



YEAR 11 HSC

ASSESSMENT TASK # 1

EXTENSION TWO

MATHEMATICS

2007

Weighting: 15% of HSC Assessment Mark.

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STUDENT NAME: /	MARK:	/ 26
	PERCENTAGE:	%
	RANK ON THIS TASK:	/ 11

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Time Allowed: 50 minutes

Directions: \* Answer all questions on separate lined paper.

\* Begin each question on a new page.

\* Show all necessary working.

\* Marks may not be awarded for careless or badly arranged work.

**Question One** ( Total - 18 marks ) **Begin your answers on a new page.**

(a) If  $z = 3 + 2i$ , plot on the same Argand diagram:

- (i)  $z$  and  $\bar{z}$  1
- (ii)  $iz$  1
- (iii)  $z(1+i)$  1

(b) (i) Find all the pairs of integers  $a$  and  $b$  such that  $(a+ib)^2 = 8+6i$  1

✓ (ii) Hence solve:  $z^2 + 2z(1+2i) - (11+2i) = 0$  2

(c) (i) If  $z = \cos \frac{\pi}{3} + i \sin \frac{\pi}{3}$ , find the value of  $z^6$  2

2 (ii) Plot on an Argand diagram all complex numbers that are the solutions of  $z^6 = 1$  2

(d) Sketch the locus of the each of the following on separate diagrams:

(i)  $\arg(z-1-2i) = \frac{\pi}{4}$  1

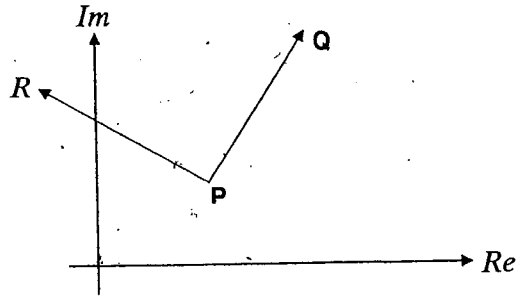
(ii)  $z\bar{z} - 3(z+\bar{z}) \leq 0$  2

3 (iii)  $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{3}$  2

**Question One continued:**

(c)

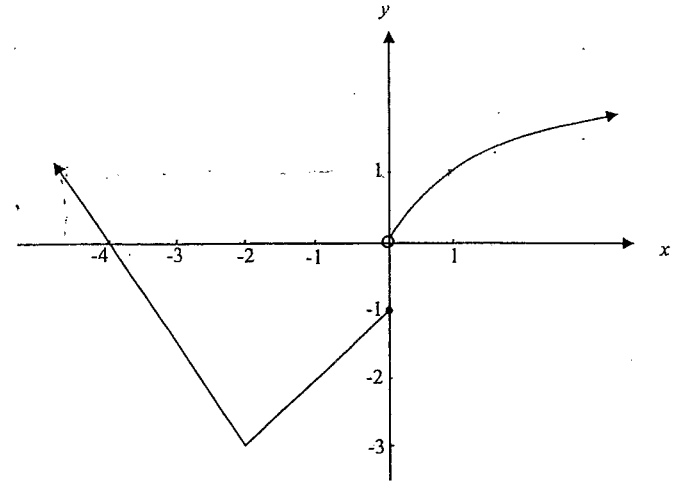
In the diagram below, P represents the complex number  $3 + 2i$  and Q represents  $7 + 8i$ .



- (i) What complex number is represented by the vector PQ? 1
  
- (ii) Suppose that R is the image of Q under an anticlockwise rotation of  $\frac{\pi}{2}$  about P. Write the complex number represented by the point R. 2

**Question Two ( Total - 8 marks ) Begin your answers on a new page.**

The diagram below shows the discontinuous function  $y = f(x)$ .



