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MARCELLIN COLLEGE RANDWICK

YEAR 11 ACCELERATED

HSC ASSESSMENT TASK # 1

MATHEMATICS

2006

Weighting: 40% of HSC Assessment Mark.

STUDENT NAME:	**MARK:**	**/ 60**

Time Allowed: **90 minutes.**

Directions:
- Answer all questions on separate lined paper.
- Show all necessary working.
- Marks may not be awarded for careless or badly arranged work.
- Begin your answers to each new question on a new answer page.

N.B. All diagrams are not to scale

Structure: 3 questions each worth 20 marks – Total 60 marks.

OUTCOMES TO BE ASSESSED:

- **H1** – Seeks to apply mathematical techniques to problems in a wide range of practical contexts
- **H5** – applies appropriate techniques from the study of calculus and series to solve problems
- **H6** – uses the derivative to determine the features of the graph of a function
- **H7** – uses the features of a graph to deduce information about the derivative
- **H8** – uses techniques of integration to calculate areas and volumes

Question 1

a. Differentiate the following with respect to x:

i. $y = x^2(1+\frac{1}{x})$ **2**

ii. $f(x) = \dfrac{x}{\sqrt{x}}$ **2**

iii. $\dfrac{d}{dx}\left(\dfrac{9-x^2}{9+x^2}\right)$ **2**

b. The tangent at $P(1,\frac{1}{2})$ on the curve $y = \dfrac{x}{x+1}$ meets the x-axis at T. Find the coordinates of T. **3**

c. The gradient of a curve is given by $\dfrac{dy}{dx} = 3x^2 - 6x - 9$

i. The curve passes through the point (1,-2), find the equation of the curve. **1**

ii. Find the coordinates of the stationary points and determine their nature. **2**

iii. Find the coordinates of the point of inflexion. **1**

iv. Sketch the curve, showing all relevant information, for $-2 \le x \le 4$. **2**

d. Two men set out from towns on roads which meet at right angles and walk towards the intersection. A is 25 km from the intersection and walks 4 km/h. B is 20 km from the intersection and walks at 3 km/h.

i. Draw a diagram showing all necessary information.

ii. Show that the distance apart, Z km, after t hours walking is given by $Z = \sqrt{25t^2 - 320t + 1025}$. **2**

iii. Hence find their minimum distance apart, and the time taken to reach this point. **3**

Question 2

a. The 3rd term of an arithmetic series is 7 and the 9th term is 19.

i. Find the first term and the common difference. **3**

ii. Find the sum of the first 10 terms. **2**

b. Find the value of $\sum_{r=4}^{17} 3(2^r)$.

c. The limiting sum of a geometric series is 30. Give a possible example of a geometric series that fits these conditions. **2**

d. The cost of building the first floor of a large building is \$155 000. Each additional storey costs \$32 000 more than the previous one. If the building is to be 24 storeys high, find:

i. the cost of building the top floor. **2**

ii. the total cost of the building. **2**

e. A woman borrows \$20 000 at 18% p.a. reducible interest, and pays it off in equal monthly instalments.

i. Express the interest rate as a monthly rate in decimal form. **1**

ii. Show that the amount she owes at the end of the second month is

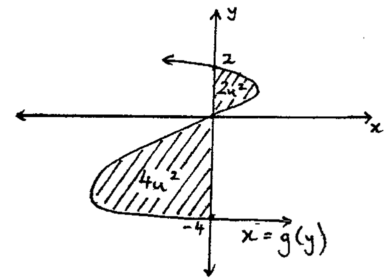
$A_2 = 20000(1.015)^2 - 1.015M - M$ where M is the monthly repayment. **1**

iii. Write an expression of A_n, the amount owed after n months. **1**

iv. Find the amount of the monthly repayment if she repays the loan in 5 years. **3**

Question 3

a. The area between each loop of the curve $x = g(y)$ and the y-axis is shown on the diagram below.



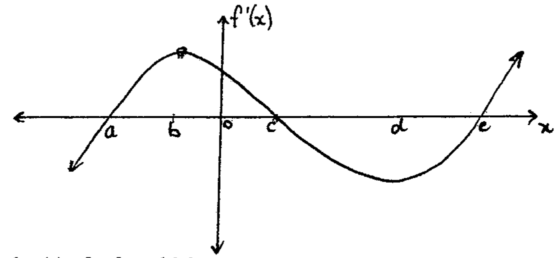
Use this information to find the value of:

i. $\int_0^2 g(y)dy$ **1**

ii. $\int_{-4}^0 g(y)dy$ **1**

iii. $\int_{-4}^2 g(y)dy$ **1**

b. The diagram below shows the graph of the derivative of a certain function $f(x)$.



