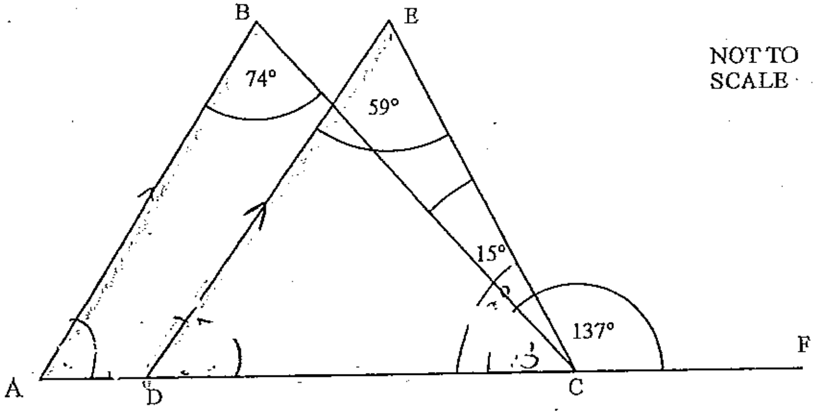
Question 13 (15 marks) Use the Question 13 writing booklet.

(a) In the figure below, EF || BC and CD || FG.



Prove that $\dfrac{AE}{AB} = \dfrac{AG}{AD}$ 2

(b) In the diagram below AF is a straight line, $\angle B = 74°$, $\angle E = 59°$, $\angle BCF = 137°$ and $\angle BCE = 15°$.



NOT TO SCALE

Prove that AB||DE 2

Question 13 continues on page 9

Question 13 (continued)

(c) Jack drops a super bouncy ball from the top of a 56 m building on to a concrete surface below. Its first rebound is 42 m, and each subsequent rebound is three quarters the height of the previous one.

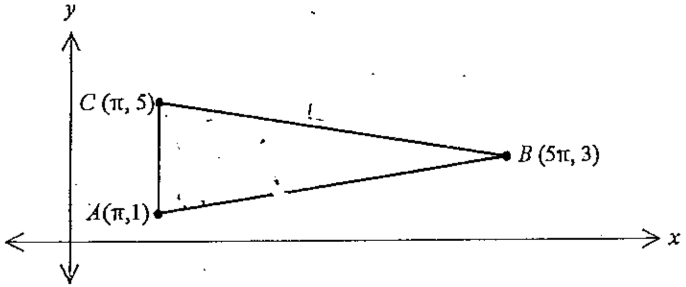
(i) How high will it rise on the fifth rebound? 2

(ii) How far will it travel in total? 1

(d) For the domain $0 \leq x \leq 6$, a function $y = f(x)$ satisfies $f'(x) < 0$ and $f''(x) < 0$. 2

Sketch a possible graph of $y = f(x)$ in this domain.

(e) The points $A(\pi, 1)$, $B(5\pi, 3)$ and $C(\pi, 5)$ form an isosceles triangle, with $AB = BC$.



(i) Find the midpoint of AB. 1

(ii) Show that the equation of the line which is perpendicular to AB and which passes through point C is: 2

$y + 2\pi x - 5 - 2\pi^2 = 0$

(iii) Calculate the distance AB. 1

(iv) Using the distances AB, BC and AC, or otherwise, find $\angle CAB$ to the nearest degree. 2

End of Question 13

Question 14 (15 marks) Use the Question 14 writing booklet.

(a) The relation $x^2 - 4x + y^2 = 5$ is rotated about the x-axis to form a solid. Find the exact volume of this solid of revolution. 2

(b) For the curve $y = x^3(3 - x)$

(i) Find any stationary points and determine their nature. 3

(ii) Draw a sketch of the curve showing the stationary points, inflexion points and intercepts on the axes. 3

(c) Georgina borrows \$650 000 to purchase her first home. She takes out a loan over 30 years, to be repaid in equal monthly instalments. The interest rate is 5.4% per annum reducible, calculated monthly.

(i) Show that the amount, $\$A_n$, owing after the nth repayment is given by the formula: 2

$$A_n = 650\,000(1.0045)^n - M\left(1 + 1.0045 + 1.0045^2 + \ldots\ldots + 1.0045^{n-1}\right)$$

(ii) Find the monthly repayment required to repay the loan in 30 years. 2

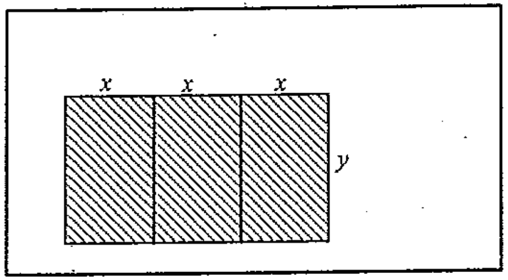
(iii) Georgina wants pay the loan off in less than 30 years. If she can afford to pay \$5 000 per month, how many months will it take her to pay off the home loan? 2

(iii) How much will Georgina save in interest if she pays \$5 000 per month? 1

End of Question 14

Question 15 (15 marks) Use the Question 15 writing booklet.

