J.M.J.Ch

MARCELLIN COLLEGE RANDWICK

YEAR 11

ACCELERATED MATHEMATICS

HSC Assessment Task #2, 2008

STUDENT NAME: ______________ **MARK:** /35

Weighting: 20 % towards HSC mark

Time Allowed: **50 minutes.**

Directions:

- Answer all questions.
- Begin each question on a new page.
- Show working where necessary.
- Marks may not be awarded for answers only.

OUTCOMES TO BE ASSESSED:

- H1 – Seeks to apply mathematical techniques to problems in a wide range of contexts
- H3 – manipulates algebraic expressions involving logarithmic and exponential functions
- H4 – expresses practical problems in mathematical terms based on simple given models
- H6 – uses the derivative to determine the features of the graph of a function
- H8 – uses techniques of integration to calculate areas and volumes

Question 1

a. How many terms of the series $9 + 18 + 36 + \ldots$ are needed to give a sum of 1143? 2

b. Evaluate $\sum_{n=1}^{10} 5 \times 2^{n-1}$. 2

c. Consider the series $-2 + 4(\pi - 3) - 8(\pi - 3)^2 + 16(\pi - 3)^3 - \ldots$

i. Explain why the geometric series has a limiting sum. 1

ii. Find the exact value of the limiting sum. 2

d. A panel beater borrows \$90 000 to purchase new machinery.

The interest is calculated monthly at the rate of 2% per month, and is compounded each month. The panel beater intends to repay the loan with interest in two equal annual instalments of $\$M$ at the end of the first and second years.

i. How much does the panel beater owe at the end of the first month? 1

ii. Write an expression involving M for the total amount owed by the panel beater after 12 months (i.e. just after the first instalment of $\$M$ has been paid.) 1

iii. Find an expression for the amount owed at the end of the second year and hence deduce that $M = \dfrac{90000(1\cdot02)^{24}}{(1\cdot02)^{12}+1}$. 2

Question 2

a. Find the value of $\log_e 3.5 - \frac{\pi}{\sqrt{e^3}}$ correct to 3 significant figures.

1

b. Differentiate $\frac{e^x}{x^2}$ leaving your answer as a fraction in simplest form.

2

c. Find the equation of the tangent to the curve $y = x\log_e x$ at the point on the curve where $x = 1$.

3

d. Consider the curve $y = (2x+1)e^{2x}$.

i. Find any stationary points and determine their nature. 2

ii. Find the coordinates of any points of inflexion. 2

iii. Sketch the curve showing all relevant information. 2

Question 3

a. Solve for x: $2\log_e(x+3) = \log_e(x+1) + \log_e(x+7)$. 2

b. Find the value of k (where $k > 0$) such that $\int_1^k \frac{t}{4t^2-1}\,dt = \frac{1}{8}\log_e 5$. 3

c. i. Copy and complete the table below for $y = \sqrt{2+e^x}$ calculating each value correct to 3 decimal places. 2

x	0	0.5	1	1.5	2
y					

ii. Use Simpson's rule with 5 function values to approximate $\int_0^2 \sqrt{2+e^x}\,dx$. 2

Answer correct to 2 decimal places.

d. Find the area bounded by the curve $y = e^{2x}$, the y-axis and the line $y = 3$ leaving your answer in exact form. 3

Question 1

a) $9 + 18 + 36 + \dots$

$a = 9 \quad r = 2$

$S_n = \dfrac{a(r^n - 1)}{r-1}$

$1143 = \dfrac{9(2^n - 1)}{1}$

$127 = 2^n - 1$

$128 = 2^n$

$n = 7$ (2)

b) $\sum_{n=1}^{10} 5 \times 2^{n-1}$

$5 + 10 + 20 + \dots$

$a = 5 \quad r = 2$

$S = \dfrac{a(r^n - 1)}{r - 1}$

$S_{10} = \dfrac{5(2^{10} - 1)}{1}$

$S_{10} = 5115$ (2)

c) i) $-\dfrac{8(\pi - 3)^2}{4(\pi - 3)}$

$r = -2(\pi - 3)$

$r \doteq -0.28$ (1)

since $-1 < r < 1$

limiting sum exists

ii) [redacted] $\dfrac{-2}{1 - [-2(\pi - 3)]}$

$= \dfrac{-2}{1 + 2\pi - 6}$

$= \dfrac{-2}{2\pi - 5}$ (2)

or $= -1.56$

d) 2% per month

two equal annual instalments

i) $A_1 = 90000(1.02)$ (1)

or $91800

ii) $A_{12} = 90000(1.02)^{12} - M$ (1)

iii) $A_{24} =$ [redacted]

$= 90000(1.02)^{24} - (1.02)^{12}M - M$

After 24 months, $0 is owed

$\therefore \; 0 = 90000(1.02)^{24} - M[1 + 1.02^{12}]$ (2)