Draw separate sketches of each of the following:

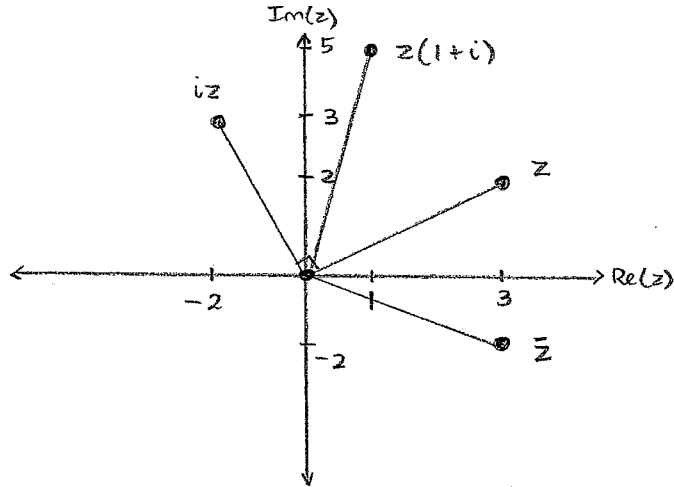
- |   |  |
|---|--|
| (a) $y =  f(x-1) $ <span style="float: right;">2</span>       | (c) $\sqrt{-f(x)}$ <span style="float: right;">2</span>  |
| (b) $y = \frac{1}{f(x)}$ <span style="float: right;">2</span> | (d) $y = \ln(f(x))$ <span style="float: right;">2</span> |

ASSESSMENT TASK NO. 1

SOLUTIONS

Question One

(a)  $z = 3 + 2i$



1 mark for each correctly plotted point.

(ii)  $z(1+i) = (3+2i)(1+i)$   
 $= 3 + 3i + 2i - 2$   
 $= 1 + 5i$

(b) (i)  $(a+ib)^2 = 8 + 6i$

$\therefore a^2 - b^2 + 2abi = 8 + 6i$

Equating real and imag. parts:

$a^2 - b^2 = 8$  and  $ab = 3$

$\therefore a = 3, b = 1$

and  $a = -3, b = -1$

) 1 mark

Q1.(b)(ii)  $z^2 + 2z(1+2i) - (1+2i) = 0$

Using QF:  $z = \frac{-2(1+2i) \pm \sqrt{4(1+2i)^2 + 4(1+2i)}}{2}$   
 $\therefore z = \frac{-2(1+2i) \pm \sqrt{32 + 24i}}{2}$

$\therefore z = \frac{-2(1+2i) \pm 2\sqrt{8+6i}}{2}$  ← 1 mark

$\therefore z = -(1+2i) \pm (3+i)$

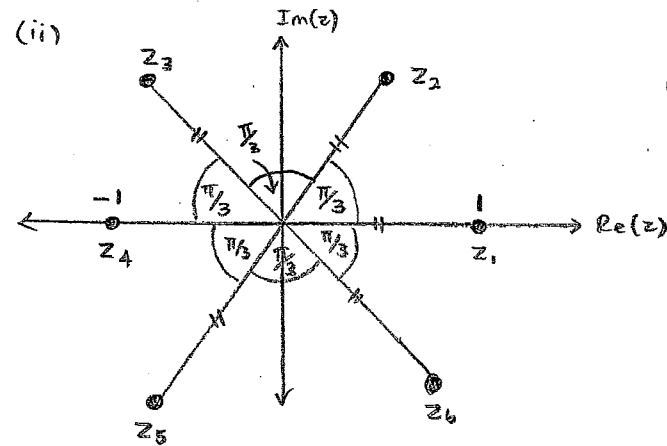
$\therefore z = -1-2i \pm (3+i)$

$\therefore z = 2-i, -4-3i$  ← 1 mark

(c) (i)  $z = \text{cis } \frac{\pi}{3}$  ← 1 mark

$\therefore z^6 = \text{cis } 2\pi$  (Using de Moivre's Theorem)

$\therefore z^6 = 1$  ← 1 mark



Marked same as Q1(d)