(a) Greg has a one hectare (Ha) block of land. He is going to fence off three identical rectangular plots within his block for his three children. Each plot will measure x m by y m as shown in the diagram below. He will retain the remainder of the block for himself and his wife. Greg can only afford 300 m of fencing to go around the children's plots.



(i) Show $y = 75 - \frac{3x}{2}$. 1

(ii) Find the value of x for which the area will be a maximum. 3

(iii) Find the maximum area of one of the children's blocks. 1

(iv) How much of Greg's 1 Ha block is left for him and his wife? 1

Question 15 continues on page 12

Question 15 (continued)

(b) The acceleration, after t seconds, of a particle moving in a straight line is given by $\ddot{x} = -\frac{14}{(t+4)^3}$.

Initially the particle is located $\frac{3}{4}$ m to the left of the origin and the initial velocity is $\frac{7}{16}$ m/s.

(i) Find the velocity v and the displacement x at any time t. 2

(ii) What is the velocity of the particle when it passes through the origin? 2

(iii) Sketch a graph of the displacement as a function of time. 2

(c) Find the value of n such that: 3

$$\frac{10^{3n} \times 25^{n+2}}{8^n} = 1$$

End of Question 15

Question 16 (15 marks) Use the Question 16 writing booklet.

(a) Connor buys a new car, which begins to depreciate immediately. The value ($$V$) of the car after t years is given by $V = A e^{-kt}$

Where A - is the initial value

k – constant of depreciation

t - time in years

If the car is worth $30 000 after 5 years and $18 000 after 10 years, find the following:

(i) The depreciation constant k 2

(ii) The initial value of the car 1

(iii) How many whole years will it take before the car's value falls below $1 000? 2

(b) A plane leaves an airport (A) and travels due north $\sqrt{3}x$ kilometres to a point K and then turns due west and travels a further x kilometres until it reaches a point P which is 380 kilometres from A. Due to storms the plane is then diverted to a new airport (B) which is 200 kilometres on a bearing of 280° from A.

(i) Draw a diagram and label it to show the above information. 1

(ii) Find the exact distance AK. 1

(iii) Show that the plane needs to travel 294 kilometres from P to the new airport (B). 2

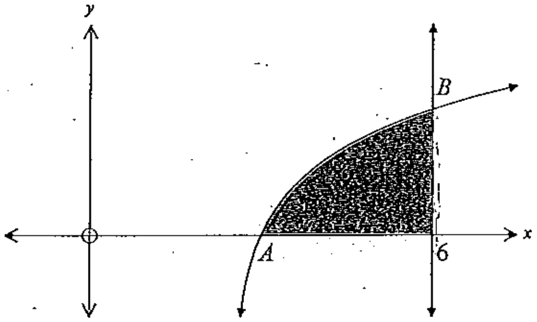
(iv) Hence or otherwise find the bearing (to the nearest degree) on which the plane flies from P to B. 1

Question 16 continues on page 14

Question 16 (continued)

(c) The diagram shows a shaded region which is bounded by the curve $y = \ln(2x - 5)$, the x axis and the line $x = 6$.

The curve $y = \ln(2x - 5)$ intersect the x axis at A and the line $x = 6$ at B.



(i) Show that the coordinates of points A and B are (3, 0) and (6, ln7) respectively. 1