$M = \dfrac{90000(1.02)^{24}}{(1.02)^{12} + 1}$

Question 2

a) $\log_e 3.5 - \dfrac{\pi}{\sqrt{e^3}}$

$= 0.552$ (1)

b) $\dfrac{e^x}{x^2}$ $\quad u \; v$

$\dfrac{d}{dx} = \dfrac{x^2 . e^x - e^x . 2x}{(x^2)^2}$ (1)

$= \dfrac{xe^x(x - 2)}{x^4}$

$= \dfrac{e^x(x-2)}{x^3}$ (1)

c) $y = x\log_e x$ where $x = 1$, $y = 0$

$y' = \log_e x + x.\frac{1}{x}$

$y' = \log_e x + 1$ (1)

when $x = 1$ $m = 1$ (1)

$\therefore \; y - 0 = 1(x - 1)$

$y = x - 1$ (1)

$0 = x - y - 1$

d) i) $y = (2x+1)e^{2x}$

$y' = e^{2x}(2) + (2x+1)2e^{2x}$

$= 2e^{2x} + 2e^{2x}(2x+1)$

$= 2e^{2x}[1 + 2x + 1]$

$= 2e^{2x}[2x + 2]$ (1)

let $y' = 0$ to find stat pts

$2e^{2x} \neq 0 \quad 2x + 2 = 0$

$x = -1$

$y'' = (2x+2)4e^{2x} + 2e^{2x}(2)$

$= 4e^{2x}[2x + 2 + 1]$

$y'' = 4e^{2x}[2x + 3]$

when $x = -1$ $y'' = 4e^{-2x}$

$\therefore \; y'' > 0$ (1)

$\therefore$ at $(-1, -e^{-2})$ a min turn pt exists

2d ii) $y'' = 4e^{2x}[2x + 3]$

let $y'' = 0$ to find stat pts

$0 = 4e^{2x}[2x + 3]$

$4e^{2x} \neq 0 \quad 2x + 3 = 0$ (1)

$x = -\frac{3}{2}$

x	$-\frac{3}{2}^-$	$-\frac{3}{2}$	$-\frac{3}{2}^+$
y''	$-$	0	$+$

$\therefore$ a pt of inflexion exists at $(-\frac{3}{2}, -2e^{-3})$ (1)

iii)

(2)

Question 3

a) $2\log_e(x+3) = \log_e(x+1) + \log_e(x+7)$

$(x+3)^2 = (x+1)(x+7)$ (1)

$x^2 + 6x + 9 = x^2 + 8x + 7$

$2 = 2x$

$x = 1$ (1)

b) $\int_1^k \dfrac{t}{4t^2 - 1}\, dt = \frac{1}{8}\log_e 5$

$\frac{1}{8}\int_1^k \dfrac{8t}{4t^2 - 1}\, dt$ (1)

$= \frac{1}{8}\left[\log_e(4t^2 - 1)\right]_1^k$

$= \frac{1}{8}\left[\log_e(4k^2 - 1) - \log_e 3\right]$

$= \frac{1}{8}\log_e \dfrac{4k^2 - 1}{3}$ (1)

$\therefore \; \dfrac{4k^2 - 1}{3} = 5$

$4k^2 - 1 = 15$

$4k^2 = 16$

$k^2 = 4$

$k = \pm 2$

Since $k > 0$

then $k = 2$. (1)

c) i)

x	0	0.5	1	1.5	2
y	1.732	1.910	2.172	2.546	3.064

(2)

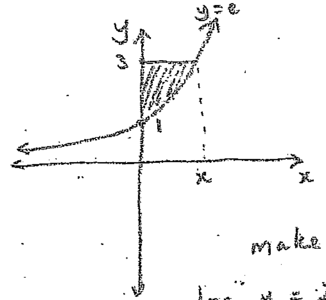
ii) $A \doteq \dfrac{1/2}{3}\{1.732 + 3.064 + 4(1.91 + 2.546) + 2(2.172)\}$ (1)

$\doteq \frac{1}{6}\{4.796 + 17.824 + 4.344\}$

$\doteq \frac{1}{6} \times 26.964$

$\doteq 4.49$ (1)

d) $y = e^{2x}$ $\quad$ y-axis $\quad y = 3$



make x the subject

$\log_e y = 2x$

$\therefore \; x = \frac{1}{2}\log_e y$

can't integrate

x-value $\quad e^{2x} = 3$

$\ln e^{2x} = \ln 3$

$2x \ln e = \ln 3$

$x = \frac{1}{2}\ln 3$

$\therefore$ Area of rect.

$= \frac{1}{2}\ln 3 \times 3$

$= \frac{3}{2}\ln 3 \; u^2$ (1)

Area under curve

$= \int_0^{\frac{1}{2}\ln 3} e^{2x}\, dx$

$= \left[\frac{1}{2}e^{2x}\right]_0^{\frac{1}{2}\ln 3}$

$= \frac{1}{2}e^{\ln 3} - \frac{1}{2}e^0$

$= \frac{3}{2} - \frac{1}{2}$ (1)

$= 1 \text{ unit}^2$

$\therefore$ shaded area $=$ area of rect $-$ area under curve

$= \frac{3}{2}\ln 3 - 1 \; u^2$ (1)