J.M.J.

MARCELLIN COLLEGE RANDWICK

YEAR 12 HSC

ASSESSMENT TASK # 2

EXTENSION ONE

MATHEMATICS

2007

Weighting: 20% of H.S.C. Assessment Mark.

STUDENT NAME: ____________________ MARK: /28

PERCENTAGE: %

RANK ON THIS TASK: /23

Time Allowed: 50 minutes.

Directions:

- Answer all questions on separate lined paper.
- Show all necessary working.
- Marks may not be awarded for careless or badly arranged work.

Outcomes examined:

PE3 – Solves problems involving permutations and combinations.

HE3 – Uses a variety of strategies to investigate Mathematical models of situations involving binomial probability.

Question One (4 marks)

Write the coefficient of:

a) x^5 in the expansion of $(5x-2)^{11}$ 2

b) x in the expansion of $\left(x+\frac{3}{x}\right)^4 (x-2)^5$ 2

Question Two (7 marks)

(a) Show $\dfrac{{}^7C_k}{{}^7C_{k-1}} = \dfrac{8-k}{k}$ 2

b) Determine the greatest coefficient in the expansion of $(3x-7)^7$ 5

Question Three (4 marks)

a) Write the expansion of $(1+x)^n$ 1

b) Show that $n\left[3^{n-1}-1\right] = \sum_{r=0}^{n-2} {}^nC_{r+2}\, 2^{r+1}(r+2)$ 3

Question Four (2 marks)

4 men and 4 women are to be seated randomly around a round table.
What is the probability that the men and women will alternate?

Question Five (3 marks)

From a group of 7 men and 5 women a team of six is to be formed.
Given each person is equally likely to be selected, what is the probability that the selected team contains at least 4 men?

Question Six (3 marks)

Consider the letters of the word TEMPERATURE.

a) In how many different ways can the letters be arranged? 1

b) What is the probability that the word formed begins with a P and that the two R's are next to each other? 2

Question Seven (5 marks)

PIN numbers are 4 digit numbers using any of the digits from 0 to 9. However, they must start with a non-zero digit and digits can be repeated.

a) What is the probability that a particular PIN number has at least one 9 among its digits? 3

b) 5 people are randomly given PIN numbers. What is the probability that exactly 3 of them have at least one 9 as one of the digits of their PINnumbers? (Answer correct to 2 decimal places) 2

END OF ASSESSMENT TASK

SOLUTIONS TO YR 12

EXTENSION I MATHS

TASK 2 2007

Question One

a) $T_{k+1} = {}^{11}C_k \, 5^{11-k} (-2)^k x^{11-k}$ ← 1 mark

Coeff. of x^5 when $k = 6$

$\therefore C_5 = {}^{11}C_6 \, 5^5 (-2)^5$

$= -46\ 200\ 000$ ← 1 mark

b) $(x + 3/x)^4 (x-2)^5$

$= \left(\frac{x^2+3}{x}\right)^4 (x-2)^5$

$= \frac{1}{x^4} (x^2+3)^4 (x-2)^5$ ← 1 mark for correct general terms

$T_{k+1} = {}^4C_k \, x^{8-2k} 3^k$

$T_{k+1} = {}^5C_k \, x^{5-k} (-2)^k$

when $k=4$ $T_5 = {}^4C_4 \, 3^4$

when $k=3$ $T_4 = {}^4C_3 \, 3^3 x^2$

when $k=2$ $T_3 = {}^4C_2 \, 3^2 x^4$

when $k=0$ $T_1 = {}^5C_0 \, x^5$

when $k=2$ $T_3 = {}^5C_2 (-2)^2 x^3$

when $k=4$ $T_5 = {}^5C_4 (-2)^4 x$

Now coeff. of x^5 in exp. of $(x^2+3)^4 (x-2)^5$ will be the same as coeff of x in exp. of $1/x^4 \cdot (x^2+3)^4 (x-2)^5$

ie. Coeff. of x $= 3^4 + {}^4C_3 \, 3^3 \, {}^5C_2 (-2)^2 + {}^4C_2 \, 3^2 \, {}^5C_4 (-2)^4$