(ii) Show that if $y = \ln(2x - 5)$, then $x = \frac{e^y + 5}{2}$. 1

(iii) Hence find the exact area of the shaded region. 3

End of Examination

Western Mathematics Exams

2015
TRIAL HSC
EXAMINATION

Mathematics

SOLUTIONS

Multiple Choice Worked Solutions

No	Working	Answer
1	$\left(\frac{2a}{3b}\right)^{-5}$ $\frac{1}{\left(\frac{2a}{3b}\right)^5}$ $=\frac{3^5b^5}{2^5a^5}$ $=\frac{243b^5}{32a^5}$	C
2	$2x^2-5x-9=0$ $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}$ $\alpha+\beta=-\frac{b}{a}=\frac{5}{2}$ $\alpha\beta=\frac{c}{a}=-\frac{9}{2}$ So $\frac{\alpha+\beta}{\alpha\beta}=\frac{-\frac{b}{a}}{\frac{c}{a}}$ $=-\frac{b}{c}$ $=-\frac{5}{9}$	C
3	$\lim\limits_{x\to\infty}\frac{3\sqrt{x}}{x-2}$ dividing by highest power of x $=\lim\limits_{x\to\infty}\frac{\frac{3\sqrt{x}}{x}}{\frac{x}{x}-\frac{2}{x}}=\lim\limits_{x\to\infty}\frac{\frac{3}{\sqrt{x}}}{1-\frac{2}{x}}$ $=\frac{0}{1-0}$ (as $x\to\infty,\frac{1}{x}\to0$ and $\frac{1}{\sqrt{x}}\to0$) $=0$	D
4	$y=3\cos 2x$ Amplitude $=3$ Period $=\frac{2\pi}{2}$ $=\pi$	B
5	~~$P(\text{red})=0.2$ $P(\text{not red})=0.8$~~ ~~$P(\text{not red for 6 balls})=(0.8)^6$~~ ~~$=0.262144$~~ ~~$=26\%$~~ $8e^{-2}=1{\cdot}08268\ldots.$ $=1{\cdot}08$ (3sf)	~~C~~ A
6	$\cos^2\left(\frac{\pi}{2}-\theta\right)\cot\theta$ $=\sin^2\theta\cot\theta$ $=\sin^2\theta\times\frac{\cos\theta}{\sin\theta}$ $=\sin\theta\cos\theta$	B
7	$\int_2^7\frac{5}{x}\,dx$ $=\left[5\ln x\right]_2^7$ $=5\ln7-5\ln2$ $=5\left[\ln7-\ln2\right]$	A
8	$x^2=4y$ $y=\frac{x^2}{4}$ $y'=\frac{2x}{4}$ $=\frac{x}{2}$ When $x=2$ $y'=1\therefore m_1=1$ So for normal $m_2=-1$ When $x=2,y=1$ $y-1=-1(x-2)$ $y-1=-x+2$ $y+x-3=0$	D
9	$\log_5 200-3\log_5 2$ $=\log_5 200-\log_5 2^3$ $=\log_5\left(\frac{200}{8}\right)$ $=\log_5 25$ $=2$	B
10	$x=2\cos2t$ $\dot{x}=-4\sin2t$ when $\dot{x}=0$ particle at rest $0=-4\sin2t$ $\sin2t=0$ $\sin\theta=0$ So $\theta=0,\pi,2\pi,3\pi,4\pi$ $2t=0,\pi,2\pi,3\pi,4\pi$ $t=0,\frac{\pi}{2},\pi,\frac{3\pi}{2},2\pi$	A

Trial HSC Examination 2015
Mathematics Course

Name____________________ Teacher__________________

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2+4=$ (A) 2 (B) 6 (C) 8 (D) 9
A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ☒ C ○ D ○

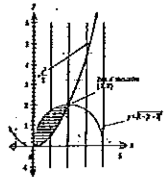
If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

correct
A ☒ B ☒ C ○ D ○

1. A ○ B ○ C ● D ○
2. A ○ B ○ C ● D ○
3. A ○ B ○ C ○ D ●
4. A ○ B ● C ○ D ○
5. A ● B ○ C ☒ D ○
6. A ○ B ● C ○ D ○
7. A ● B ○ C ○ D ○
8. A ○ B ○ C ○ D ●
9. A ○ B ● C ○ D ○
10. A ● B ○ C ○ D ○

Question 11		2015	
	Solution	Marks	Allocation of marks
(a)	x^2-2x-7 $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\frac{2\pm\sqrt{2^2-4\times1\times-7}}{2\times1}$ $x=\frac{2\pm\sqrt{4+28}}{2}$ $x=\frac{2\pm\sqrt{32}}{2}$ $x=\frac{2\pm\sqrt{16}\times\sqrt{2}}{2}$ $x=\frac{2\pm4\sqrt{2}}{2}$ $x=1\pm2\sqrt{2}$	2	1 for substitution into formula (or use of squares) 1 for simplification of surds.
(b)	$\int\frac{3x}{x^2+1}\,dx$ $=\frac{3}{2}\int\frac{2x}{x^2+1}\,dx$ $=\frac{3}{2}\ln\left(x^2+1\right)+C$	1	1 for correct answer.
(c)	$\frac{2}{\sqrt{7}+3}-\frac{3\sqrt{7}}{\sqrt{7}-3}=\frac{2\sqrt{7}-6-21-9\sqrt{7}}{7-9}$ $=\frac{-7\sqrt{7}-27}{-2}$ $=\frac{7\sqrt{7}+27}{2}$	2	1 (rational denominator) 1 for simplification
(d)	$\frac{x}{x+4.2}=\frac{5.6}{8.2}$ $8.2x=5.6(x+4.2)$ $8.2x=5.6x+23.52$ $2.6x=23.52$ $x=9.045$ $x=9.0$ (correct [illegible])	2	1 for correct ratio 1 for solving equation

Question 11		2015	
	Solution	Marks	Allocation of marks
(e)	$x^2-5y+5=0$ $x^2=5y-5$ $x^2=5(y-1)$ $\therefore$ Vertex at $(0,1)$ $4a=5$ $a=\frac{5}{4}$ $\therefore$ focal length $=\frac{5}{4}$ Focus $\left(0,1+\frac{5}{4}\right)$ $\therefore$ Focus $=\left(0,\frac{9}{4}\right)$	2	1 for vertex 1 for focus
(f)	$S_n=\frac{n}{2}\{2a+(n-1)d\}$ $S_{30}=\frac{30}{2}\{2\times5+(29\times4)\}$ $=1890$ $S_9=\frac{9}{2}[2\times5+(8\times4)]$ $=189$ Sum 10th - 30th terms $=S_{30}-S_9$ $=1890-189$ $=1701$	2	1 for correct substitution into formula 1 for answer (1 mark only if S_n is used)
(g)	$\int_0^{\ln6}e^x\,dx=\left[e^x\right]_0^{\ln6}$ $=e^{\ln6}-e^0$ $=6-1$ $=5$	2	1 for integration 1 for answer

Question 11		2015	
	Solution	Marks	Allocation of marks
(h)		2	1 for correct functions 1 for correct shading of intersection Point of intersection not required for marks.