Find the value(s) of x for which :

i. the function $f(x)$ has stationary points. **2**

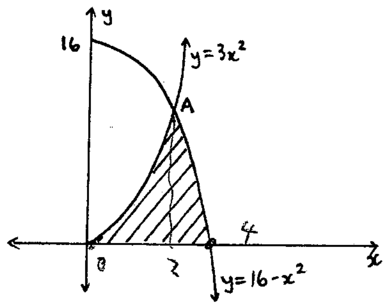
ii. the function $f(x)$ is decreasing. **2**

iii. the function $f(x)$ is concave down. **2**

c. Find k such that $\int_0^k (5-2x)dx = 4$. **3**

d. The curve $y = \frac{1}{x}$ is rotated about the x-axis between $x = 1$ and $x = 2$. Use Simpson's rule with three function values to estimate the volume of the solid formed. **3**

e. The graphs of $y = 16 - x^2$ and $y = 3x^2$ are drawn below.



i. Find the coordinates of point A. **2**

ii. Find the area of the region contained by the two curves and the x-axis. **3**

Question 1 (20) Peter Simpson

A I $x^2\left[1+\frac{1}{x}\right]$ $\frac{2x}{x}$

$u = x^2$

$\frac{du}{dx} = 2x$

$v = 1+\frac{1}{x}$ x^{-1}

$\frac{dv}{dx} = -\frac{1}{x^2}$ $-x^{-2}$ $-\frac{1}{x^2}$

$\left(1+\frac{1}{x}\right)2x + x^2\left(-\frac{1}{x^2}\right)$

$= 2x+2-1$

$= 2x+1$

II $u = x$

$\frac{du}{dx} = 1$

$v = x^{1/2}$

$\frac{dv}{dx} = \frac{1}{2}x^{-1/2}$

$= \frac{1}{2\sqrt{x}}$

$\frac{x^{1/2}\times 1 - x\left(\frac{x^{-1/2}}{2}\right)}{x}$

$= x^{1/2} - \frac{x^{1/2}}{2}$

$= x^{1/2}\left[1-\frac{1}{2}\right]$

$= \sqrt{x}\left[\frac{1}{2}\right]$

$x \div x^{1/2}$

$x^{1-1/2}$

$= \frac{1}{2}(x)^{-1/2}$ 2

$= \frac{1}{2\sqrt{x}}$

III

Question 1 Pete Simpson

III $u = 9 - x^2$

$\frac{du}{dx} = -2x$

$v = 9 + x^2$

$\frac{dv}{dx} = 2x$

$$\frac{(9+x^2)(-2x) - (9-x^2)(2x)}{(9+x^2)^2}$$

$= -18x - 2x^3 - [18x - 2x^3]$

$$= \frac{-18x - 2x^3 - 18x + 2x^3}{(9+x^2)^2}$$

2

$$= \frac{-36x}{(9+x^2)^2}$$

B $P(1, \frac{1}{2})$ $\frac{x}{x+1}$

$u = x$

$\frac{du}{dx} = 1$

$v = x + 1$

$\frac{dv}{dx} = 1$

$$\frac{x+1-x}{(x+1)^2}$$

$$f'(x) = \frac{1}{(x+1)^2}$$

sub $(1, \frac{1}{2})$ into $f'(x)$

$$\frac{1}{(1+1)^2}$$

$= \frac{1}{4}$

$m = \frac{1}{4}$ $P(1, \frac{1}{2})$

$y - \frac{1}{2} = \frac{1}{4}(x-1)$ 3

$4y - 2 = x - 1$

~~$x = 4y$~~ $4y = x + 1$

~~...~~ let ~~...~~ $y = 0$

Pete Simpson

Question 1

C I $f(x) = x^3 - 3x^2 - 9x + c$

$-2 = (1)^3 - 3(1)^2 - 9(1) + c$

$1 - 3 - 9 + c = -2$

$c = 9$ 1

$\therefore f(x) = x^3 - 3x^2 + 9x + 9$

II $3x^2 - 6x - 9 = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$ $x = -1$

$y = -18$ $y = 14$

$f''(x) = 6x - 6$

$f''(3) = 18 - 6$

$= 12$

$\therefore f''(3) > 0$

$\therefore$ minimum at $x = 3$

$f''(-1) = -6 - 6$ 2

$f''(-1) = -12$

$f''(-1) < 0$

$\therefore$ maximum at $x = -1$

$-1 - 3 + 9 + 9$

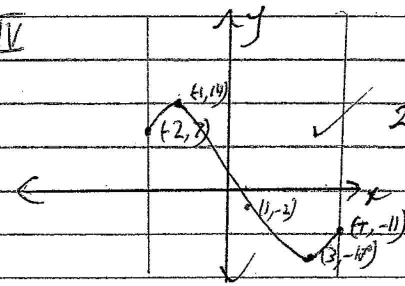
III $f''(x) = 6x - 6$

$6x = 6$

$x = 1$ 1

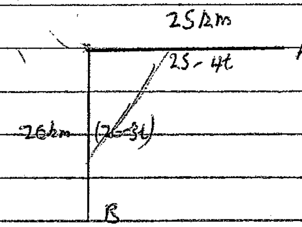
$y = -2$

IV



2

D



II $Z = \sqrt{(25-4t)^2 + (20-3t)^2}$

$= \sqrt{625 - 200t + 16t^2 + 400 - 120t + 9t^2}$

$= \sqrt{1025 - 320t + 25t^2}$ 2

$= \sqrt{25t^2 - 320t + 1025}$

III $(25t^2 - 320t + 1025)^{-1/2}(50t - 320)$

$$= \frac{50t - 320}{25t^2 - 320t + 1025} = 0$$

$50t - 320 = 0$ 3

$50t = 320$

$t = 6.4$ hrs or 384 minutes

$\therefore Z = 1$ km

Pete Simpson

Question 2 (18)