$z_1 = 1$

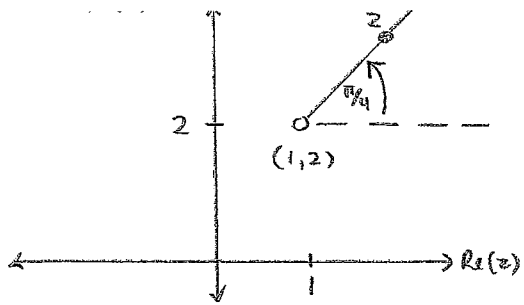
$z_2 = \text{cis } \pi/3$

$z_3 = \text{cis } 2\pi/3$

$z_4 = -1$

$z_5 = \text{cis } (-2\pi/3)$

$z_6 = \text{cis } (-\pi/3)$



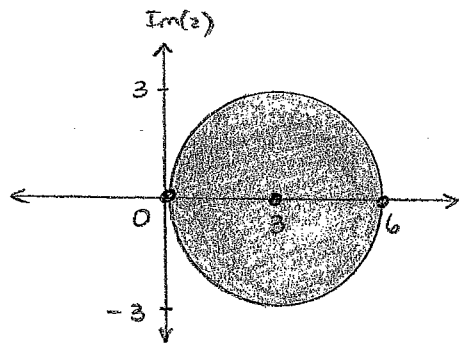
For Q1 (d) (i) - (iii),  
1 mark deducted  
for each incorrectly  
drawn or omitted  
part of correct graph

(ii) let  $z = x + yi$

$$\begin{aligned} \text{New } z\bar{z} &= (x+yi)(x+yi) \\ &= x^2 + y^2 \end{aligned}$$

and  $z + \bar{z} = 2x$

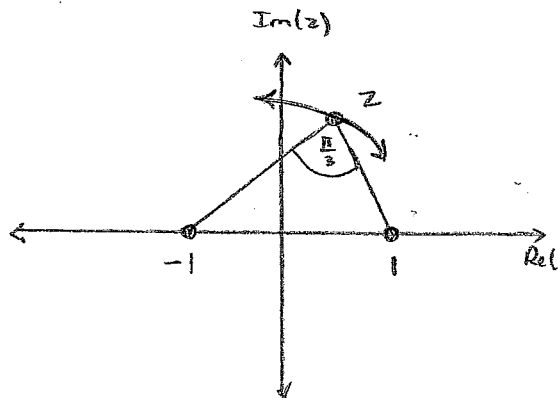
$$\begin{aligned} \therefore z\bar{z} - 3(z + \bar{z}) &= x^2 - 6x + y^2 \\ &= (x-3)^2 - 9 + y^2 \end{aligned}$$