Question 12		2015	
	Solution	Marks	Allocation of marks
(a)	(i) $y = \sin^2(4x)$ $y' = 2\sin 4x \cdot \cos 4x \cdot 4$ $= 8\sin 4x \cos 4x$	1	1 for correct use of chain rule
	(ii) $y = x^3 e^{3x}$ $y' = x^3(3e^{3x}) + e^{3x}(3x^2)$ $= 3x^2 e^{3x}(x+1)$	1	1 for correct use of product rule
	(iii) $y = \frac{e^x}{(x+3)^2}$ Quotient Rule $u = e^x \quad v = (x+3)^2$ $u' = e^x \quad v' = 2(x+3)$ $y' = \frac{vu' - uv'}{v^2}$ $= \frac{(x+3)^2(e^x) - (e^x)(2(x+3))}{(x+3)^4}$ Further simplification gives $= \frac{(x+3)(e^x) - (e^x)(2)}{(x+3)^3}$ $= \frac{e^x(x+3-2)}{(x+3)^3}$ $= \frac{e^x(x+1)}{(x+3)^3}$ but is not required.	2	Give 2 marks for a solution which shows a correct use of the quotient rule with correct differentials found including use of the chain rule (or expansion). Full simplification by cancelling is not required. Give 1 mark for a solution which has a minor error in the use of the quotient rule or finding the differentials
(b)	$\sqrt{3}\cos x = \sin x$ $\sqrt{3} = \frac{\sin x}{\cos x}$ $\tan x = \sqrt{3}$ $x = \frac{\pi}{3}$ tan positive 1st, 3rd quadrant $\therefore \quad x = \frac{\pi}{3}, \frac{4\pi}{3}$	2	1 for determining equation 1 for all solutions

<table>
<tr><td>Question 12</td><td></td><td>2015</td><td></td></tr>
<tr><td></td><td>Solution</td><td>Marks</td><td>Allocation of marks</td></tr>
<tr><td>(c)</td><td>$\int_0^1 \tan x\,dx$
Using Simpson's Rule
$h = \frac{1-0}{4} = \frac{1}{4}$
$\int_0^1 \tan x\,dx =$
$\frac{h}{3}\left\{[\tan 0 + \tan 1] + 4\left[\tan\frac{1}{4} + \tan\frac{3}{4}\right] + 2\left[\tan\frac{1}{2}\right]\right\}$
$= \frac{1}{12}(7.3977\ldots)$
$= 0.62$</td><td>2</td><td>1 for substitution into Simpson's rule

1 for evaluating correct answer
(only 1 mark if radians not used)</td></tr>
<tr><td>(d)</td><td><table>
<tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
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<tr><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td></tr>
</table></td><td></td><td>Simplification of fractions is not needed</td></tr>
<tr><td></td><td>(i) P(7 or 11) = $\frac{8}{36}$
$= \frac{2}{9}$</td><td>1</td><td>1 for correct answer</td></tr>
<tr><td></td><td>(ii) P(≥3) = $\frac{12}{36}$
$= \frac{1}{3}$</td><td>1</td><td>1 for correct answer</td></tr>
</table>

a) iv) $y = \frac{\cos x}{x^2}$ $\quad u = \cos x \quad v = x^2$

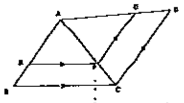
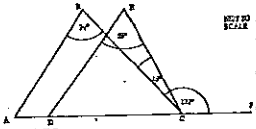
$u' = -\sin x \quad v' = 2x$

$y' = \frac{-x^2 \sin x - 2x\cos x}{x^4}$ ✓

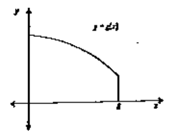
$y' = \frac{-x\sin x - 2\cos x}{x^3}$ ✓

Question 12		2015	
	Solution	Marks	Allocation of marks
(e)	$3x^2 + x + 1 \equiv A(x-1)(x+2) + B(x+1) + C$ RHS $= A(x^2 + 2x - x - 2) + Bx + B + C$ $= Ax^2 + Ax - 2A + Bx + B + C$ $= x^2A + x(A+B) + (-2A + B + C)$ Equating coefficients $A = 3$ $A + B = 1$ ① $-2A + B + C = 1$ ② From ① $A + B = 1$ $3 + B = 1$ $\therefore \quad B = -2$ From ② $-2A + B + C = 1$ $-6 - 2 + C = 1$ $C = 9$	2	1 for expansion and determining coefficients 1 for solving to find the values of A, B C.
(f)	(i) $y = x\cos x$ $0 = x\cos x$ $\therefore \quad x = 0$ or $\cos x = 0$ $\cos x = 0$ $x = 0, \frac{\pi}{2}, \frac{3\pi}{2}$ ∴ first after origin is $\frac{\pi}{2}$ $\therefore P\left(\frac{\pi}{2}, 0\right)$	1	1 for any reasonable explanation.