A I $T_3 = 7$ $T_9 = 19$

$T_n = a + (n-1)d$

$T_3 = a + 2d = 7$ $T_9 = a + 8d = 19$

$a = 7 - 2d$ $a = 19 - 8d$

$7 - 2d = 19 - 8d$

$6d = 12$ 3

$d = 2$

$\therefore a = 3$

II $S_n = \frac{n}{2}[2a + (n-1)d]$

$= 5[6 + 9(2)]$ 2

$= 120$

B $A = 48$ 48 + 96 + 192

$n = 14$

$r = 2$

$S_n = \frac{a(r^n - 1)}{r - 1}$ 3

$= \frac{48(2^{14} - 1)}{1}$

$= 786\,384$

C $\frac{a}{1-r} = 30$

$a = 30 - 30r$

$a - 30r + 30 = 0$

D I 155 000 + 187 000 + 219 000

$T_n = a + (n-1)d$

$= 155\,000 + (23)32\,000$

$= 891\,000$

II $S_n = 12[310\,000 + 23(32\,000)]$

$= 12{,}552\,000$

Pete Simpson

Question 2

E $20\,000$ I 0.015 1

II $A_1 = 20\,000(1.015) - m$

$A_2 = [20000(1.015) - m]1.015 - m$ 1 $a = 1$ $r = 1.015$ $n = 60$

$A_2 = 20000(1.015)^2 - (1.015)m - m$

II $A_n = 20000(1.015)^n - m[1 + 1.015 + (1.015)^2 + \dots + (1.015)^{n-1}]$ 1

II $A_{60} = 20000[1.015]^{60} - m\left[\frac{1((1.015)^{60} - 1)}{0.015}\right]$

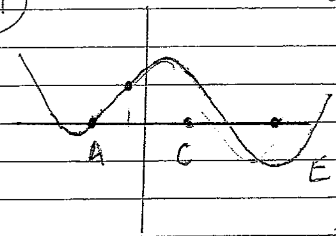
$m\left[\frac{1((1.015)^{60} - 1)}{0.015}\right] = 20000[1.015]^{60}$ 3

$m = \cancel{50.79}$ 507.87

Question 3 Peter Simpson

(19)

A I 2 ✓
II −4 ✓ 3
III −2 ✓



B I ~~B/D~~ A, C, E ✓ 2
II $C < f x < E$ and f(x) a ✓ 2
III $B < f(x) < D$ ✓ 2

C $\int_0^h (5-2x)$

$\left[5x - x^2\right]_0^h = 4$ ✓

$5h - h^2 = 4$ 3

$h^2 - 5h + 4 = 0$

$(h+4)(h-1) = 0$ ✓

$h = 4 \quad h = 1$

$\left(\frac{1}{x}\right)^2$

	x	$f(x)^2$	
x_1	1	1	
x_2	1.5	~~9/4~~	4/9
x_3	2	~~4~~	1/4

~~$\pi \left[\frac{1}{3}\left[1 + 4 + 4\left(\frac{9}{4}\right) + 2(0)\right]\right]$~~ ✓

~~$= \pi \left[\frac{\pi}{3}\left[5 + 9\right]\right]$~~ 1

~~$= \frac{\pi}{3}\left[14\right.$~~

~~$= \frac{14\pi}{3}$~~

$\frac{\pi}{3}\left[1 + \frac{1}{4} + 4\left(\frac{4}{9}\right) + 2(0)\right]$

$= \frac{\pi}{3}\left[\frac{5}{4} + \frac{16}{9}\right]$

$= \frac{109\pi}{108}$

$\frac{109\pi}{216}$

Question 3 Peter Simpson

E I $y = 3x^2$

$y = 16 - x^2$

$3x^2 = 16 - x^2$

$4x^2 = 16$ ✓ 2

$x^2 = 4$

$x = \pm 2$

Since a is in 1st quadrant

$x = 2$ ✓

$\therefore y = 12$

$y = 16 - x^2$

$x^2 = 16$

$x = \pm 4$

II $\int_0^2 3x^2 + \int_2^4 16 - x^2$

$= \left[x^3\right]_0^2 + \left[16x - \frac{x^3}{3}\right]_2^4$ ✓

$= 8 + \left[\frac{128}{3}\right] - \left[32 - \frac{2^3}{3}\right]$ 3

$8 + \frac{128}{3} - \frac{88}{3}$ ✓

$= 8 + \frac{40}{3}$ ✓

$= \frac{64}{3} u^2$