J.M.J.

MARCELLIN COLLEGE RANDWICK

YEAR 11

ACCELERATED MATHEMATICS

HSC TASK 1

2005

Weighting: 15% of HSC Assessment Mark.

STUDENT NAME: ______ **MARK:**

PERCENTAGE:

RANK ON THIS TASK:

Time Allowed: **1 Hour**

Directions:

- Answer both questions on separate lined paper.
- Show all necessary working. Where more than one mark is allocated to a question, full marks will not be awarded for answers only.
- Marks may not be awarded for careless or badly arranged work.

Outcomes examined:

- **P6** – Relates the derivative of a function to the slope of its graph
- **P7** – Determines the derivative of a function through routine application of the rules of differentiation
- **H1** – Seeks to apply mathematical techniques to problems in a wide range of practical contexts
- **H5** – Applies appropriate techniques from the study of calculus to solve problems
- **H6** – Uses the derivative to determine the features of the graph of a function
- **H8** – Uses techniques of integration to calculate areas and volumes
- **H9** – Communicates using mathematical language, notation and graphs

Question 1 (15 marks) **Marks**

a) Differentiate with respect to x

i) $(3-2x)^5$ **1**

ii) $\dfrac{2x-5}{x^2-3}$ **2**

b) Evaluate $\int_{-1}^{2} \dfrac{dx}{2x+5}$ **2**

c) The gradient function of a curve is given by $\dfrac{dy}{dx} = 6x^2 - 12x - 18$.
The curve passes through the point (-2,-3).

i) Find the equation of the curve. **2**

ii) Find the coordinates of the stationary points and determine their nature. **2**

iii) Find the coordinates of the point of inflexion **2**

iv) Sketch the curve showing its main features in the domain $-2 \le x \le 5$. **2**

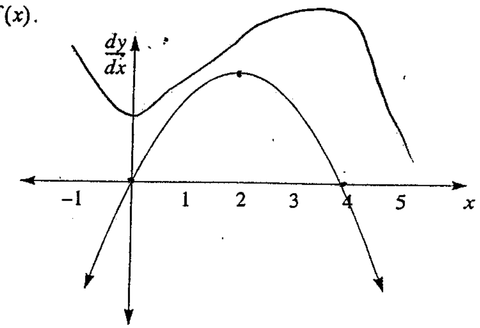
d) Find the domain for which the curve $y = x^3 - 8x^2 + 6x - 3$ is concave upwards. **2**

Question 2 (15 marks) **Marks**

a) Find the equation of the tangent to the curve $y = x^2 + 2$ at the point (-1,3) **2**

b) The curve $ax^3 + bx$ passes through the point (1,7). The tangent at this point is parallel to the line $y = 2x - 6$.
Find the values of a and b. **3**

c) The diagram shows the graph of $\frac{dy}{dx}$ where $\frac{dy}{dx}$ is the gradient function of the curve $y = f(x)$.



i) Find the values of x for which $f(x)$ is a decreasing function. **1**

ii) For what value of x does $y = f(x)$ have a maximum turning point. Justify your answer. **2**

d) The curve $y = 6x - x^2$ and the line $y = 2x$ intersect.

i) Draw a neat sketch showing points of intersection. **2**

ii) Find the area bounded by the line and the curve. **2**

e) Find the volume of the solid of revolution formed when $y = \sqrt{x}$ is rotated about the y-axis between $y = 1$ and $y = 3$ **3**

MATI +IO .

$\frac{30}{30}$

M+IO

~~13~~

15

Q1.

a)i) $(3-2x)^5$

$5\times -2(3-2x)^4$

$-10(3-2x)^4$ ✓ 1

ii) $\frac{2x-5}{x^2-3}$ $u = 2x-5$ $v = x^2-3$

$u' = 2$ $v' = 2x$

$y' = \frac{vu' - uv'}{v^2}$

$= \frac{2(x^2-3) - 2x(2x-5)}{(x^2-3)^2}$

$= \frac{2x^2-6-4x^2+10x}{(x^2-3)^2}$

$= \frac{-2x^2+10x-6}{(x^2-3)^2}$ ✓ 2

$= \frac{-2x^2+10x-6}{x^4-6x^2+9}$

careful.

b) $\int_{-1}^{2} \frac{1}{2x+5}\,dx$

$\int_{-1}^{2} (2x+5)^{-1}\,dx$

$= \left[\frac{(2x+5)^0}{0}\right]_{-1}^{2}$ 2

= no answer

c) i) $\frac{dy}{dx} = 6x^2 - 12x - 18$

$y = \frac{6x^3}{3} - \frac{12x^2}{2} - \frac{18x}{1} + c$

$y = 2x^3 - 6x^2 - 18x + c.$ subs. $(-2,-3)$ (1)

$-3 = 2(-2)^3 - 6(-2)^2 - 18(-2) + c.$

$-3 = -4 + c.$ 2

$c = 1.$ subs. $c \rightarrow$ (1) ✓

$y = 2x^3 - 6x^2 - 18x + 1$

ii) $\frac{dy}{dx} = 6x^2 - 12x - 18$

$\frac{dy}{dx} = 0$

$0 = x^2 - 2x - 3$

$0 = (x+1)(x-3)$ ✓

$x = -1$ or $x = 3.$

$\frac{d^2y}{dx^2} = 12x - 12$?

