J.M.J.

MARCELLIN COLLEGE RANDWICK

YEAR 12 HSC

ASSESSMENT TASK # 2

EXTENSION ONE

MATHEMATICS

2005

TOPICS: BINOMIAL THEOREM & PERMS & COMBS.

Weighting: 10% of H.S.C. Assessment Mark.

STUDENT NAME: ___ MARK:

PERCENTAGE:

RANK ON THIS TASK:

Time Allowed: **40 minutes.**

Directions:

- Answer all questions on separate lined paper.
- Show all necessary working.
- Marks may not be awarded for careless or badly arranged work.

Outcomes examined:

PE3 – Solves problems involving permutations and combinations.

HE3 – Uses a variety of strategies to investigate Mathematical models of situations involving binomial probability.

Question One (2 marks)

Write the 5^{th} term in the expansion of: $(a+b)^{11}$

Question Two (3 marks)

8 people are to be seated randomly around a round table. What is the probability that a particular husband and wife will not be seated next to each other?

Question Three (5 marks)

a) How many seven – letter arrangements of the letters of the word COLLEGE are possible?

b) In how many of the seven – letter arrangements in a) above will the 2 L's be next to each other?

c) How many seven – letter arrangements can be made from the letters of the word COLLEGE if the 2^{nd} letter must be a G and the 1^{st} letter cannot be an L?

Question Four (2 marks)

Find the term independent of x in the expansion of $(x^2 + \frac{1}{x})^{18}$

Question Five (4 marks)

A machine produces electronic components for computers. Sampling shows that the probability of a particular component being faulty is 12%. In a random sample of 16 components, what is the probability that:

a) exactly 1 component is faulty ? (Answer correct to 3 decimal places)

b) less than 3 components are faulty? (Answer correct to 3 decimal places)

Question Six (5 marks)

a) Write down the binomial expansion of $(1 + x)^n$

b) By differentiating $x^3 (1 + x)^n$ and its binomial expansion, show that:

$$\sum_{r=0}^{n} (r+3)\ {}^{n}C_{r} = (n + 6) . 2^{n-1}$$

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SOLUTIONS | **MARKING SCHEME**

Question One

$T_{k+1} = {}^nC_k\, a^{n-k} b^k$

$\therefore T_5 = {}^{11}C_4\, a^7 b^4$

or $330a^7b^4$

1 mark for general term

) 1 mark for correct term in either form

Question Two

No. of ways of arranging 8 people in a circle = 5040

1 mark for total number of ways.

Treat husband and wife as 1 $\therefore$ 7 people = 720 ways

But husband & wife could alternate positions.

$\therefore$ Total ways together = 1440

1 mark for total ways together

$\therefore P(\text{not together}) = \frac{5040 - 1440}{5040}$

$= \frac{5}{7}$

1 mark for correct probability

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SOLUTIONS | **MARKING SCHEME**

Question Three

a) $\frac{7!}{2!2!} = 1260$

1 mark for correct no. of ways.

b) Treat 2 L's as one.

∴ Total number of ways with L's next to each other $= \frac{6!}{2!}$

$= 360$

1 mark for correct no. of ways.

c) No. of possible letters as 1st letter = 3

No. of possible letters as 2nd letter = 1

Remaining 5 letters:

If C or O used as 1st letter, $\frac{5!}{2!2!} = 30$

If an E used as 1st letter, $\frac{5!}{2!} = 60$

1 mark for no. of letters possible as "1st letter"

1 mark for total no. of ways letters after the 1st 2 can be arranged.

∴ Total ways $= 3 \times 1 \times 90$

$= 270$.

1 mark for multiplying the 3 different restrictions correctly together.

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SOLUTIONS | **MARKING SCHEME**

Question Four

$T_{k+1} = {}^{18}C_k (x^2)^{18-k} (x^{-1})^k$

$= {}^{18}C_k \, x^{36-2k} \, x^{-k}$

$= {}^{18}C_k \, x^{36-3k}$

1 mark for correct general term

Now term indep. of x

when $k = 12$

$\therefore$ Term indep. of $x = {}^{18}C_{12}$

$= 18564$

1 mark for correct term

Question Five

a) P(exactly 1 faulty)

$= {}^{16}C_1 (0.12)(0.88)^{15}$

$= 0.282$ (3dp)

1 mark for correct binomial expression

1 mark for correct prob (3dp)

b) P(less than 3 are faulty)

$= P(\text{No F}) + P(1\text{F}) + P(2\text{F})$

$= {}^{16}C_0 (0.88)^{16} +$ ANSWER ABOVE $+ {}^{16}C_2 (0.12)^2 (0.88)^{14}$

1 mark for correct breakdown of possibilities

$= 0.700$ (3dp)

1 mark for correct prob. (3dp)

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SOLUTIONS | **MARKING SCHEME**

Question Six

a) $(1+x)^n$

$= {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \ldots + {}^nC_n x^n$

1 mark for correct expansion

b) $x^3(1+x)^n = {}^nC_0 x^3 + {}^nC_1 x^4 + {}^nC_2 x^5 + \ldots + {}^nC_n x^{n+3}$

1 mark for this statement

Differentiating both sides:

$x^3 n(1+x)^{n-1} + 3x^2(1+x)^n = 3\,{}^nC_0 x^2 + 4\,{}^nC_2 x^3 + \ldots + (n+3)\,{}^nC_n x^{n+2}$

$x^2(1+x)^{n-1}[xn + 3(1+x)] = \sum_{r=0}^{n} (r+3)\,{}^nC_r\, x^{r+2}$

1 mark for correct simplification

1 mark for correct differentiation of both sides of equality

Let $x = 1$

$\therefore 2^{n-1}[n+6] = \sum_{r=0}^{n} (r+3)\,{}^nC_r$

1 mark for letting $x = 1$ and correctly equating to reqd result.