$= 8721$ ← 1 mark

Question Two

a) $$\frac{^7C_k}{^7C_{k-1}} = \frac{7!}{k!(7-k)!} \times \frac{(k-1)!(8-k)!}{7!}$$

$$= \frac{7!}{k(k-1)!(7-k)!} \times \frac{(k-1)!(8-k)(7-k)!}{7!}$$

$$= \frac{8-k}{k}$$

1 mark for correctly deriving result

b) Consider the exp. of $(3x+7)^7$

Now $T_{k+1} = {^7C_k}\, 3^{7-k}\, 7^k\, x^{7-k}$

and $T_k = {^7C_{k-1}}\, 3^{8-k}\, 7^k\, x^{8-k}$

$$\therefore \frac{C_{k+1}}{C_k} = \frac{^7C_k\, 3^{7-k}\, 7^k}{^7C_{k-1}\, 3^{8-k}\, 7^k}$$

$$= \frac{8-k}{k} \cdot \frac{7}{3}$$ (from part (a) above)

$$= \frac{56-7k}{3k}$$

1 mark for correct to this result

Now Greatest coeff. when $C_{k+1}/C_k > 1$

ie $\frac{56-7k}{3k} > 1$

$\therefore k < 5\,3/5$ ← 1 mark

$\therefore$ Greatest coeff. when $k = 5$

continued next page

But in the exp. of $(3x-7)^7$, the coeff. when $k=5$ is negative ← 1 mark for correctly disregarding $k=5$ with reason.

$\therefore$ G.C. when $k=4$ or $k=6$

Test $k=4$

$^7C_4\, 3^3\, 7^4 = 2\ 268\ 945$ ← 1 mark for checking both $k=4$ and $k=6$

Test $k=6$

$^7C_6\, 3^1\, 7^6 = 2\ 470\ 629$

$\therefore$ G.C. is $2\ 470\ 629$ (ie. when $k=6$)

1 mark for correct greatest coeff.

Question Three

a) $(1+x)^n = {^nC_0} + {^nC_1}x + {^nC_2}x^2 + \dots + {^nC_n}x^n$

1 mark

b) Differentiate both sides:

ie. $n(1+x)^{n-1} = {^nC_1} + 2\,{^nC_2}x + \dots + n\,{^nC_n}x^{n-1}$

1 mark

let $x=2$

$\therefore n(3^{n-1}) = {^nC_1} + 2^2\,{^nC_2} + 2^3.3\,{^nC_3} + n\,2^{n-1}\,{^nC_n}$

1 mark

$\therefore n(3^{n-1}) = n + {^nC_2}\,2^2.3 + {^nC_3}\,2^3.4 + \dots + {^nC_n}\,n\,2^{n-1}$

$$\therefore n(3^n-1) - n = \sum_{r=0}^{n-2} {^nC_{r+2}}\, 2^{r+1}(r+2)$$

$$\therefore n[3^{n-1}-1] = \sum_{r=0}^{n-2} {^nC_{r+2}}\, 2^{r+1}(r+2)$$

1 mark for correctly deriving

Question Four

$3!\ 4!$ ← No. of ways of placing women among the already seated men

no. of ways of seating 4 men around a circular table

1 mark

Total ways without restrictions $= 6!$

$\therefore$ Prob(M + W alternate) $= \frac{3!\,4!}{6!}$

$= \frac{1}{5}$ ← 1 mark

Question Five

$P(4 \text{ men } 2 \text{ women}) = \frac{{}^7C_4\ {}^5C_2}{{}^{12}C_6} = \frac{350}{924}$

$P(5 \text{ men } 1 \text{ woman}) = \frac{{}^7C_5\ {}^5C_1}{{}^{12}C_6} = \frac{105}{924}$

$P(\text{all } 6 \text{ men}) = {}^7C_6 / {}^{12}C_6 = \frac{7}{924}$

2 marks (1 mark deducted for each error)

$\therefore$ Total prob(at least 4 men) $= 462/924$

$= \frac{1}{2}$ ← 1 mark

Question Six

a) $\frac{11!}{2!\,3!\,2!} = \frac{39916800}{24}$

$= \frac{9979200}{6}$

$= 1663200$ ← 1 mark

Question Six continued

b) No. of ways beg. with P with 2 R's together $= \frac{9!}{2!\,3!}$ ← 1 mark

$= 30240$

$\therefore P(\quad) = \frac{30240}{1663200}$

$= \frac{1}{55}$ ← 1 mark

Question Seven

a) Total no. of possible PIN no.s $= 9 \times 10 \times 10 \times 10$

$= 9\,000$ ← 1 mark

Prob.(at least 1 9) $= 1 - P(\text{no } 9\text{'s})$ ← 1 mark

$= 1 - \left(\frac{8}{9} \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10}\right)$

$= 1 - \frac{5832}{9000}$

$= \frac{44}{125}$ ← 1 mark

b) $P(\text{exactly } 3) = {}^5C_3 \left(\frac{44}{125}\right)^3 \left(\frac{81}{125}\right)^2$ ← 1 mark

$= 0.183137804$

$= 0.18$ (2dp) ← 1 mark for correct prob. (2dp) only if binomial prob. used.