J.M.J.

MARCELLIN COLLEGE RANDWICK

YEAR 12 PRELIMINARY

ASSESSMENT TASK # 2

EXTENSION I MATHEMATICS

2007

ighting: 40% of Preliminary Assessment Mark.

UDENT NAME:		MARK:	/ 29
		PERCENTAGE:	%
		RANK ON THIS TASK:	/ 24 3

ne Allowed: 50 minutes

ections:

- Answer all questions on separate answer paper.
- Show all necessary working.
- Marks may not be awarded for careless or badly arranged work.

comes examined:

- PE3 – Solves problems involving polynomials and parametric representations.
- PE4 – Uses the parametric representation together with differentiation to identify geometric properties of parabolas.

QUESTION ONE (2 MARKS)

Marks

$P(x) = x^3 + x^2 - 21x - 45$

(a) Show that $x + 3$ is a factor of $P(x)$ — 1

(b) Hence factorize $P(x)$ in terms of its linear factors — 1

QUESTION TWO (6 MARKS)

Consider the equation $x^3 + 2x - 6 = 0$

(a) Show a root exists between $x = 1.4$ and $x = 1.5$ — 2

(b) Using the method of 'halving the interval', determine whether $x = 1.4$ or $x = 1.5$ is the best approximation to the root correct to 1 decimal place — 1

(c) Use Newton's Method once with an initial approximation of $x = 1.45$ to determine a better approximation to the root (correct to 2 decimal places) — 3

QUESTION THREE (11 MARKS)

$P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are variable points on the parabola $x^2 = 4ay$

(a) Show the equation of PQ is given by: — 2

$$y = \frac{(p+q)x}{2} - apq$$

(b) If PQ is a focal chord, find the value of pq — 1

(c) Show the equation of the normal to $x^2 = 4ay$ at P is given by: — 3

$$py - ap^3 = 2ap - x$$

(d) State the equation of the normal at Q — 1

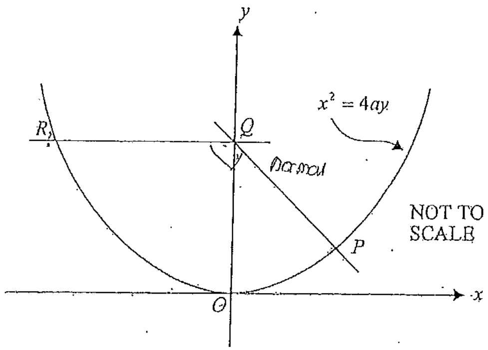
(e) Find the locus of the point of intersection of the normals at P and Q — 4

QUESTION FOUR (4 MARKS)

Marks

The polynomial $2x^3 + ax^2 + bx + 6$ has $x - 1$ as a factor and leaves a remainder of -12 when divided by $x + 2$. Find the values of a and b. 4

QUESTION FIVE (6 MARKS)



The diagram above shows the graph of the parabola $x^2 = 4ay$. The normal to the parabola at the variable point $P(2at, at^2)$, $t > 0$, cuts the y axis at Q. Point R lies on the parabola.

(i) Show that the equation of the normal to the parabola at P is $x + ty = at^3 + 2at$. 2

(ii) Find the coordinates of R given that QR is parallel to the x axis and $\angle PQR > 90°$ 2

(iii) Let M be the midpoint of RQ. Find the Cartesian equation of the locus of M. 2

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EXTENSION I MATHS

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Question One

a) $P(-3) = 0$ $\therefore$ by Factor Theorem, $x + 3$ is a factor of $P(x)$ 1

b)
$$
\begin{array}{r}
x^2 - 2x - 15 \\
x + 3 \,\big)\, x^3 + x^2 - 21x - 45 \\
x^3 + 3x^2 \\
\hline
-2x^2 - 21x - 45 \\
-2x^2 - 6x \\
\hline
-15x - 45 \\
-15x - 45 \\
\hline
0
\end{array}
$$

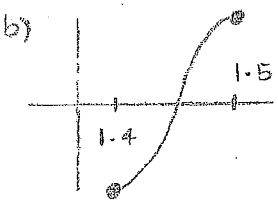
$\therefore P(x) = (x+3)(x-5)(x+3)$ 1

Question Two let $P(x) = x^3 + 2x - 6$

a) $P(1.4) = -0.456$ and $P(1.5) = 0.375$ 1

Since $P(x)$ is continuous, $P(1.4) < 0$ and $P(1.5) > 0$, a root exists between $x = 1.4$ and $x = 1.5$. 1

b)



$P(1.45) = -0.051375$

$\therefore x = 1.5$ is the best approx. (1 dp) 1

c) $a_1 = a_0 - \dfrac{P(a_0)}{P'(a_0)}$ 1

$= 1.45 - \dfrac{P(1.45)}{P'(1.45)}$

$P(1.45) = -0.051375$

$P'(x) = 3x^2 + 2$ 1

$\therefore P'(1.45) = 8.3075$

$\therefore a_1 = 1.46$ (2dp) 1

Question Three

a) m of PQ = $\frac{p+q}{2}$ ①

Eqn of PQ: $y - ap^2 = \frac{p+q}{2}(x - 2ap)$

$\therefore 2y - 2ap^2 = px + qx - 2ap^2 - 2apq$

$\therefore 2y = (p+q)x - 2apq$ ①

$\therefore y = \frac{(p+q)x}{2} - apq$

b) If PQ is a focal chord it passes thru $(0, a)$

$\therefore a = -apq$

$\therefore pq = -1$ ①

c) $x^2 = 4ay$

$\therefore y = \frac{x^2}{4a}$

$\therefore y' = \frac{2x}{4a}$

$\therefore y'(2ap) = p$ ①

$\therefore$ m of T = p

$\therefore$ m of N = $-\frac{1}{p}$ ①

Eqn of Normal at P:

$y - ap^2 = -\frac{1}{p}(x - 2ap)$

$\therefore py - ap^3 = -x + 2ap$ ①

ie. $py - ap^3 = 2ap - x$ as req'd

d) $qy - aq^3 = 2aq - x$ ①

Question Three continued...

(e) $py - ap^3 = 2ap - x$ (1)

$qy - aq^3 = 2aq - x$ (2)

(1) − (2): $py - qy - ap^3 + aq^3 = 2ap - 2aq$

$\therefore py - qy = ap^3 - aq^3 + 2ap - 2aq$

$\therefore (p-q)y = a(p-q)(p^2 + pq + q^2) + 2a(p-q)$

$\therefore y = a(p^2 + pq + q^2) + 2a$

subst $y \to (1)$

$ap(p^2 + pq + q^2) + 2ap - ap^3 = 2ap - x$

$\therefore ap^3 + ap^2q + apq^2 + 2ap - ap^3 = 2ap - x$

$\therefore x = -ap^2q - apq^2$

$\therefore x = -apq(p+q)$

$\therefore$ Point of Intersection of Normals has coords:

① ①

$\left[-apq(p+q),\ a(p^2 + pq + q^2 + 2)\right]$

Now $x = -apq(p+q)$ and $y = a(p^2 + pq + q^2 + 2)$

$\therefore p + q = \frac{x}{-apq}$

But $pq = -1$

$\therefore p + q = \frac{x}{a} \longrightarrow$

$\therefore \frac{y}{a} = p^2 + q^2 + pq + 2$

①

$\therefore \frac{y}{a} = (p+q)^2 - 2pq + pq + 2$

$\therefore \frac{y}{a} = \left(\frac{x}{a}\right)^2 + 3$

$\therefore \frac{y}{a} = \frac{x^2}{a^2} + 3$

$\therefore ay = x^2 + 3a^2$

$\therefore x^2 = ay - 3a^2$ ①

Question Four

let $P(x) = 2x^3 + ax^2 + bx + 6$

so $P(1) = 0$ ie $a + b + 8 = 0$

$\therefore a + b = -8 \quad (1)$ 1

and $P(-2) = -12$ ie. $4a - 2b - 10 = -12$

$\therefore 2a - b = -1 \quad (2)$ 1

Solving simultaneously:

$(1) + (2)$: $3a = -9$

$\therefore a = -3$ 1

$\therefore b = -5$ 1

Question Five

(i) $y = \dfrac{x^2}{4a}$

$\therefore y' = \dfrac{x}{2a}$

$\therefore y'(2at) = t$

$\therefore$ m of T at P $= t$

$\therefore$ m of N at P $= -\dfrac{1}{t}$ 1

Eqn of Normal:

$y - at^2 = -\dfrac{1}{t}(x - 2at)$

$\therefore ty - at^3 = -x + 2at$

ie. $x + ty = at^3 + 2at$ 1

ii) let $x = 0$ $\therefore ty = at^3 + 2at$

$\therefore y = at^2 + 2a$

$\therefore$ Coords of Q are $(0, at^2 + 2a)$ 1

when $y = at^2 + 2a$ $\quad x^2 = 4a(at^2 + 2a)$

$\therefore x = \pm 2a\sqrt{t^2 + 2}$ 1

From diagram, R has coords $\left(-2a\sqrt{t^2 + 2},\ at^2 + 2a\right)$

(ii) continued ...

we also know the x coord of R is negative as $\angle PQR > 90°$ and hence R must be on negative side of x axis.

(iii) Midpoint RQ $= \left(\dfrac{-2a\sqrt{t^2 + 2}}{2},\ \dfrac{2(at^2 + 2a)}{2}\right)$

$= \left(-a\sqrt{t^2 + 2},\ at^2 + 2a\right)$ 1

Now $x = -a\sqrt{t^2 + 2}$ and $y = at^2 + 2a$

$\therefore x^2 = a^2t^2 + 2a^2$

$\therefore t^2 = \dfrac{x^2 - 2a}{a}$

subst into $y = at^2 + 2a$

$\therefore y = a\left(\dfrac{x^2 - 2a}{a}\right) + 2a$

$\therefore y = x^2$ 1

$t^2 = \dfrac{x^2 - 2a^2}{a^2}$

$y = a\left(\dfrac{x^2 - 2a^2}{a^2}\right) + 2a$

$y = \dfrac{x^2 - 2a^2}{a} + 2a$

$ay = x^2 - 2a^2 + 2a^2$

$ay = x^2$

$x^2 = ay$