$$(x-3)^2 + y^2 \leq 9$$

(iii)  $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{3}$

$$\therefore \arg(z-1) - \arg(z+1) = \frac{\pi}{3}$$



Q1 (e) i)  $PQ = PO + OQ$

$$= -3 - 2i + 7 + 8i$$

$$= 4 + 6i \quad \leftarrow 1 \text{ mark}$$

ii)  $\vec{PR} = i\vec{PQ}$

$$= i(4 + 6i)$$

$$= -6 + 4i$$

$\leftarrow 1 \text{ mark}$

$$\vec{OR} = \vec{OP} + \vec{PR}$$

$$= 3 + 2i - 6 + 4i$$

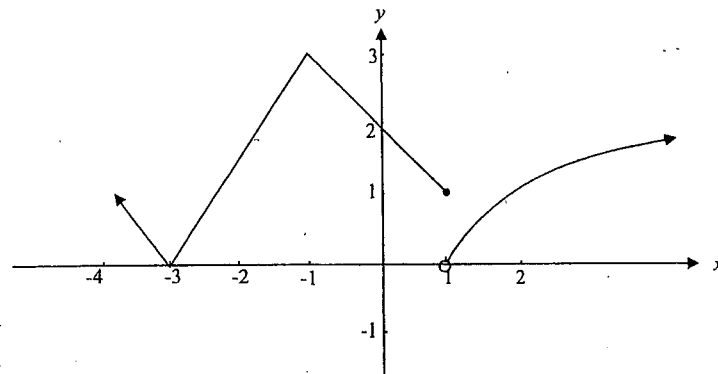
$$= -3 + 6i$$

$\therefore$  the complex no. represented

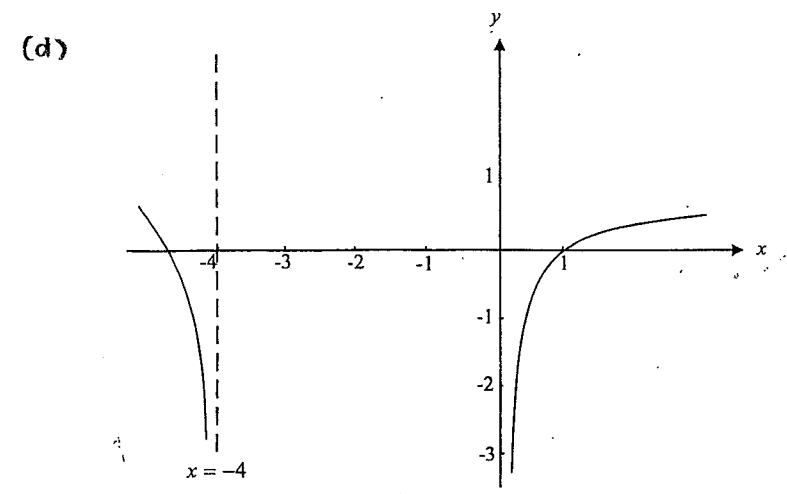
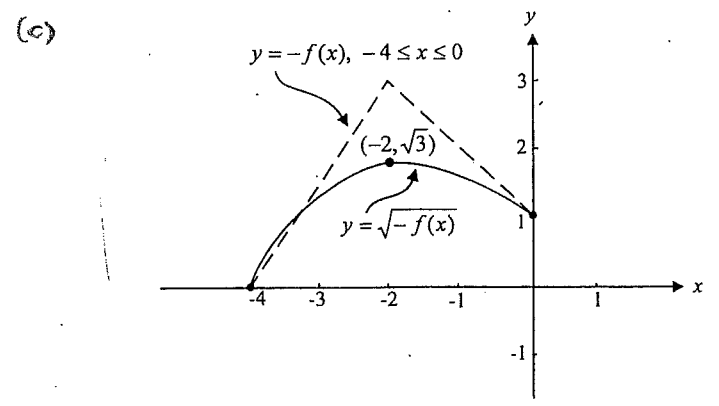
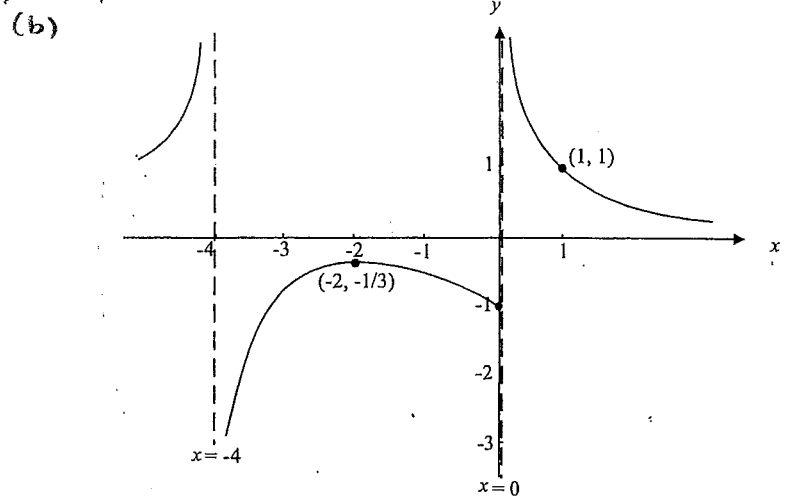
by R is  $-3 + 6i \quad \leftarrow 1 \text{ mark}$

Question Two

(a)



For all parts (a) - (d) of Q2, one mark deducted  
for each incorrectly drawn or omitted part of  
correct graph



(1)  $z = \cos \frac{\pi}{3}$   
 $z^6 = (\cos \frac{\pi}{3})^6$  (de Moivre)  
 $= \cos 2\pi$   
 $= \cos 0$   
 $= 1$

(ii)