Question 12		2015	
	Solution	Marks	Allocation of marks
	(ii) $y = x\cos x$ $u = x \quad v = \cos x$ $u' = 1 \quad v' = -\sin x$ $y' = \cos x - x\sin x$ when $x = \frac{\pi}{2}$ $y' = \cos\frac{\pi}{2} - \frac{\pi}{2}\sin\frac{\pi}{2}$ $= -\frac{\pi}{2}$ Equation of tangent $y - 0 = -\frac{\pi}{2}\left(x - \frac{\pi}{2}\right)$ $y = \frac{-\pi x}{2} + \frac{\pi^2}{4}$	2	1 for gradient of tangent 1 for equation of tangent.

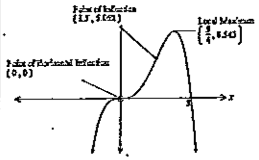
Question 13		2015	
	Solution	Marks	Allocation of marks
(a)	 AE/AB = AF/AC (ratio of intercepts) AF/AC = AG/AD (ratio of intercepts) ∴ AE/AB = AG/AD (equating ratios)	2	1 for use of ratios of intercepts 1 for conclusion
(b)	 Prove AB‖DE ∠BAD + 74° = 137° (exterior ∠ Δ) ∴ ∠BAD = 63° ∠EDC + 55° = 137° − 19° (exterior ∠ Δ) ∠EDC = 63° ∠BAD = ∠EDC (both = 63°) ∴ AB ‖ DE (equal corresponding ∠'s)	2	1 for showing angles are 63° 1 for stating lines parallel with reason

Question 13		2015	
	Solution	Marks	Allocation of marks
(c)	(i) Taking 42 as the first term as it is the first completed rebound.  $T_n = ar^{n-1}$ For the rise on the 5th bounce $a = 42 \quad r = \frac{3}{4} \quad n = 5$ $T_5 = 42\left(\frac{3}{4}\right)^{5-1}$ $T_5 = 42\left(\frac{3}{4}\right)^4$ $= 13.29\text{m}$ Note if take 56 as first term, need to find the 6th term.	2	1 for determining the series 1 for finding correct term (only 1 mark if found T6 after using a = 56)
	(ii) $\lvert r\rvert < 1 \quad r = \frac{3}{4}$ Consider one bounce up and down as a term, so $t_1 = 84$ $\therefore S_\infty = \frac{a}{1-r}$ $= \frac{84}{1-\frac{3}{4}}$ $= \frac{84}{\frac{1}{4}}$ $= 336\text{m}$ Total distance travelled will be 336 + 56 = 392m Alternately take 42 as first term and double result from 56 m	1	1 for correct answer

Question 13		2015	
	Solution	Marks	Allocation of marks
(d)	$f'(x) < 0$ ∴ negative gradient (decreasing) $f''(x) < 0$ ∴ concave down 	2	Graph needs to have negative gradient (1 mark) and concave down (1 mark).
(e)	(i) $A(\pi, 1) \quad B(5\pi, 3) \quad C(\pi, 5)$ Midpoint $M = \left[\frac{5\pi + \pi}{2}, \frac{1+3}{2}\right]$ $= (3\pi, 2)$	1	1 for correct answer
	(ii) $m_{AB} = \frac{3-1}{5\pi - \pi}$ $= \frac{2}{4\pi} = \frac{1}{2\pi}$. For Perpendicular line $m_1 \times m_2 = -1$ $\frac{1}{2\pi} \times m_2 = -1$ $m_2 = -2\pi$ Equation of line $y - 5 = -2\pi(x - \pi)$ $y - 5 = -2\pi x + 2\pi^2$ $y + 2\pi x - 5 - 2\pi^2 = 0$	2	1 for gradient of the perpendicular 1 for finding the equation of the line
	(iii) $\overline{AB} = \sqrt{(5\pi - \pi)^2 + (3-1)^2}$ $= \sqrt{(4\pi)^2 + (2)^2}$ $= \sqrt{16\pi^2 + 4}$	1	1 for correct answer