$\frac{d^2y}{dx^2} = 12x - 12$

Sub. $x=-1$ $\frac{d^2y}{dx^2} = 12(-1) - 12$

$= -24$ $\frac{d^2y}{dx^2} < 0$ ∴ max pt ✓

sub $x=3$ $\frac{d^2y}{dx^2} = 12x - 12$

$= 24$ $\frac{d^2y}{dx^2} > 0$ ∴ min pt. ✓

M.+10

M-+6

$y = 2x^3 - 6x^2 - 18x + 1$ subs $x = -1$

$y = 2(-1)^3 - 6(-1)^2 - 18(-1) + 1$

$y = 11$. $(-1, 11)$ is max turning pt ✓ 2

$y = 2(3)^3 - 6(3)^2 - 18(3) + 1$

$y = -53$ $(3, -53)$ is min turning pt

(iii) $\frac{d^2y}{dx^2} = 12x - 12$

$\frac{d^2y}{dx^2} = 0$

$0 = 12x - 12$. 2

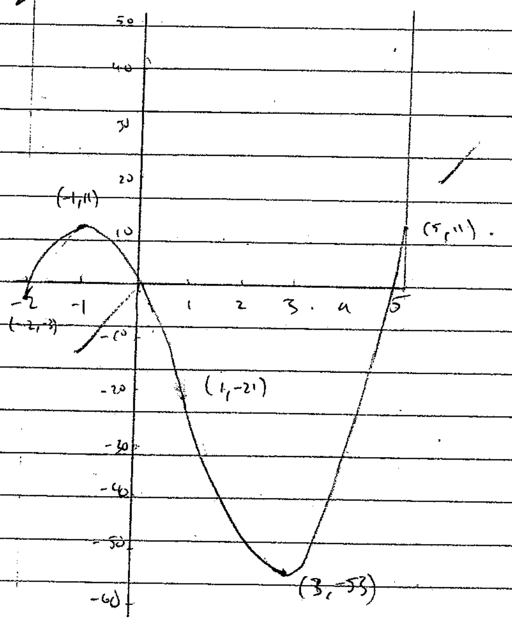
$x = 1$ · is pt of inflexion

$y = -21$. (sub x into normal formula)

$(1, -21)$ is pt of inflexion.

$(-1, 11)$ max.
$(3, -53)$ min.
$(1, -21)$ inflexion

iv)



2

d) N? found it.

Q1

d) $y = x^3 - 8x^2 + 6x - 3$

$y' = 3x^2 - 16x + 6$

$y'' = 6x - 16$

$y'' > 0$.

$6x - 16 > 0$.

$6x > 16$ ✓ 2

$x > 2\frac{2}{3}$.

Q2. 18

a) $y = x^2 + 2$

$y' = 2x$ sub. $x = -1$

$y' = -2$

$y - 3 = -2(x + 1)$. 2

$y - 3 = -2x - 2$

$2x + y - 1 = 0$. ✓

b) $y = ax^3 + bx$

$y' = 3ax^2 + b$

$y' = 2$

$2 = 3a(1)^2 + b$ ✓

$y = 2x - 6$, $m = 2$.

$y = ax^3 + bx$

$7 = a(1)^3 + b(1)$ (subs $(1, 7)$)

$7 = a + b$.

$b = 7 - a$. (2)

Subs (2)

$2 = 3a + 7 - a$

$-5 = 2a$

$a = -2\frac{1}{2}$. 3

sub $a \Rightarrow$ (2)

$b = 7 - (-2\frac{1}{2})$. ✓

$b = 7 + 2\frac{1}{2}$.

$b = 9\frac{1}{2}$.

M. the

c) i) $x < 0,\ x > 4$ ✓

ii) when x crosses the axis from pos. grad to neg grad 2

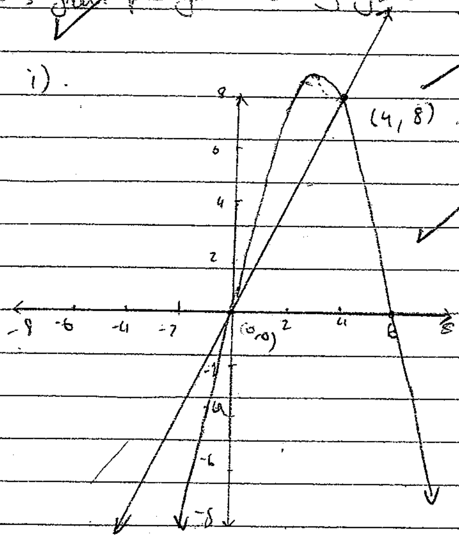
$x = 4$.

d) $y = 6x - x^2$. $\quad 2x = 6x - x^2$ $\quad$ i).

$0 = 6x - x^2$. $\quad 0 = 4x - x^2$

$x(6 - x)$ $\quad x = 4$ or 0

$x = 0$ or $x = 6$



2.

ii) $\int_0^4 6x - x^2\,dx - \int_0^4 2x\,dx$

$\int_0^4 6x - x^2 - 2x\,dx$

$\int_0^4 4x - x^2\,dx$

$= \left[\frac{4x^2}{2} - \frac{x^3}{3}\right]_0^4$

$= \left[2x^2 - \frac{x^3}{3}\right]_0^4$

$= \left(2(4)^2 - \frac{(4)^3}{3}\right) - (0)$ ✓ 2

$= 10\frac{2}{3}\ u^2$

e) $y = \sqrt{x}$

$y^2 = x$.

$V = \pi\int_1^3 (y^2)^2$ ✓ 3

$V = \pi\left[\frac{y^5}{5}\right]_1^3$

$= \pi\left(48\frac{3}{5} - \frac{1}{5}\right)$

$= \pi . 48\frac{2}{5}$