Question 13		2015	
	Solution	Marks	Allocation of marks
	(iv) $\overline{AB} = \sqrt{16\pi^2 + 4}$ $\overline{BC} = \overline{AB} = \sqrt{16\pi^2 + 4}$ (given) $\overline{AC} = 4$ (vertical line) $\therefore \cos A = \frac{b^2 + c^2 - a^2}{2bc}$ $\cos A = \frac{16\pi^2 + 4 + 16 - (16\pi^2 + 4)}{2 \times 4 \times \left(\sqrt{16\pi^2 + 4}\right)}$ $\cos A = \frac{16}{8\sqrt{16\pi^2 + 4}}$ $A = 81°$ (nearest degree) Alternate Solution Can also be done using the inclination of a line $m_{AB} = \frac{1}{2\pi}$ $\tan\theta = m_{AB}$ (where θ is the inclination to the positive x axis) $\tan\theta = \frac{1}{2\pi}$ $\theta = \tan^{-1}\left(\frac{1}{2\pi}\right) = 9°$ Now AC is vertical so inclination = 90° $\angle BAC = 90° - 9° = 81°$	2	1 for use of cosine rule 1 for answer 1 for angle with x axis 1 for answer

Question 14		2015	
	Solution	Marks	Allocation of marks
(a)	$x^2-4x+4+y^2=9$ is a circle $x^2-4x+4+y^2=5+4$ $(x-2)^2+y^2=9$ ∴ Centre (2,0) Radius = 3 So $V=\frac{4}{3}\pi r^3$ $=\frac{4}{3}\times\pi\times3^3$ $=36\pi$ units³ OR $x^2-4x+y^2=5$ $y^2=5-x^2+4x$ Intercepts $y=0$ so $x^2-4x-5=0$ $(x+1)(x-5)=0$ Intercepts are $x=-1$ or $x=5$ $V=\int_a^b y^2dx$ $=\int_{-1}^{5}5-x^2+4x\,dx$ $=\pi\left[5x-\frac{x^3}{3}+2x\right]_{-1}^{5}$ $=\pi\left(25-\frac{125}{3}+50\right)-\left(-5+\frac{1}{3}+2\right)$ $=36\pi$ units³	2	1 for circle and finding end points 1 for volume either method
(b)	(i) $y=x^3(3-x)=3x^3-x^4$ $y'=9x^2-4x^3$ Stationary points where $y'=0$ $9x^2-4x^3=0$ $x^2(9-4x)=0$ $x=0$ or $x=\frac{9}{4}$ $y=0$ or $y=8.543$ $y''=18x-12x^2$ $x=0, y''=0$ so possible inflexion test $x=-1,\ y''=-30\ x=1\ y''=6$ so change of concavity so (0,0) is horizontal inflexion $x=\frac{9}{4}\quad y''=-20\frac{1}{4}$ ∴ concave down so $\left(\frac{9}{4},8.543\right)$ is a local maximum	3	1 for the two x values of stationary pts 1 for second derivative used to determine possible nature. 1 for checking inflexion and naming the two points and their nature.

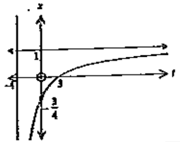
Question 14		2015	
	Solution	Marks	Allocation of marks
	(ii) Use second derivative to check for other turning points. $y''=18x-12x^2$ $y''=0$ when $18x-12x^2=0$ $6x(3-2x)=0$ $x=0$ or $x=\frac{3}{2}$ $x=0$ is horizontal inflexion found in part i) $x=\frac{3}{2},\ y=5\frac{1}{16}$ $x=2,\ y''=-12$ $x=1,\ y''=6$ ∴ change of concavity so inflexion at $\left(\frac{3}{2},5\frac{1}{16}\right)$ Intercepts on x axis $x^3(3-x)=0$ $x=0$ or $x=3$ 	3	1 for determining other inflexion 1 for general shape of sketch 1 for showing all features
(c)	(i) $P=\$650000\quad r=5.4\div100\div12=0.0045$ $A=P(1+r)^n-M$ $A_1=650000(1.0045)^1-M$ $A_2=A_1(1.0045)^1-M$ $A_2=[650000(1.0045)^1-M](1.0045)-M$ $A_2=650000(1.0045)^2-M(1.0045)-M$ $A_2=650000(1.0045)^2-M[1+1.0045]$ $A_3=(650000(1.0045)^2-M[1+1.0045])(1.0045)-M$ $A_3=650000(1.0045)^3-M[1+1.0045+1.0045^2]$ . . . $A_n=650000(1.0045)^n-M[1+1.0045+\ldots+1.0045^{n-1}]$	2	1 for setting up initial terms as examples 1 for following pattern to establish required formula

Question 14		2015	
	Solution	Marks	Allocation of marks
	(ii) Months = 30 × 12 = 360 repayments $A_{360}=0$ (loan repaid) $A_n=650000(1.0045)^n-M[1+1.0045+\ldots+1.0045^{n-1}]$ $0=650000(1.0045)^n+M[1+1.0045+\ldots+1.0045^{n-1}]$ $M[1+1.0045+\ldots+1.0045^{n-1}]=650000(1.0045)^n$ $M=\frac{650000(1.0045)^n}{1+1.0045+\ldots+1.0045^{n-1}}$ The denominator is a geometric series with $a=1, r=1.0045$ and $n=360$ $S_n=\frac{a(r^n-1)}{r}$ $S_{360}=\frac{1((1.0045)^{360}-1)}{0.0045}$ $S_{360}=\frac{(1.0045)^{360}-1}{0.0045}$ ∴ $M=\frac{(650000(1.0045)^{360})\times0.0045}{(1.0045)^{360}-1}$ $M=\$3549.55$	2	1 for expression for M 1 for substituting into sum of series and finding M (can use rounded answer for S_n)
	(iii) $A_n=650000(1.0045)^n-5000S_n$ $A_n=\$0$ paid off $5000S_n=650000(1.0045)^n$ $5000\left[\frac{(1.0045)^n-1}{0.0045}\right]=650000(1.0045)^n$ $5000(1.0045)^n-5000=2925(1.0045)^n$ $5000(1.0045)^n-2925(1.0045)^n=5000$ $(1.0045)^n(5000-2925)=5000$ $(1.0045)^n=\frac{5000}{2075}$ $\ln(1.0045^n)=\ln\left[\frac{5000}{2075}\right]$ $n\ln(1.0045)=\ln\left[\frac{5000}{2075}\right]$ $n=\frac{\ln\left[\frac{5000}{2075}\right]}{\ln1.0045}$ $n=195.81$ $=196$ months	2	1 for using sum to establish equation 1 for solving to find n

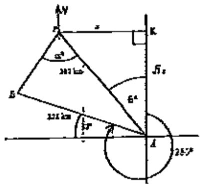
Question 14		2015	
	Solution	Marks	Allocation of marks
	(iv) Total of loan over 30 years 360 × \$3 649.53 = \$1 313 592 Total of loan by paying \$5000/month 196 × \$5 000 = \$980 000 Interest Saving \$1 313 592 − \$980 000 = \$333 592	1	1 for answer

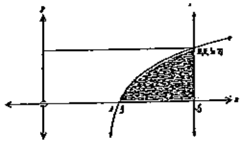
Question 15		2015	
	Solution	Marks	Allocation of marks
(a)	(i) $3x+3x+4y=300$ $4y=300-6x$ $y=75-\frac{3x}{2}$	1	1 for correct expression
	(ii) $A=3x\times y$ $A=3x\left[75-\frac{3x}{2}\right]$ $A=225x-\frac{9x^2}{2}$ Maximum Area find A' $A'=225-\frac{18x}{2}$ $=225-9x$ $A'=0$ $0=225-9x$ $9x=225$ $x=25m$ When $x=25m$ $y=37.5m$ Test maximum point $A''=-9$ <0 ∴ Maximum Area ∴ $x=25m$ will produce the maximum area.	3	1 for A' 1 for x 1 for test that it is maximum
	(iii) $A=25\times37.5$ $=937.5m^2$	1	1 for area
	(iv) $3\times937.5=2812.5m^2$ 1Ha = $10000m^2$ $10000-2812.5=7187.5m^2$ So Greg and his wife will have 7187.5 m² left.	1	1 for answer

Question 15		2015	
	Solution	Marks	Allocation of marks
(b)	(i) $\ddot{x}=-\frac{14}{(t+4)^3}$ $=-14(t+4)^{-3}$ $\dot{x}=\int-14(t+4)^{-3}dt$ $=7(t+4)^{-2}+c_1$ when $t=0$ $v=\frac{7}{16}$ $\frac{7}{16}=7(0+4)^{-2}+c_1$ $c_1=0$ ∴ $\dot{x}=\frac{7}{(t+4)^2}$ $x=\int\frac{7}{(t+4)^2}dt$ $=\frac{-7}{t+4}+c_2$ when $t=0$ $x=-\frac{3}{4}m$ $-\frac{3}{4}=\frac{-7}{0+4}+c_2$ $c_2=1$ ∴ $x=\frac{-7}{t+4}+1$	2	1 for velocity 1 for displacement
	(ii) When $x=0$ the particle is at the origin $0=\frac{-7}{t+4}+1$ $\frac{7}{t+4}=1$ $7=t+4$ $t=3$ When $t=3$ $\dot{x}=\frac{7}{(t+4)^2}$ $=\frac{7}{(3+4)^2}$ $=\frac{1}{7}$ m/s	2	1 for value of t 1 for velocity

Question 15		2015	
	Solution	Marks	Allocation of marks
	(iii) $x=1-\frac{7}{t+4}=\frac{t+4-7}{t+4}=\frac{t-3}{t+4}$ $t\neq-4$, so $t=-4$ would be a vertical asymptote. $t=0\Rightarrow x=-\frac{3}{4}$ $x=0,\Rightarrow\frac{t-3}{t+4}=0\Rightarrow t=3$ As $t\to+\infty,\frac{t-3}{t+4}\to1$ from below e.g. $t=1000,\ x=\frac{1000-3}{1000+4}=\frac{997}{1004}=0.993$ So $x=1$ is an horizontal asymptote. 	2	1 for intercepts 1 for sketch with asymptote
(c)	$\frac{10^{3n}\times25^{n+2}}{8^n}=1$ LHS $=\frac{(10^3)^n\times(5^2)^{n+2}}{(2^3)^n}$ $=\frac{(1000)^n\times(5^2)^{n+2}}{(2^3)^n}$ $=\frac{(2^3\times5^3)^n\times5^{2n+4}}{(2^{3n})}$ $=\frac{(2^{3n}\times5^{3n})\times(5^2)^{n+2}}{(2^{3n})}$ $=5^{3n}\times5^{2n+4}$ ∴ $5^{5n+4}=1$ $5^0=1$ ∴ $5n+4=0$ $n=-\frac{4}{5}$	3	1 for expanding the terms 1 for collecting powers of 2 and of 5 1 for solving for n

Question 16		2015	
	Solution	Marks	Allocation of marks
(a)	$V=Ae^{-kt}$ $30000=Ae^{-5k}$ ……(1) $18000=Ae^{-10k}$ ……(2) ② + ① $\frac{18000}{30000}=\frac{Ae^{-10k}}{Ae^{-5k}}$ $e^{-5k}=\frac{3}{5}$ $-5k=\ln\left(\frac{3}{5}\right)$ $k=-\frac{\ln\left(\frac{3}{5}\right)}{5}$ $k=0.102165124$	2	1 for eliminating A 1 for value of k
	(ii) $V=Ae^{-0.102\ldots\times t}$ when $t=5$ $V=\$30000$ $30000=Ae^{-0.102\times5}$ $A=\$50000$	1	1 for value of A
	(iii) $V=\$50000e^{-0.102t}$ $50000e^{-0.102t}<1000$ $e^{-0.102t}<\frac{1}{50}$ $-0.102t<\ln\left(\frac{1}{50}\right)$ $t>\frac{\ln\left(\frac{1}{50}\right)}{-0.102}$ $t>38.23$ ∴ It will take 39 years to fall below \$1000	2	1 for correct inequality for t 1 for value of t

Question 16		2015	
	Solution	Marks	Allocation of marks
(b)	(i) 	1	1 for diagram
	(ii) $x^2+(\sqrt{3}x)^2 = 386^2$ $x^2+3x^2 = 386^2$ $4x^2 = 386^2$ $x^2 = \frac{386^2}{4} = \frac{386}{2}$ $x = 193$ The distance AK $= \sqrt{3}x = \sqrt{3} \times 193 = 193\sqrt{3}$	1	1 for answer
	(iii) $\tan\theta = \frac{x}{\sqrt{3}x}$ $\tan\theta = \frac{1}{\sqrt{3}}$ $\theta = 30°$ $\angle PAB = 90 - 30 - 18$ $= 55°$ $(PB)^2 = 200^2 + 386^2 - (2 \times 200 \times 386 \times \cos 55°)$ $= 16556.23$ $PB = 254.44$ ≈ 254 km	2	1 for angle θ. 1 for distance
	(iv) Find $\angle APB$ $\frac{\sin\alpha}{200} = \frac{\sin 55°}{254.44}$ $\alpha = \sin^{-1}\left[\frac{200 \sin 55°}{254.44}\right]$ $= 31°21'$ $= 31°$ $\theta = 30$ so $\angle APK = 60°$ Bearing $= 90° + 60° + 31°$ $= 181°T$	1	1 for bearing

(c)	(i) A is on the x axis so $y = 0$ $\ln(2x-5) = 0$ $2x-5 = e^0 = 1$ $2x = 6$ $x = 3$ A is the point (3, 0) For B, $x = 6$ so $y = \ln(2 \times 6 - 5)$ $y = \ln 7$ B is the point (6, ln 7) 	1	1 for use of logs to show both values
	(ii) Given $\ln(2x-5)$ change subject to x. $2x-5 = e^y$ $2x = e^y+5$ $x = \frac{e^y+5}{2}$	1	1 for changing the subject

	(iii) Can't integrate $\ln(2x-5)$ so use the area between the curve and the y axis and subtract from the rectangle shown. Area to y axis $= \int_0^{\ln 7} \frac{e^y+5}{2}\,dy$ $= \left[\frac{e^y+5y}{2}\right]_0^{\ln 7}$ $= \frac{(e^{\ln 7}+5(\ln 7))}{2} - \frac{e^0+5\times 0}{2}$ $= \frac{(7+5(\ln 7)-1)}{2}$ $= \frac{(6+5\ln 7)}{2}$ Area Rectangle $= 6 \times \ln 7 = 6\ln 7$ Shaded area $= 6\ln 7 - \frac{(6+5\ln 7)}{2}$ $= \frac{(12\ln 7 - (6+5\ln 7))}{2}$ $= \frac{7\ln 7 - 6}{2}$ square units		1 for correct integral 1 for finding area to y axis 1 for shaded area